

Electrostatics and Current Electricity

CHAPTER 4

Capacitor and Capacitance

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4

Capacitor and Capacitance

INTRODUCTION

Capacitor (or condenser) is a device that stores (or condenses) electrostatic field energy. Capacitors provide temporary storage of energy in circuits and can be made to release the energy when required. The property of a capacitor that characterizes its ability to store energy is called its capacitance. When energy is stored in a capacitor, an electric field exists within the capacitor. The stored energy can be associated with the electric field. Indeed, energy can be associated with the existence of any electric field. The study of capacitors and capacitance leads us to an important aspect of an electric field, that is, its energy.

CAPACITANCE

ISOLATED CONDUCTOR

An electrical conductor (such as metals) holds its charge at its surface. When an isolated spherical conductor of radius R is given with a charge Q , its potential is given by

$$V = \frac{KQ}{R}$$

This tells us that the potential of an isolated spherical conductor is directly proportional to its charge. This holds good for an isolated conductor of any shape and size.

Since $V \propto Q$, the ratio of the charge given to an isolated conductor and its potential rise, that is, Q/V is a constant, which is defined as the capacitance of the conductor denoted by C . So

$$C = Q/V$$

If $V = 1$ V, $C = Q$. Hence, we define the capacitance of an isolated conductor as the charge required to rise the potential of the conductor by 1 V. In the SI system, the unit of capacitance is farad.

$$1(\text{F}) = \frac{1 \text{ coulomb (C)}}{1 \text{ volt (V)}} = 1 \text{ CV}^{-1}$$

In practice, the following smaller units of capacitance are also used:

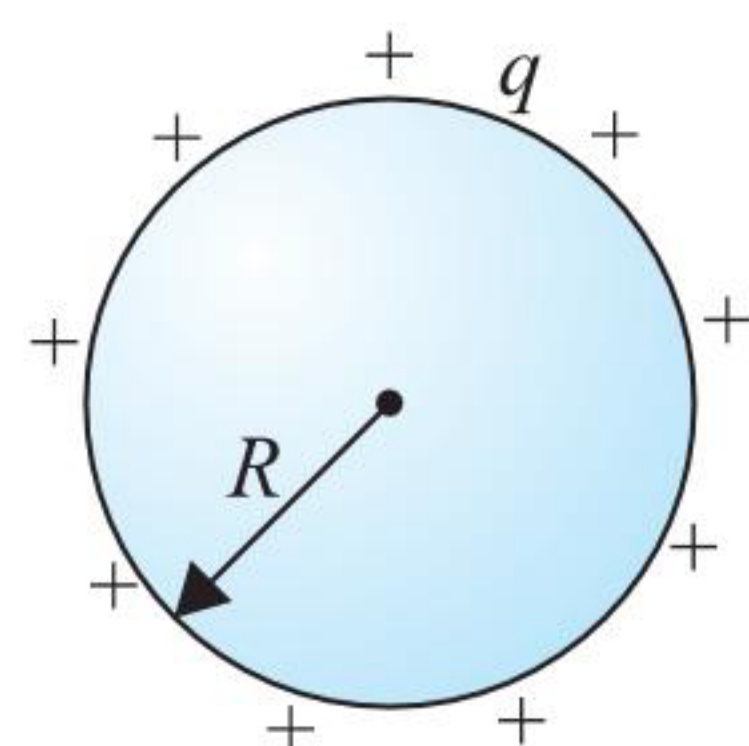
$$1 \text{ microfarad } (\mu\text{F}) = 10^{-6} \text{ farad}$$

$$1 \text{ micro microfarad } (\mu\mu\text{F}) = 10^{-12} \text{ farad}$$

$$1 \mu\mu\text{F} \text{ is also known as } 1 \text{ picofarad (pF)}$$

CAPACITANCE OF A SPHERICAL CONDUCTOR OR CAPACITOR

As we have already said that a single conductor can also act as a capacitor, here we will find the capacitance of a single isolated sphere. For this, let a charge q be given to a



spherical conductor of radius R , then potential on it is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

The other conductor is supposed to be at infinity, whose potential will be taken as zero. So the potential difference between the sphere and the conductor at infinity becomes $V - 0 = V$.

Then capacitance is

$$C = \frac{q}{V} = 4\pi\epsilon_0 R$$

Thus, the capacitance of a spherical conductor is $C = 4\pi\epsilon_0 R$.

Note: The earth is a spherical conductor of radius $R = 6.4 \times 10^6$ m. The capacitance of earth is

$$C = \left(\frac{1}{9 \times 10^9} \right) (6.4 \times 10^6) \approx 711 \times 10^{-6} \text{ F} = 711 \mu\text{F}$$

Therefore, we can say that farad is a big unit. Thus, no body in the universe can have a capacitance of 1 F.

ILLUSTRATION 4.1

Two uniformly charged spherical drops at potential V coalesce to form a larger drop. If capacity of each smaller drop is C then find capacity and potential of larger drop.

Sol. When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is radius and q is charge on smaller drop then $C = 4\pi\epsilon_0 r$ and $q = CV$

$$\text{Equating volume we get } \frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3}r$$

$$\text{Capacitance of larger drop } C' = 4\pi\epsilon_0 R = 2^{1/3}C$$

$$\text{Charge on larger drop } Q = 2q = 2CV$$

$$\text{Potential of larger drop } V' = \frac{Q}{C'} = \frac{2CV}{2^{1/3}C} = 2^{2/3}V$$

SYSTEM OF CONDUCTORS

Let us bring two conductors (called plates or electrodes) and connect them with the two terminals of a battery. The battery pulls some electrons from conductor 1 and injects these to conductor 2 simultaneously. As a result, conductor 1 will get a positive charge $+Q$, say, and hence conductor 2 will receive a negative charge $-Q$. Then, all E lines coming from conductor 1 will terminate at conductor 2. In this way, the total field and field energy will be confined in the space between the conductors.

When we bring a negatively charged conductor 1 near a positively charged conductor 2, its potential decreases. It means that more charge is required to raise the potential of the conductor 1 by 1 V. In other words, the capacitance of each conductor increases in the presence of the others. Hence, in a system of two conductors, each conductor can have (store) more charge than when they are isolated.

The potential difference between the conductors is $V = V_+ - V_-$, where V_+ and V_- are the potentials of the positive and the negative conductors, respectively. Since, $|V_+|$ and $|V_-|$ vary linearly with the charge Q , we have $V \propto Q$.

Then the ratio of Q and V , that is, Q/V is a constant, which can be defined as the capacitance of the system of two conductors. The ratio of the amount of charge supplied by the battery to each conductor and the potential difference between the conductors is defined as the capacitance of the system of two conductors. Thus,

$$C = \frac{Q}{V_+ - V_-} = \frac{\text{Charge flown through the battery}}{\text{Rise in potential difference}}$$

Note: The capacitance can be increased by reducing potential keeping the charge constant. Consider a conducting plate M which is given a charge Q such that its potential rises to V then the capacitance of the plate is given by

$$C = \frac{Q}{V}$$

If we place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure) and if N is earthed from the outer side (see figure) then the potential difference across the plates $V' = V_+ - V_-$. Then new capacitance of the system of plates,

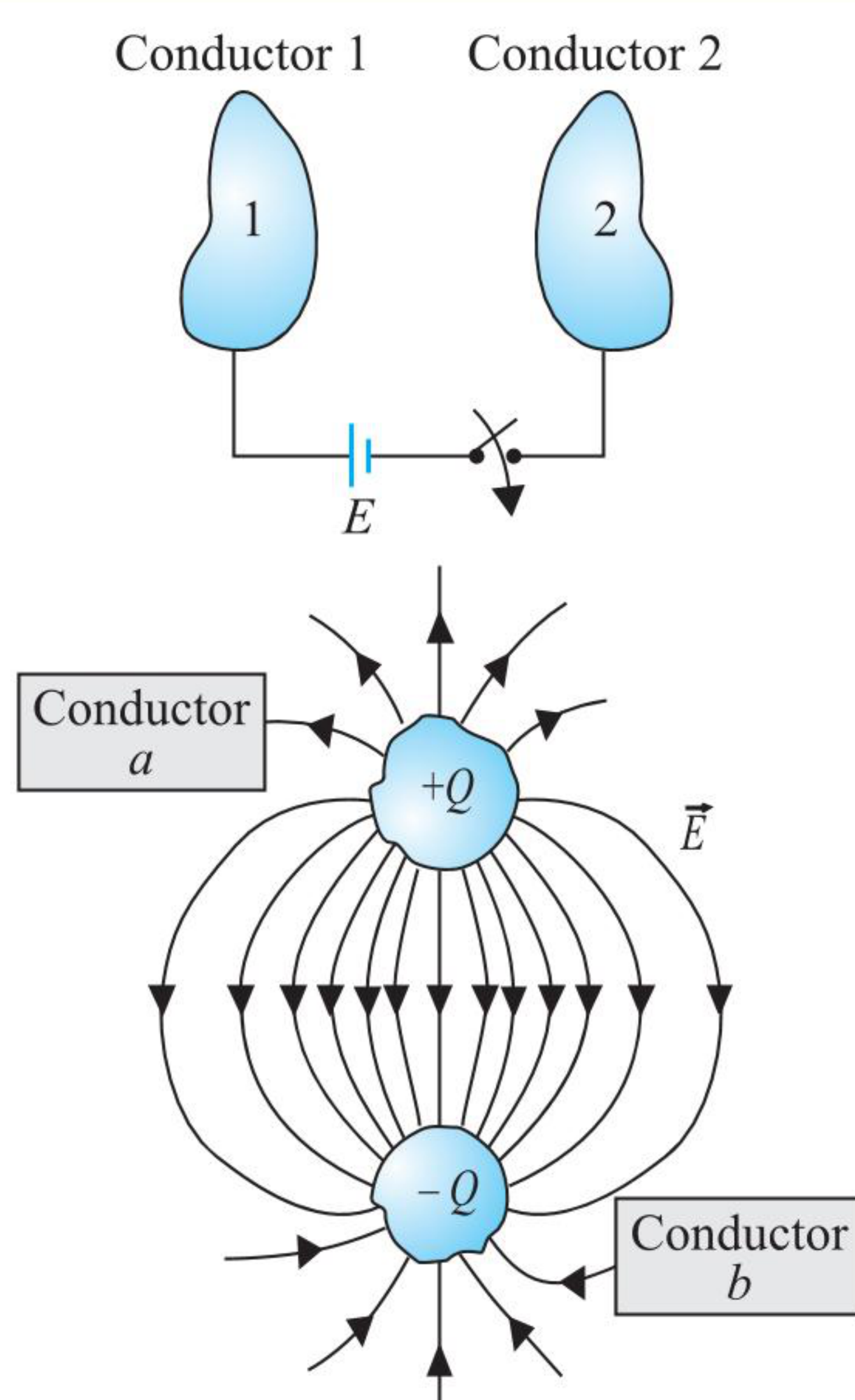
$$C' = \frac{Q}{V'} = \frac{Q}{V_+ - V_-} \Rightarrow C' \gg C$$

It means if an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable. This is the principle of a parallel plate capacitor.

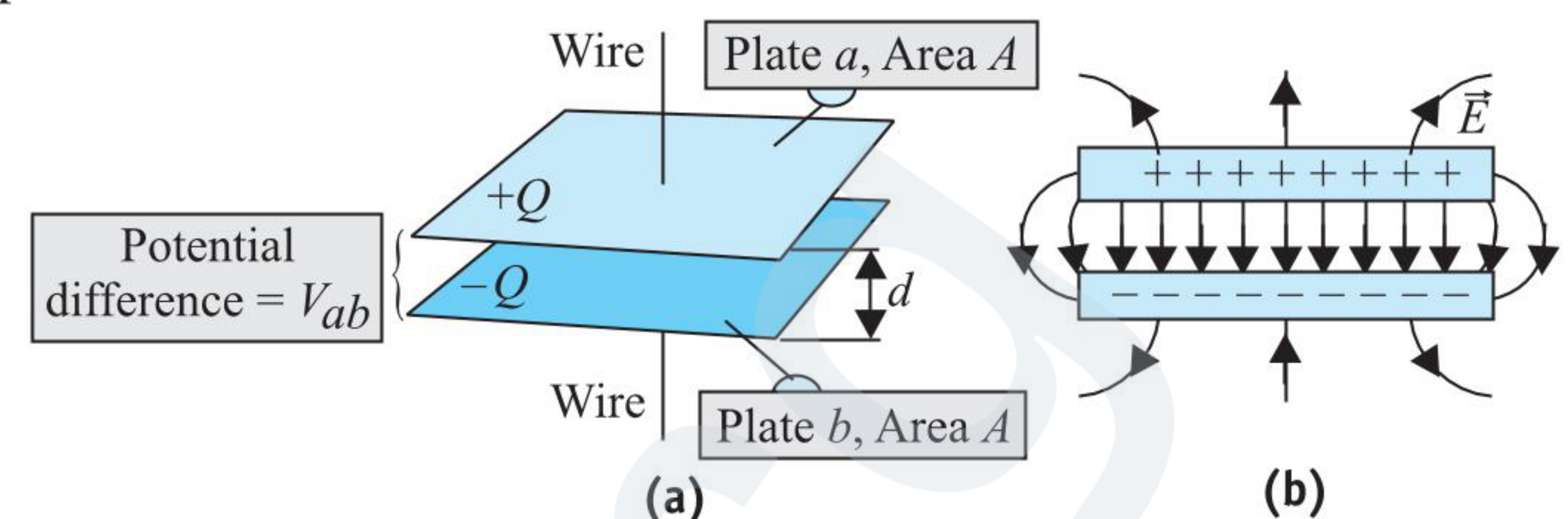
PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two large plates placed parallel to each other with a separation d smaller in comparison to the length and breadth of the plates.

In an ideal capacitor, electric field resides in the region within the plates. No electric field is outside the plates (neglecting fringing effect for ideal case). So the entire energy resides within



the capacitor and no energy is, therefore, outside the capacitor. Electric field is directed from the positive plate to the negative plate in such a way that the lines emerge perpendicularly from the positive plate and terminate perpendicularly to the negative plate.



(a) A charged parallel plate capacitor.

(b) When the separation of the plates is small compared to their size, the fringing of the electric field E at the edges is slight.

Electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

Therefore, potential difference between the plates is

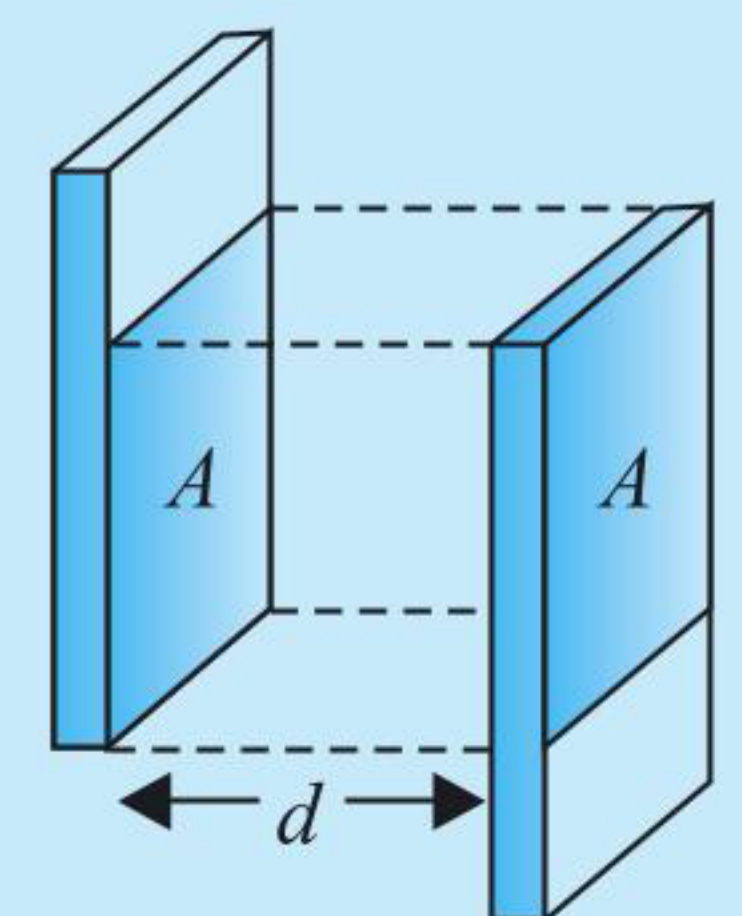
$$V = Ed = \frac{Qd}{A\epsilon_0}$$

and capacitance is

$$C = \frac{Q}{V_+ - V_-} = \frac{\epsilon_0 A}{d}$$

Important Points:

- If we place a dielectric slab completely fills the space between the plates of parallel plate capacitor the capacitance becomes $C = \frac{\epsilon_0 K A}{d}$; where K = dielectric constant of the slab.
- If the plates of a parallel plate capacitor are not facing each other completely or if one of the plates of parallel plate capacitor slides relatively then the capacitance decrease as overlapping area decreases as shown in figure.
- Non-uniformity of electric field at the boundaries of the plates is negligible if the separation between the plates is very small as compared to the length of the plates.

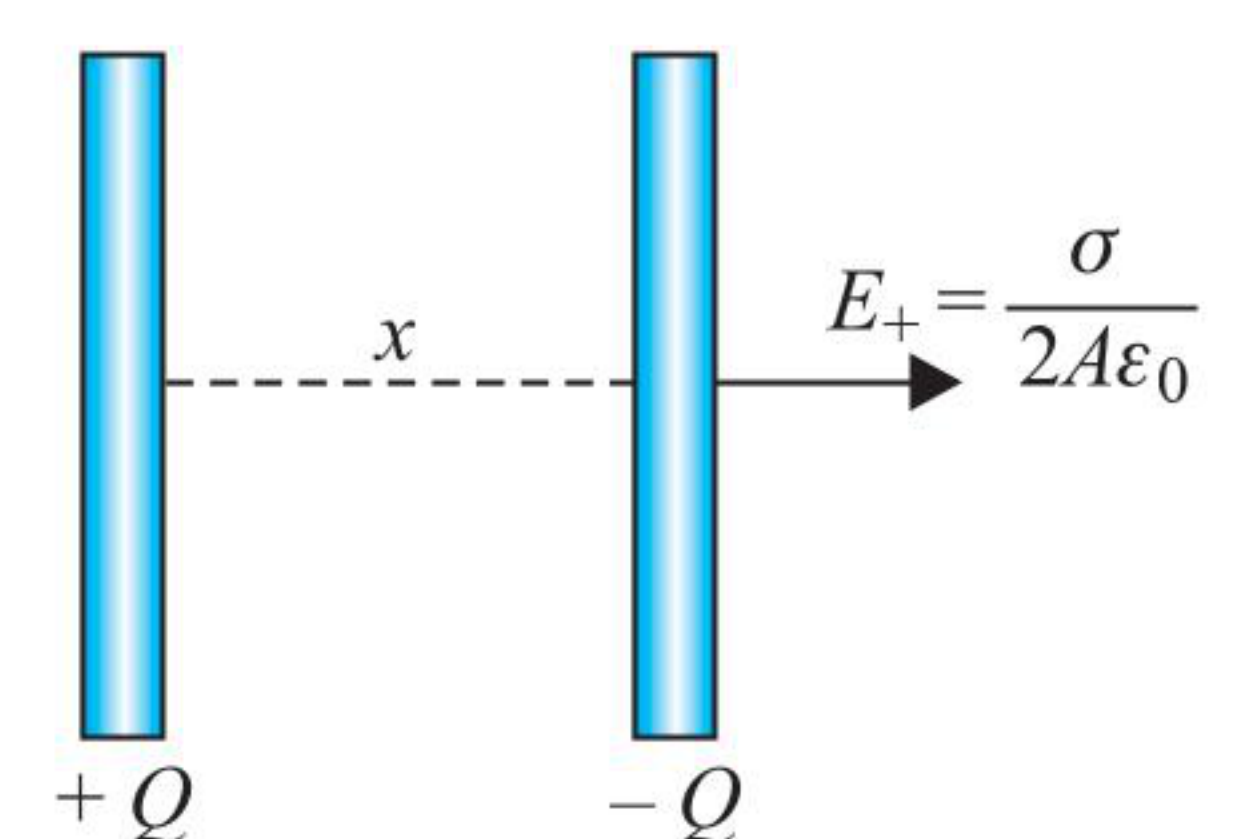


FORCE BETWEEN THE PLATES OF A PARALLEL PLATE CAPACITOR

Consider a parallel plate capacitor with plate area A . Suppose a positive charge $+Q$ is given to one plate and a negative charge $-Q$ to the other plate. The electric field due to the positive plate is

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}$$

at all points if the plate is large. The negative charge $-Q$ on the other plate finds itself in the field of this positive charge. Therefore, the force on this plate is



$$F = EQ = \frac{Q^2}{2A\epsilon_0}$$

This force will be attractive.

Important Point:

- If we place a dielectric slab between the plates of the capacitor, the force between the plates of the capacitor will remain unchanged as the electric field at the position of the plates remains unchanged.

SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric spherical conducting shells of radii a and b , say $b > a$. The outer shell is earthed. Place a charge $+Q$ on the inner shell. It will reside on the outer surface of the shell. A charge $-Q$ will be induced on the inner surface of the outer shell. A charge $+Q$ will flow from the outer shell to earth.

Consider a Gaussian spherical surface of radius r such that $a < r < b$. From Gauss's law, the electric field at distance $r > a$ is

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

The potential difference is

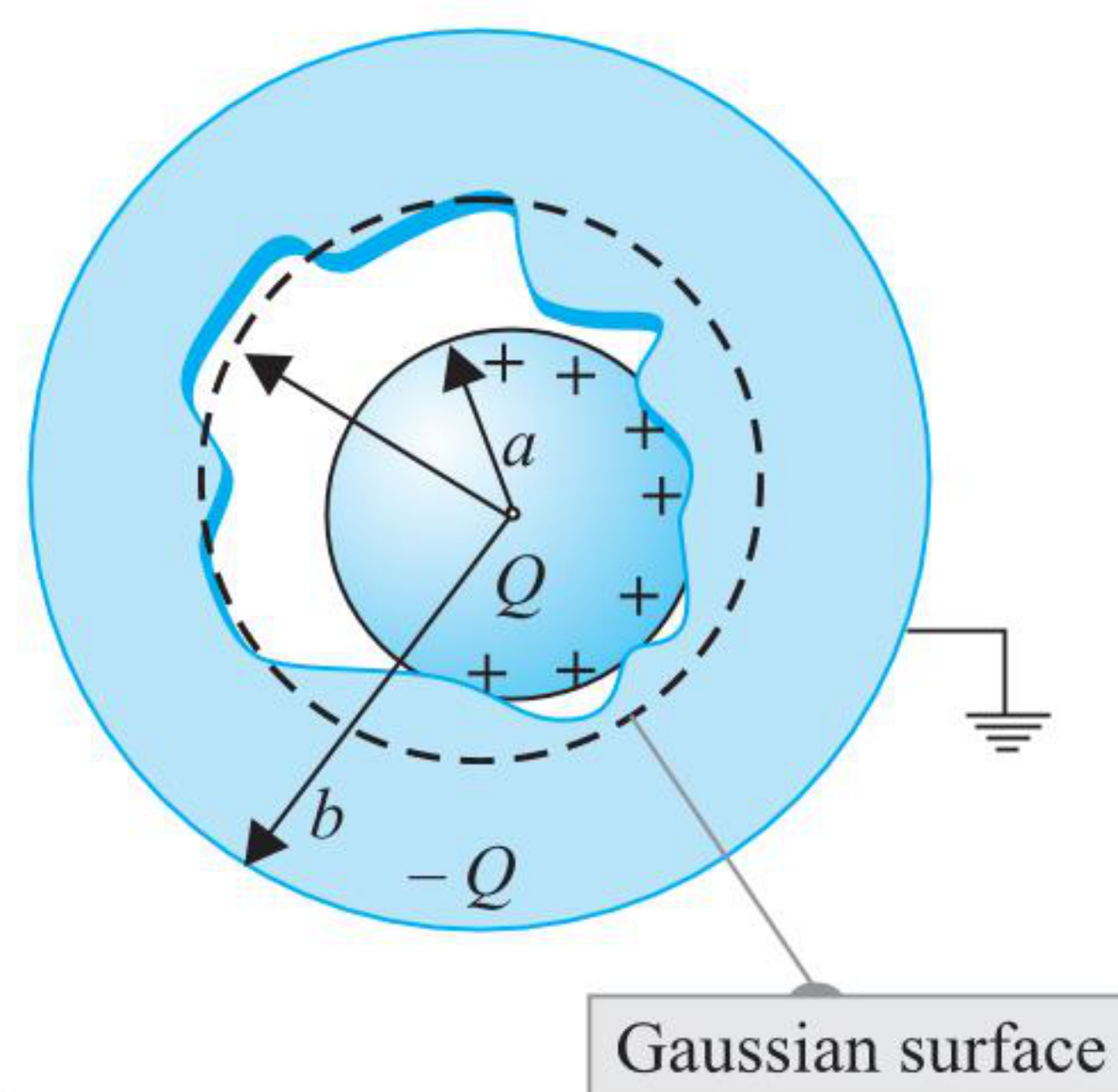
$$V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{r} = - \int_a^b \frac{Q}{4\pi\epsilon_0 r^2} dr$$

Since $V_b = 0$, we have

$$V_a = \int_a^b \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q(b-a)}{4\pi\epsilon_0 ab}$$

Therefore, capacitance is

$$C = \frac{Q}{V_a - V_b} = \frac{Q}{V_a} = \frac{4\pi\epsilon_0 ab}{b-a}$$



For the curved part

$$\phi = \int \vec{E} \cdot d\vec{s} = \int E ds = E \int ds = E 2\pi r y$$

Charge inside the Gaussian surface is

$$q = \frac{Qy}{L}$$

From Gauss's law

$$\phi = E 2\pi r y = \frac{Qy}{L\epsilon_0} \text{ or } E = \frac{Q}{2\pi\epsilon_0 L r}$$

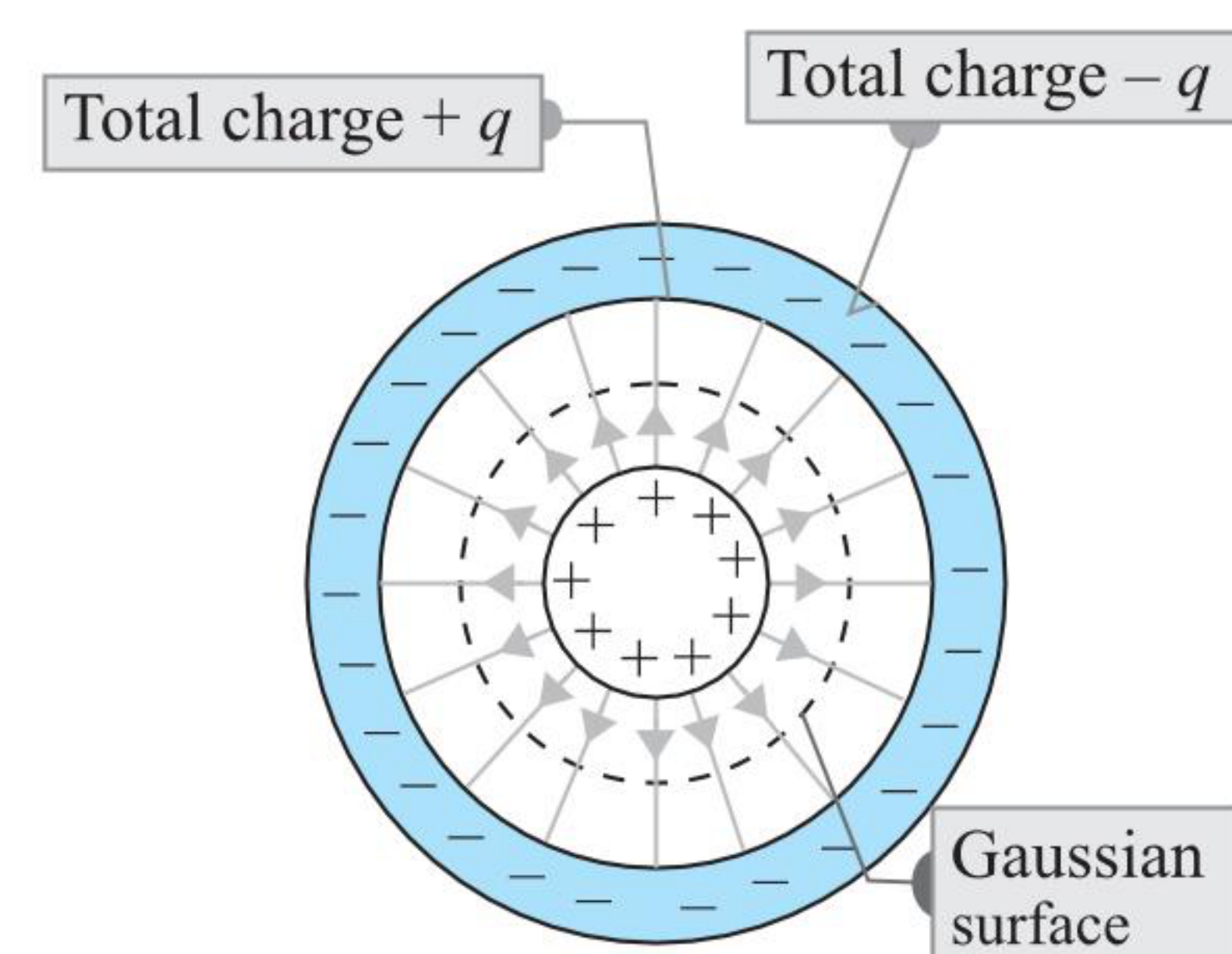
Potential difference is

$$V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{r} = - \int_a^b \frac{Q}{2\pi\epsilon_0 L r} dr = - \frac{Q}{2\pi\epsilon_0 L} \int_a^b \frac{1}{r} dr$$

$$\text{or } V_a = \frac{Q}{2\pi\epsilon_0 L} \ln \frac{b}{a} \quad (\text{since } V_b = 0)$$

Capacitance is

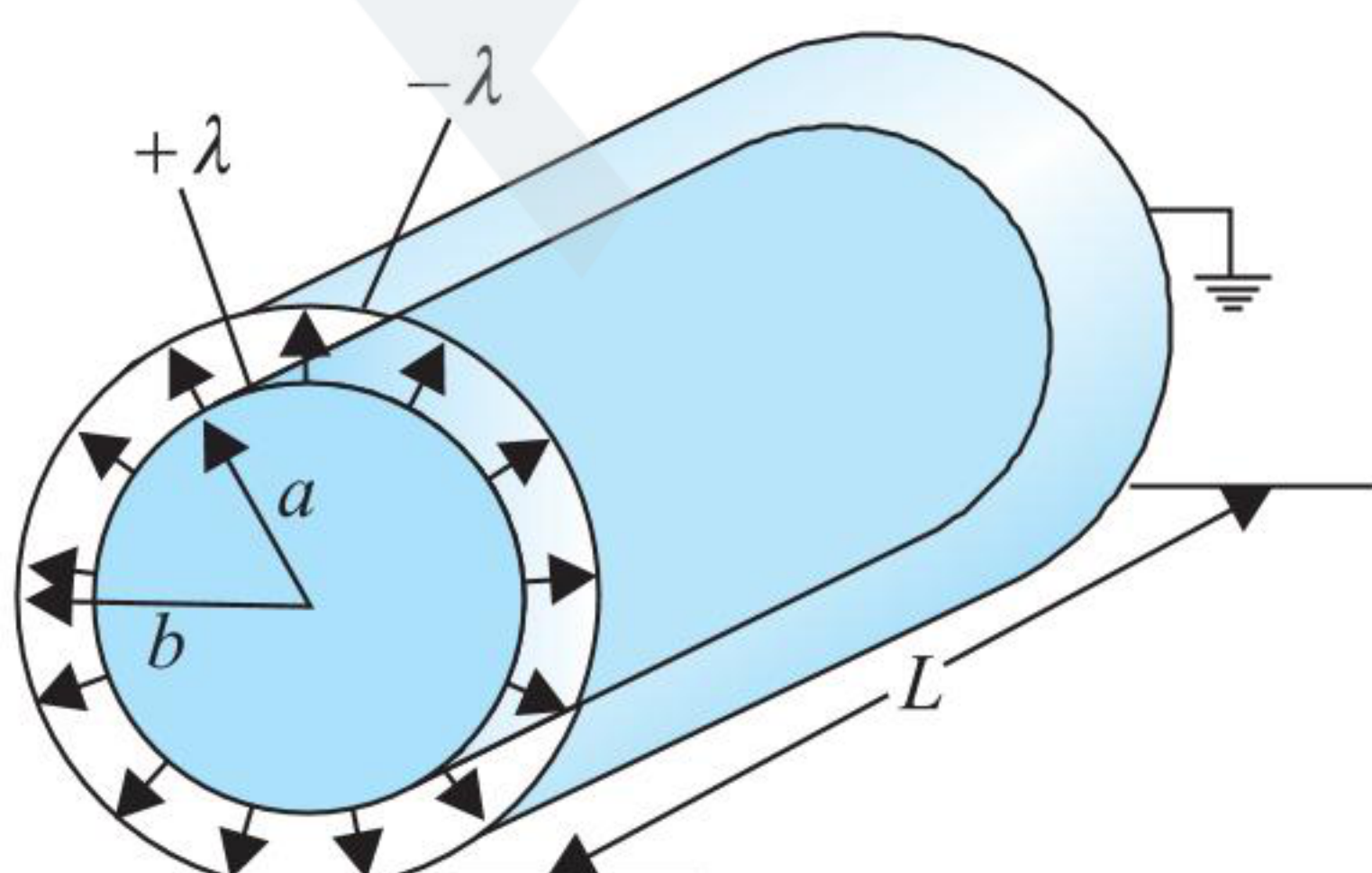
$$C = \frac{Q}{V_a - V_b} = \frac{Q}{V_a} = \frac{2\pi\epsilon_0 L}{\ln \left(\frac{b}{a} \right)}$$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate).

CYLINDRICAL CAPACITOR

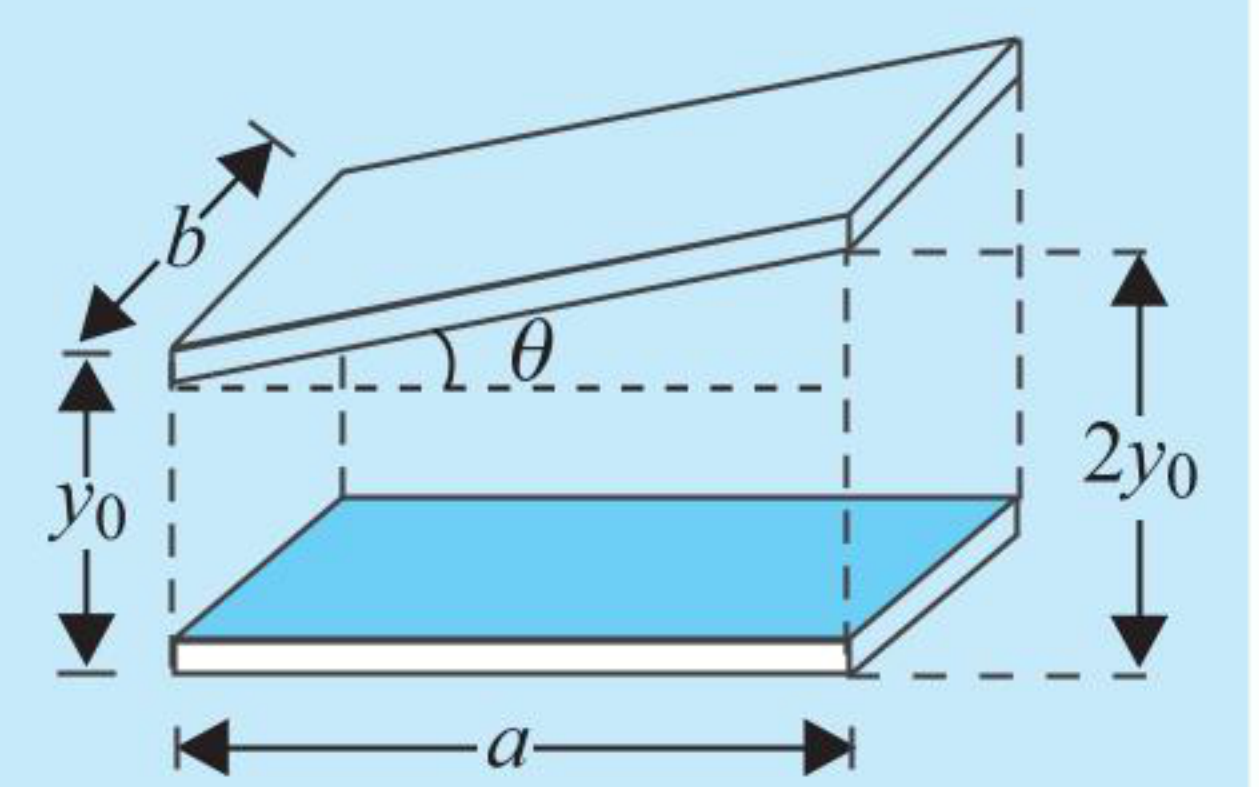
A cylindrical capacitor consists of two coaxial cylinders of radii a and b , say $b > a$. The outer one is earthed. The cylinders are long enough so that we can neglect fringing of electric fields at the ends. The electric field at a point between the cylinders will be radial, and its magnitude will depend on the distance from the central axis. Consider a Gaussian surface of length y and radius r such that $a < r < b$. Flux through the plane surface is zero because the electric field and the area vector are perpendicular to each other.



In the long cylindrical capacitor, the linear charge density λ is assumed to be positive in this figure. The magnitude of charge in a length of either cylinder is λL .

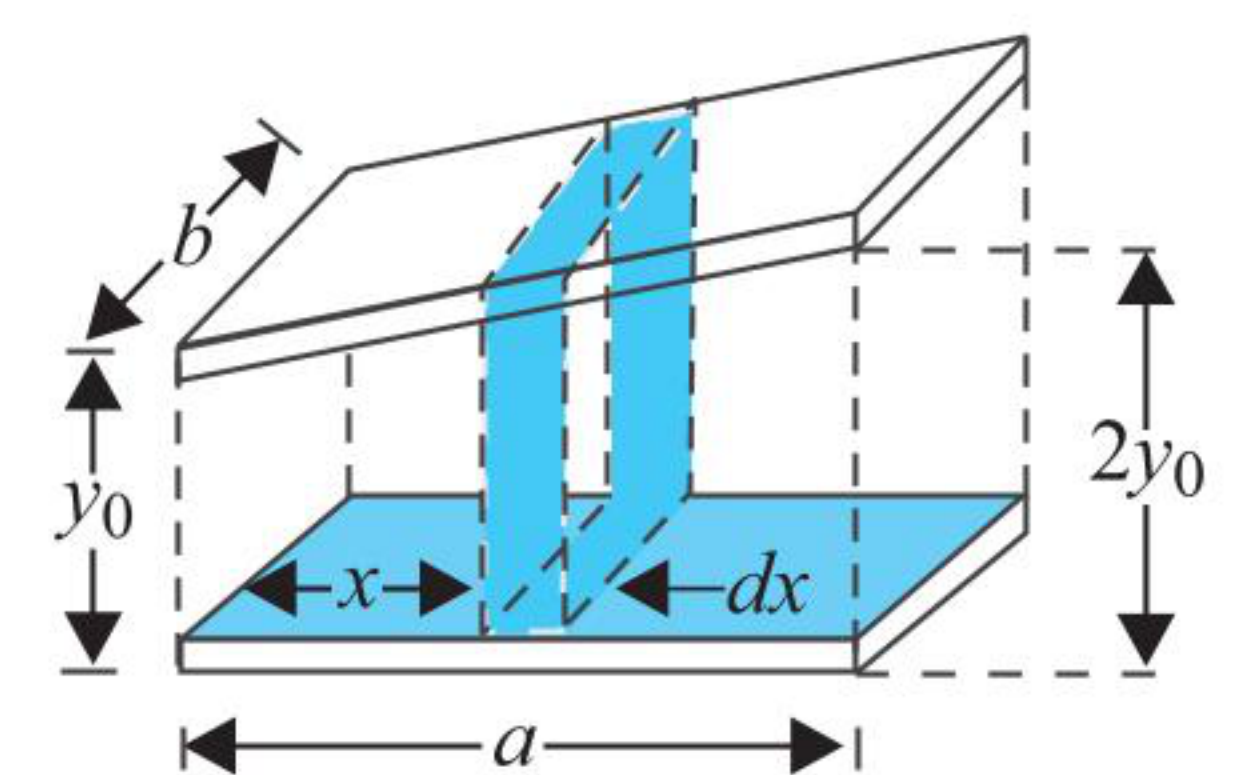
ILLUSTRATION 4.2

A capacitor has rectangular plates of length a and width b . The top plate is inclined at a small angle as shown in figure. The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right, where y_0 is much less than a or b . Calculate the capacitance of the system.



Sol. We consider a differential strip of width dx and length b to approximate a differential capacitor of area $b dx$ and separation

$$d = y_0 + \left(\frac{y_0}{a} \right) x.$$



All such differential capacitors are in parallel arrangement. Thus,

$$dC = \frac{\epsilon_0 (b dx)}{y_0 + \left(\frac{y_0}{a} \right) x} \text{ or } C = \int dC$$

$$\begin{aligned} \text{or } C &= \epsilon_0 b \int_0^a \frac{dx}{y_0 + \frac{y_0}{a}x} \\ &= \frac{\epsilon_0 b}{(y_0/a)} \left[\ln \left(\frac{y_0 + \frac{y_0}{a} \times a}{y_0} \right) \right] = \frac{\epsilon_0 ab}{y_0} \ln 2 \end{aligned}$$

We can determine the expression for capacity in terms of θ as

$$\begin{aligned} d &= (y_0 + x \tan \theta) \\ C &= \int dC = \int_0^a \frac{\epsilon_0 b dx}{(y_0 + x \tan \theta)} \\ &= \frac{\epsilon_0 b}{\tan \theta} \ln \frac{(y_0 + a \tan \theta)}{y_0} \end{aligned}$$

For small θ ,

$$\tan \theta \approx \theta \text{ or } C = \frac{\epsilon_0 b}{\theta} \ln \left(1 + \frac{a\theta}{y_0} \right)$$

Now, we can use the expansion

$$\log(1+x) = x - \frac{1}{2}x^2 + \dots$$

For $x < 1$, we can neglect higher powers. Thus,

$$C = \frac{\epsilon_0 b}{\theta} \left[\frac{a\theta}{y_0} - \frac{1}{2} \left(\frac{a\theta}{y_0} \right)^2 \right] = \frac{\epsilon_0 ab}{y_0} \left[1 - \frac{a\theta}{2y_0} \right]$$

ENERGY STORED IN A CHARGED CONDUCTOR OR CAPACITOR

The charging of a capacitor is associated with a continuous pumping of positive charge from conductor 2 to conductor 1 by the battery (figure). If a charge $+dq$ is brought from conductor 2 to conductor 1 against the electric field E of the charged conductors, the work done by the battery is

$$dW = (Ed) dq = Vdq$$

$$\text{where } V = \frac{q}{C} \Rightarrow dW = \frac{q}{C} dq$$

Then the total work done by the battery in sending a charge Q is

$$W = \int dW = \int_0^Q \frac{q dq}{C} = \frac{Q^2}{2C}$$

The work is done at the expense of the chemical energy of the battery, which is stored in the capacitor in the form of electrostatic field energy U . Then, $U = Q^2/2C$. Putting $Q = CV$, the other two expressions of energy can be written as

$$U = \frac{1}{2} QV = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

ALTERNATIVE METHOD

A charged capacitor is a group of two charged conductors. The charge of one conductor is $Q_1 = +Q$ and potential $V_1 = V_+$, and the

charge of the other conductor is $Q_2 = -Q$ and potential $V_2 = V_-$. As we know, the energy possessed by a charged conductor is

$$U = \frac{1}{2} qV$$

where q and V are the charge and potential of the conductor, respectively. Then the total energy possessed by the two conductors (plates) is

$$\begin{aligned} U &= \frac{1}{2} Q_1 V_1 + \frac{1}{2} Q_2 V_2 \\ &= \frac{1}{2} (+Q)(V_+) + \frac{1}{2} (-Q)(V_-) = \frac{Q}{2} (V_+ - V_-) \end{aligned}$$

where $V_+ - V_- = V$ (potential difference between the conductors plates). Thus,

$$U = \frac{1}{2} QV \quad (\text{as obtained earlier})$$

Important Points:

- Work done by battery, $W_{\text{Battery}} = \text{Charge supplied by battery} \times \text{E.M.F. of the battery}$
 $QV = CV^2 = \frac{Q^2}{C}$

But the energy stored in capacitor $U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$, it means 50% energy supplied by the battery is lost in form of heat.

Hence heat developed in the circuit $H = \frac{Q^2}{2C}$ or $\frac{(\Delta Q)^2}{2C}$

- If many capacitors are connected in the circuit then heat developed in the circuit $H = \sum \frac{(\Delta Q)^2}{2C}$

ENERGY DENSITY IN A PARALLEL PLATE CAPACITOR

As derived earlier, the electrostatic energy stored in a parallel plate capacitor is $U = \frac{1}{2} CV^2$

where $C = (\epsilon_0 A/d)$ and $V = Ed$. So

$$U = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) (Ed)^2 = \frac{1}{2} \epsilon_0 E^2 (Ad)$$

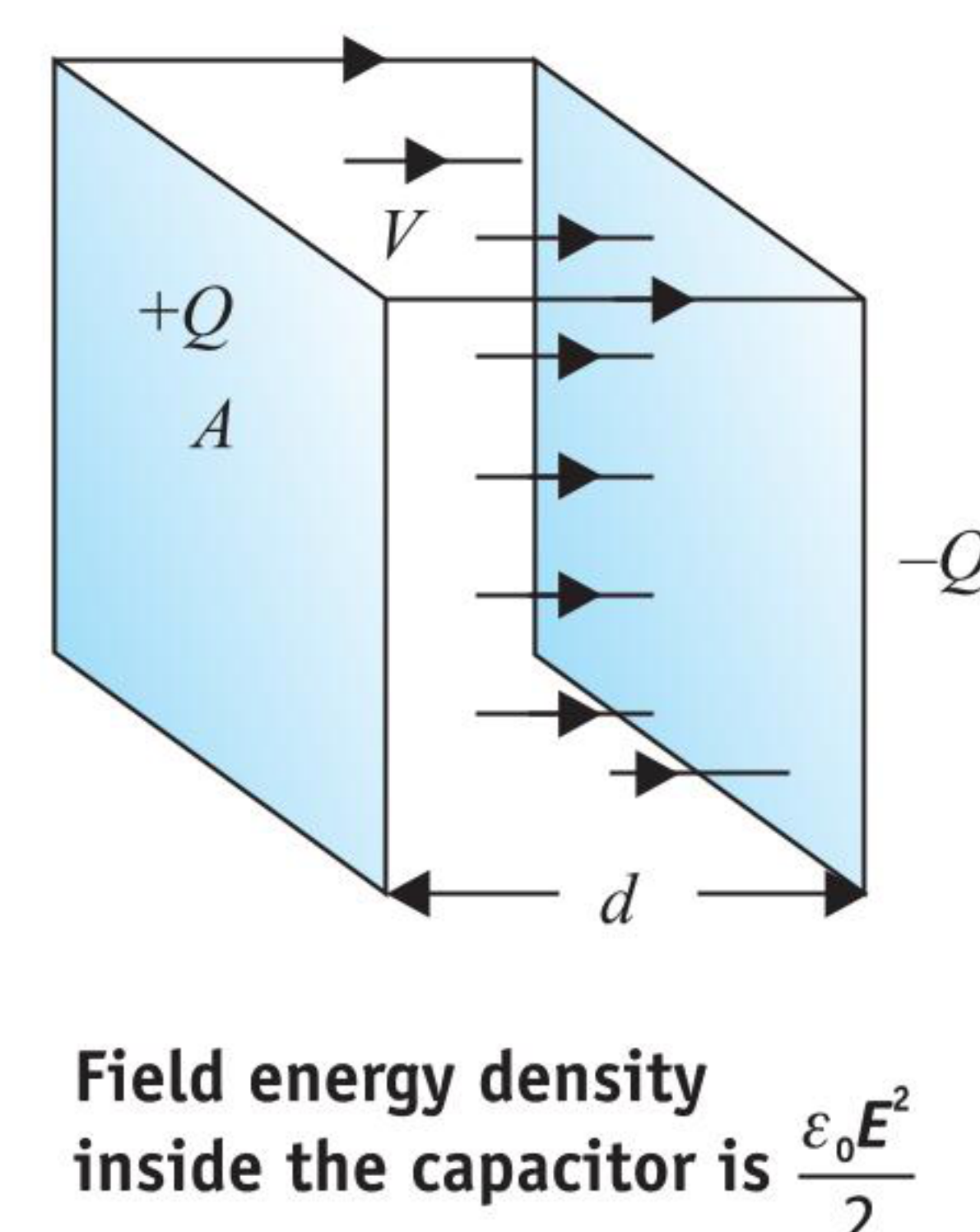
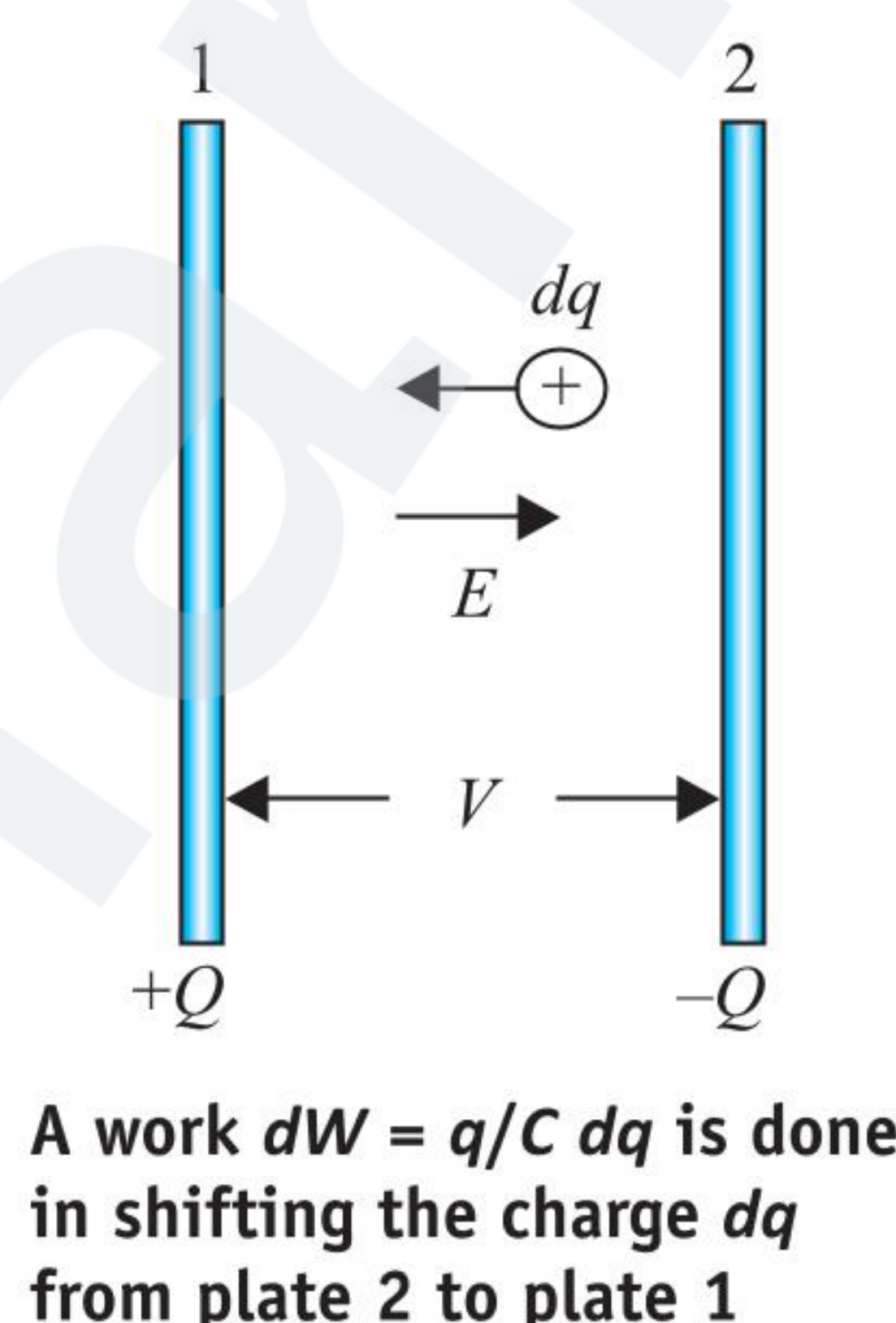
where $Ad = V$ (volume of the capacitor).

Then, $U/V = dU/dV$, that is, the energy stored per unit volume can be given by

$$\frac{dU}{dV} = u = \frac{1}{2} \epsilon_0 E^2$$

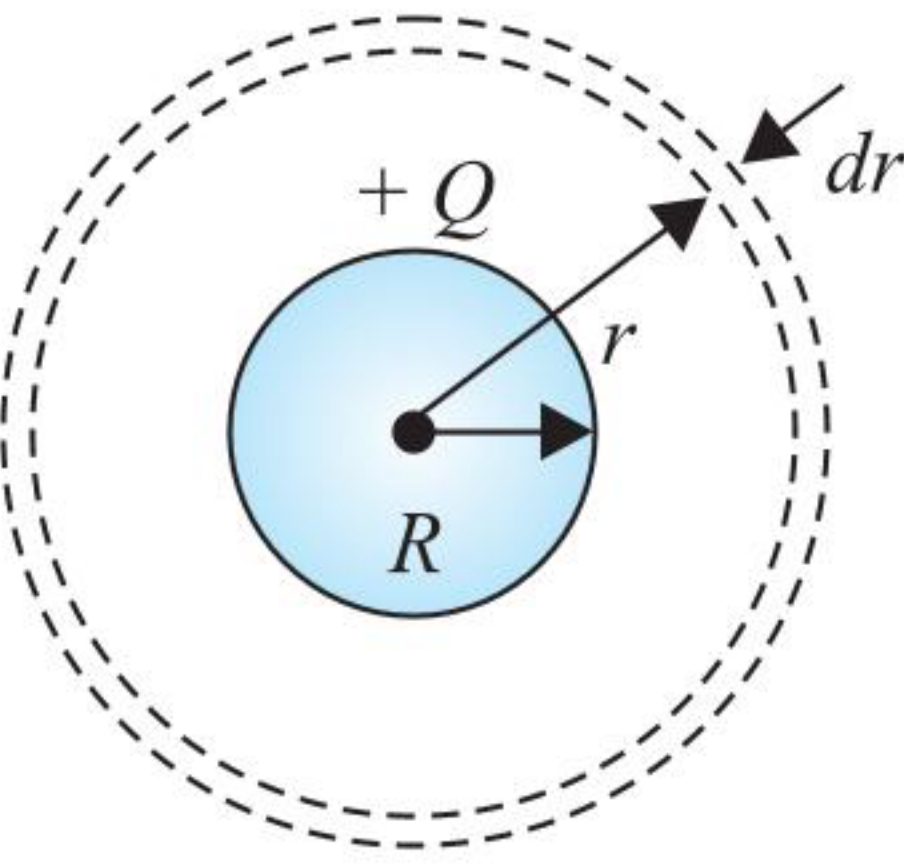
Although we have proved the above result for a parallel plate capacitor, but in general, this is true for any kind of capacitor or any other kind of electric field.

The potential energy of a spherical capacitor made of a single sphere can be found using the concept of energy density. Energy density is given by



$$u = \frac{1}{2} \epsilon_0 E^2$$

$$\therefore dU = u dV = \frac{1}{2} \epsilon_0 \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \right)^2 4\pi r^2 dr$$

$$\text{or } U = \frac{1}{8\pi\epsilon_0} Q^2 \int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\epsilon_0 R}$$


Note: It is a common misconception that electric field energy is a new kind of energy, different from the electric potential energy described before. It is simply a different way of interpreting electric potential energy. We can regard the energy of a given system of charges as being a shared property of all the charges, or we can think of the energy as being a property of the electric field that the charges create. Either interpretation leads to the same value of the potential energy.

ILLUSTRATION 4.3

A capacitor of capacitance C , which is initially uncharged, is connected with a battery of emf ϵ . Find the heat dissipated in the circuit during the process of charging.

Sol. Initial charge in the capacitor is $Q = 0$. When the capacitor is connected across battery, the final potential difference across the capacitor will be E . Final charge in the capacitor is $Q_f = C\epsilon$. Initial potential energy stored is $U_i = 0$. Final potential energy stored is $U_f = (1/2)C\epsilon^2$. So the charge supplied by the battery is

$$\Delta Q = Q_f - Q_i = (C\epsilon - 0) = C\epsilon$$

Work done by the battery in charging the capacitor is

$$W_{\text{battery}} = (\Delta Q) \epsilon = (C\epsilon) \epsilon = C\epsilon^2$$

We know that $W_{\text{battery}} = \Delta U + \text{heat developed}$. So

$$H = W_{\text{battery}} - \Delta U = C\epsilon^2 - \frac{1}{2}C\epsilon^2$$

$$\therefore \text{Heat developed} = \frac{1}{2}C\epsilon^2$$

ILLUSTRATION 4.4

A capacitor of capacitance C , which is initially charged up to a potential difference ϵ , is connected with a battery of emf $\epsilon/2$ such that the positive terminal of the battery is connected with the positive plate of the capacitor. After a long time

- find the total charge flow through the battery
- find the total work done by the battery
- Find the heat dissipated in the circuit during the process of charging

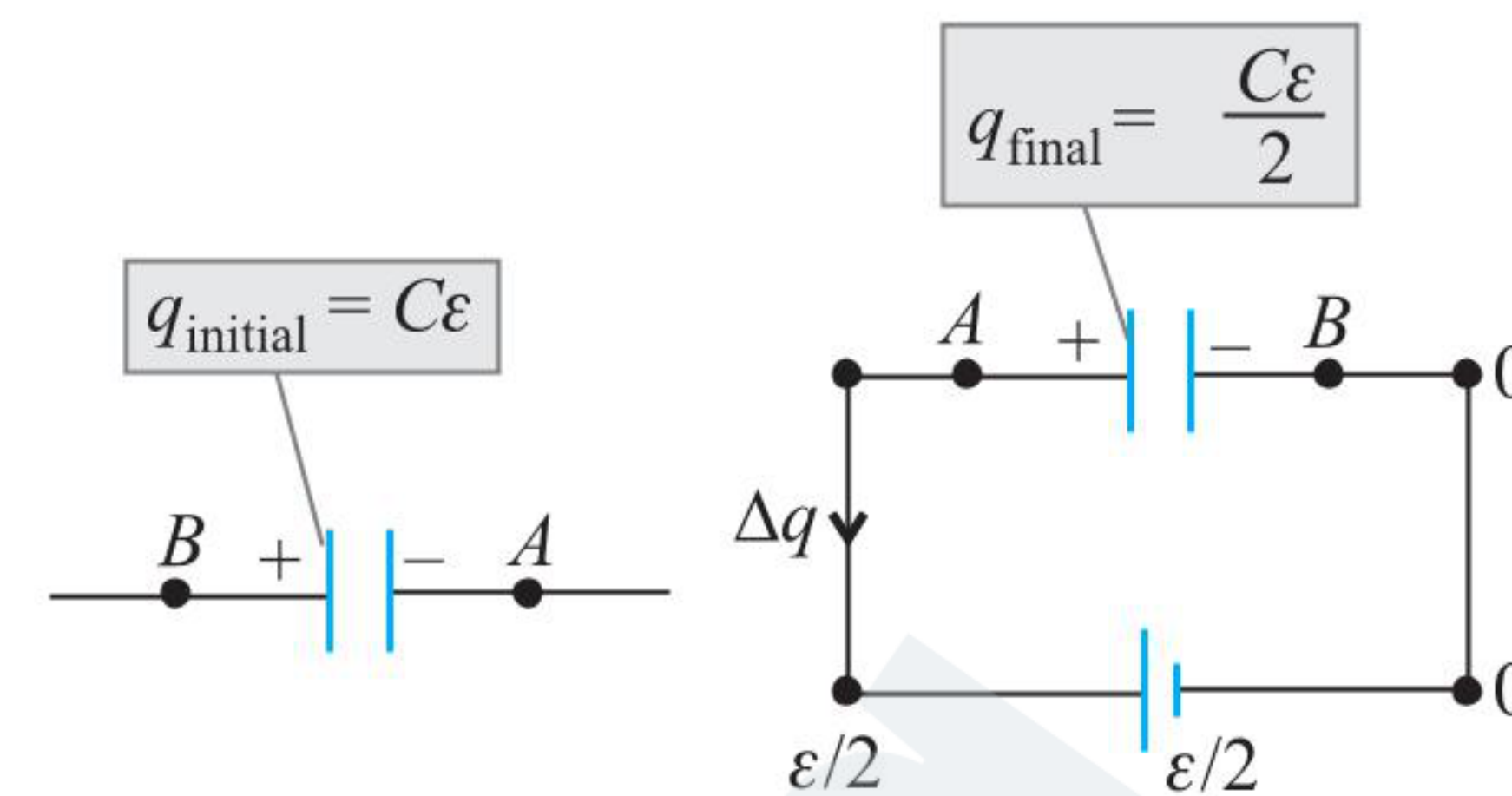
Sol.

- When the capacitor is connected to the battery of emf $\epsilon/2$, the final charge in the capacitor is $C\epsilon/2$. Charge flowing through the capacitor is the charge supplied by the battery

$$\Delta q = (q_{\text{final}} - q_{\text{initial}})$$

$$= (C\epsilon/2 - C\epsilon) = -C\epsilon/2$$

(The charge is flowing from the capacitor to the battery.)



- Work done by the battery is

$$W_{\text{battery}} = (\Delta q) \frac{\epsilon}{2} = \left(-\frac{C\epsilon}{2} \right) \frac{\epsilon}{2} = -\frac{C\epsilon^2}{4}$$

Change in the potential energy of the capacitor is $U_{\text{final}} - U_{\text{initial}}$. So

$$\Delta U = \frac{1}{2} C \left(\frac{\epsilon}{2} \right)^2 - \frac{C\epsilon^2}{2} = \frac{1}{8} C\epsilon^2 - \frac{1}{2} C\epsilon^2 = -\frac{3C\epsilon^2}{8}$$

- Work done by the battery = Change in potential energy of the capacitor + heat produced. So heat produced is

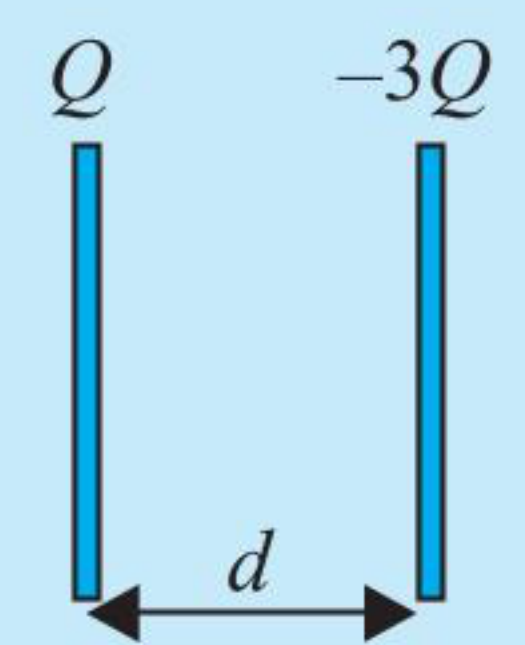
$$H = W_{\text{battery}} - \Delta U = \frac{3C\epsilon^2}{8} - \frac{C\epsilon^2}{4} = \frac{C\epsilon^2}{8}$$

Approach 2: The heat produced, $H = \frac{(\Delta Q)^2}{2C} = \frac{(-C\epsilon/2)^2}{2C} = \frac{C\epsilon^2}{8}$

ILLUSTRATION 4.5

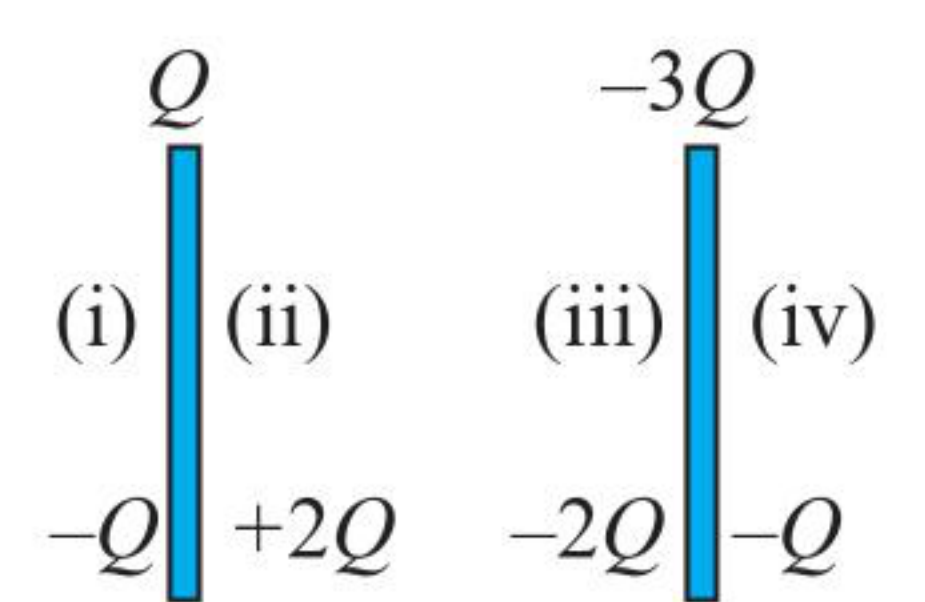
A parallel plate capacitor has capacitance C . If the charges of the plates are Q and $-3Q$, find the

- charges at the inner surfaces of the plates
- potential difference between the plates
- charge flown if the plates are connected
- energy lost by the capacitor in (iii.)
- charge flown if any plate is earthed



Sol. The charge on surface (ii), $q_2 = \frac{Q - (-3Q)}{2} = 2Q$

- Hence the charges on the inner surfaces are $2Q$ and $-2Q$.
- $V = \frac{\text{Charge of capacitor}}{\text{Capacitance}} = \frac{2Q}{C}$



- If the plates are connected, $2Q$ and $-2Q$ will be neutralized, so $2Q$ charge will flow from the left to the right plate.
- Energy lost = initial energy stored in capacitor

$$= \frac{(2Q)^2}{2C} = \frac{2Q^2}{C}$$

- If the left plate is earthed, $2Q$ charge flows from the ground to the left plate. If the right plate is earthed, $2Q$ charge flows from the ground to right plate.

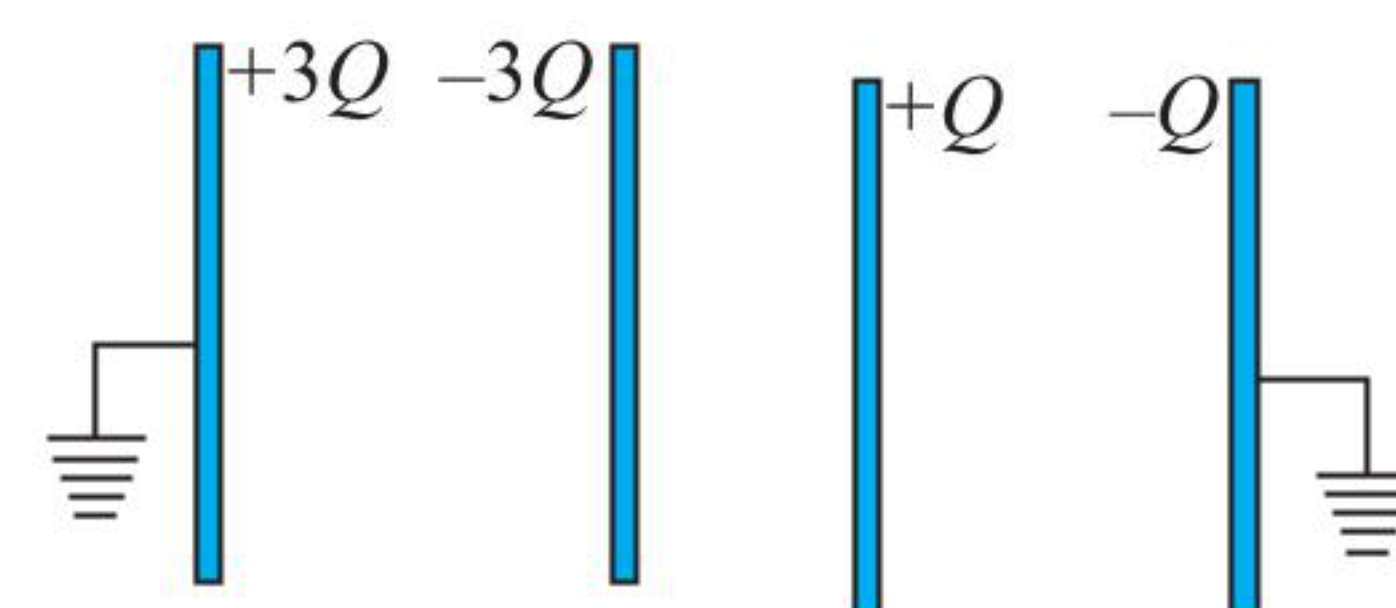
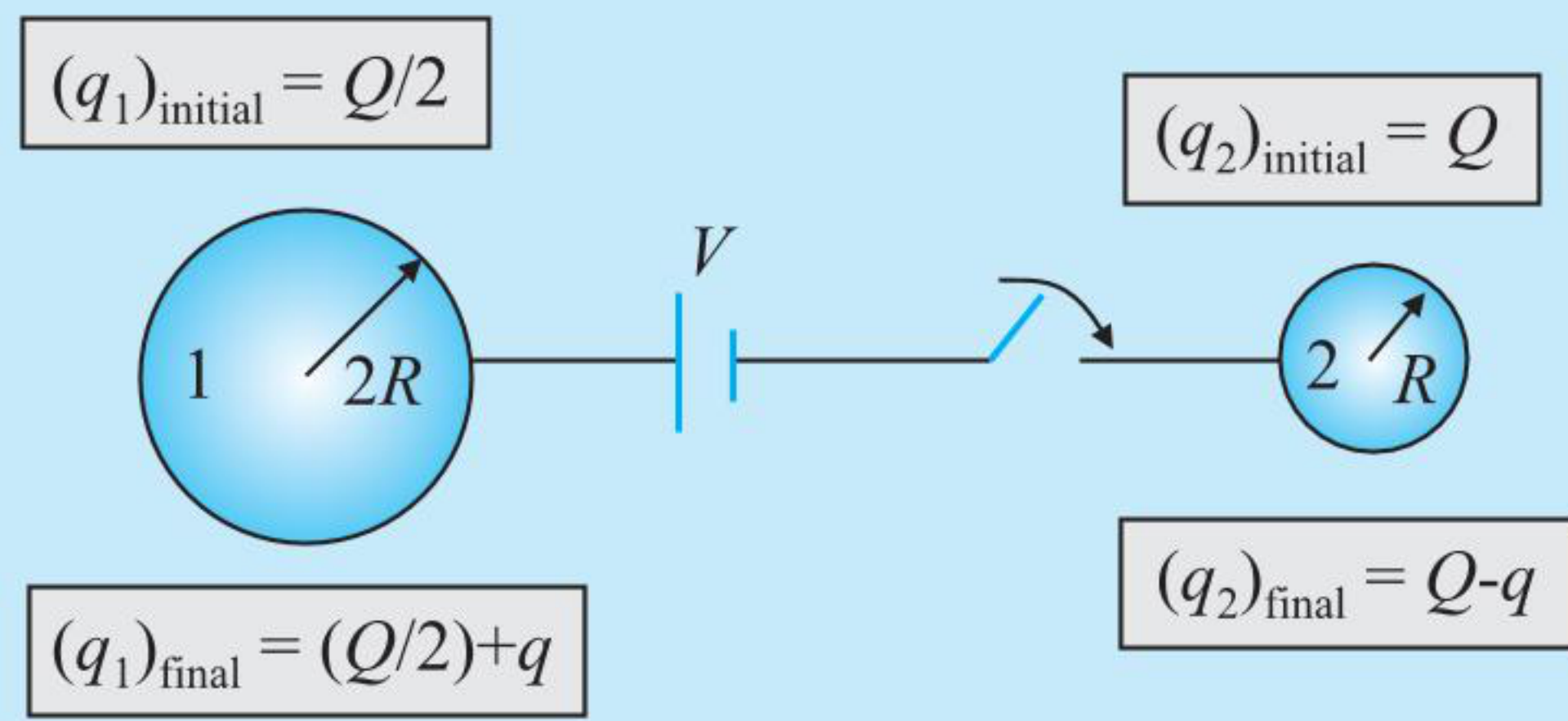


ILLUSTRATION 4.6

There are two spheres of radii R and $2R$ having charges Q and $Q/2$, respectively. These two spheres are connected with a cell of emf V volts as shown in figure. When the switch is closed, find the final charge on each sphere.



Sol. When the switch is closed, the potential difference between the spheres should be V . Let q charges flow from the sphere of radius R .

$$\frac{(q_1)_{\text{final}}}{C_1} - \frac{(q_2)_{\text{final}}}{C_2} = V$$

$$C_1 = 4\pi\epsilon_0(2R), C_2 = 4\pi\epsilon_0 R$$

Then

$$\frac{\left(\frac{Q}{2} + q\right)}{4\pi\epsilon_0(2R)} - \frac{(Q - q)}{4\pi\epsilon_0(R)} = V$$

$$\text{or } \frac{Q}{2} + q - 2Q + 2q = 4\pi\epsilon_0(2R)V$$

$$\text{or } q = \frac{8\pi\epsilon_0 RV}{3} + \frac{Q}{2}$$

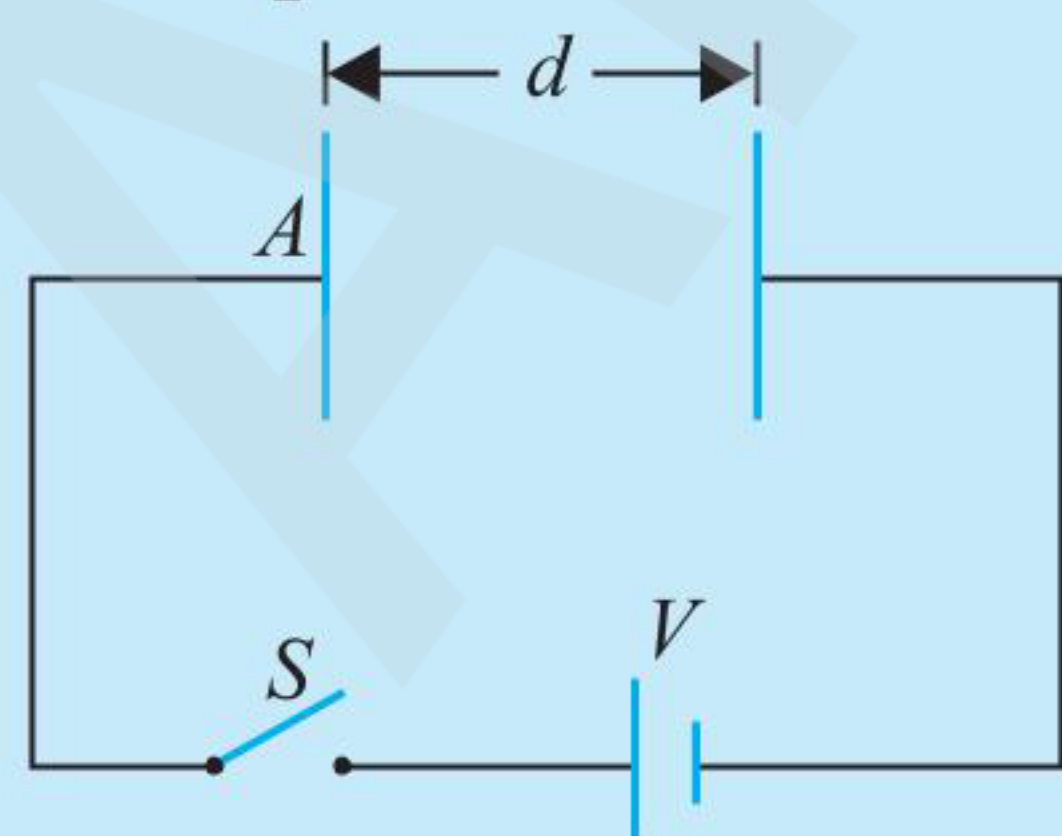
So the final charges on each of the spheres are

$$Q_1 = Q - q = \frac{-8\pi\epsilon_0 RV}{3} + \frac{Q}{2}$$

$$\text{and } Q_2 = \frac{Q}{2} + q = Q + \frac{8\pi\epsilon_0 RV}{3}$$

ILLUSTRATION 4.7

A parallel plate capacitor is connected across a battery of emf V as shown in figure. The plates of the capacitor have area A and separation between the plates is d . Now switch S is closed to connect the capacitors with battery. Now the distance between the plates is slowly reduced to $d/2$. Calculate the work done by the external agent in the process.



Sol. As the plates are moved slowly the charge flow from battery to the capacitor plates is very slow. Hence the energy dissipated must be zero.

The initial charge in capacitor, $q_{\text{initial}} = CV$

Where the capacitance is $C_{\text{initial}} = \frac{\epsilon_0 A}{d} = C$

Initial energy stored in capacitor $U_{\text{initial}} = \frac{1}{2} CV^2$

Now the distance between the plates is slowly reduced to $\frac{d}{2}$

The capacitance will become $C_{\text{final}} = \frac{\epsilon_0 A}{d/2} = 2C$

Final energy stored in capacitor $U_{\text{final}} = \frac{1}{2} (2C)V^2 = CV^2$

The final charge in capacitor, $q_{\text{final}} = 2CV$

Hence work done by battery, $W_{\text{battery}} = \text{Charge supplied} \times \text{E.M.F of the battery}$

$$\Rightarrow W_{\text{battery}} = (\Delta q) \cdot V = (2CV - CV)V = CV^2$$

Now applying conservation of energy principle

$$W_{\text{battery}} + W_{\text{external}} = \Delta U$$

$$CV^2 + W_{\text{external}} = \left(CV^2 - \frac{1}{2} CV^2 \right)$$

$$\Rightarrow W_{\text{external}} = -\frac{1}{2} CV^2$$

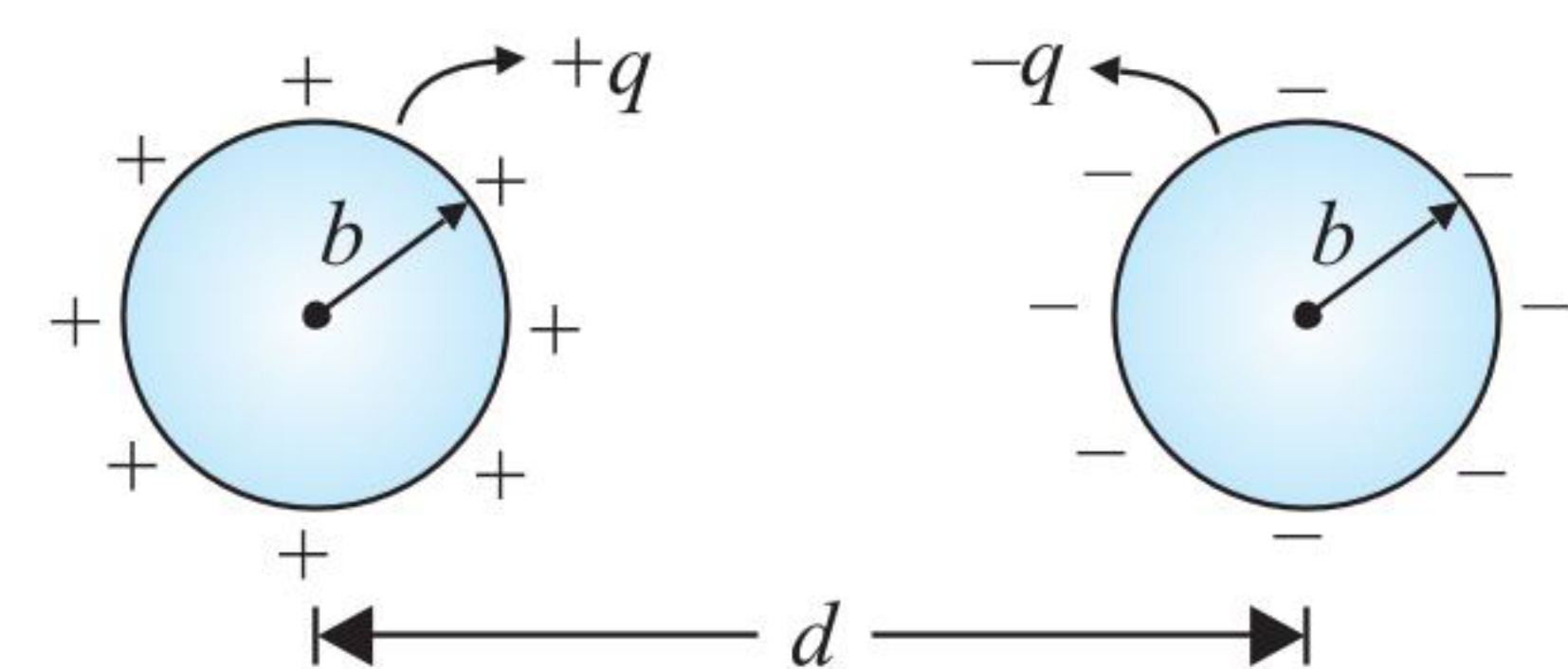
ILLUSTRATION 4.8

Two conducting sphere of radii a and b are placed at separation d ($d \gg a$ and b) such that charge distribution on both the spheres remains spherically symmetric. If a charge $+q$ is provided to the sphere of radius a and $-q$ is given to the sphere of radius b . Find the electrostatic energy (U) of the system and calculate the capacitance of the system.

Sol. The electrostatic self-energy of two spheres,

$$U_1 = \frac{q^2}{8\pi\epsilon_0 a} \text{ and } U_2 = \frac{q^2}{8\pi\epsilon_0 b}$$

It is to be noted that electrostatic self-energy is always positive. We can write the interaction energy (U_{12}) of two spheres assuming them to be point charges.



$$U_{12} = \frac{q(-q)}{4\pi\epsilon_0 d} = \frac{-q^2}{4\pi\epsilon_0 d}$$

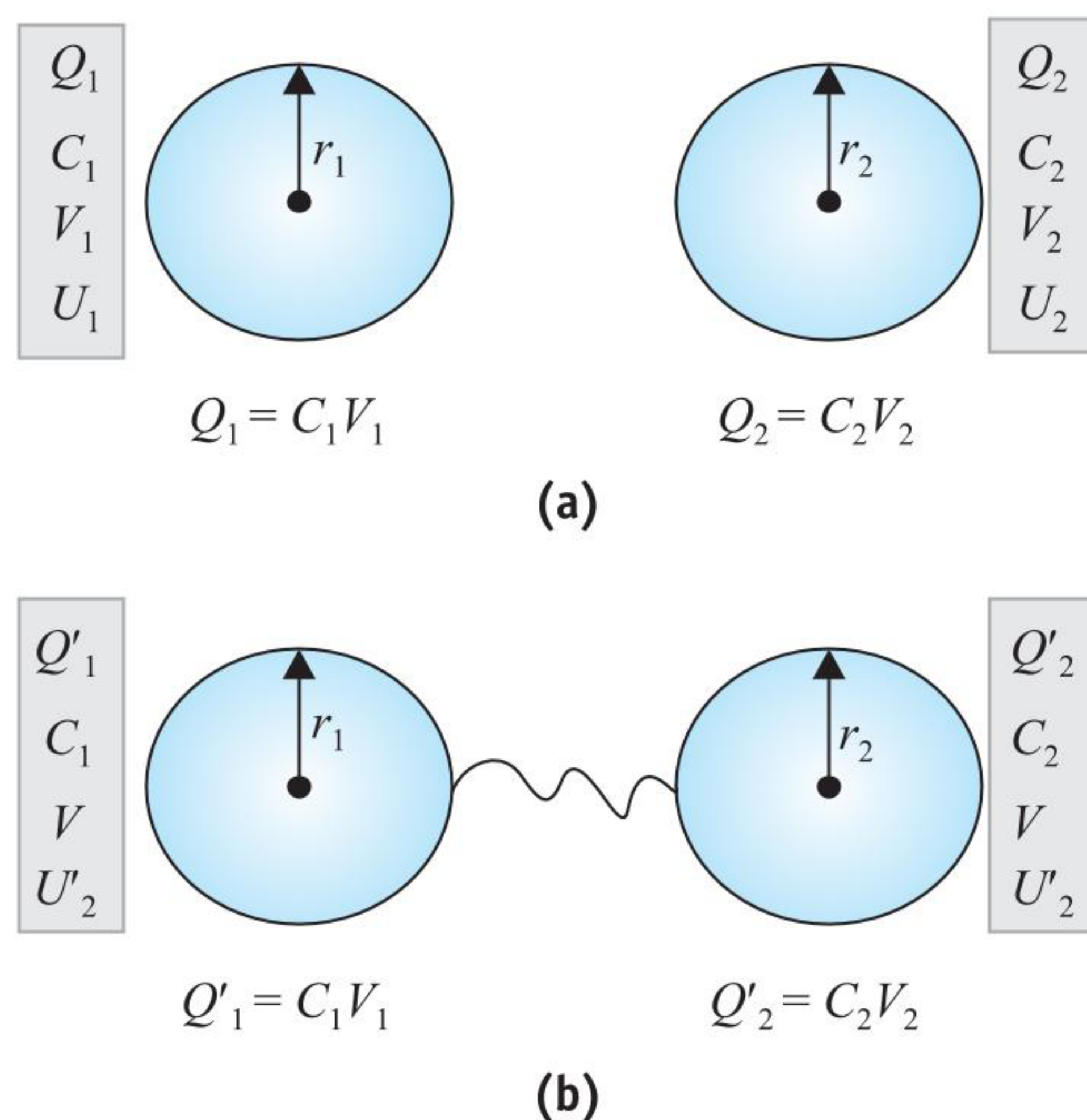
$\therefore$ Total energy of the system, $U = U_1 + U_2 + U_{12}$

$$\Rightarrow U = \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d} \right)$$

$$\text{As, } U = \frac{q^2}{2C} \text{ hence, } C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d} \right)}$$

SHARING OF CHARGE

When two conductors are joined together through a conducting wire, charge begins to flow from one conductor to another till both have the same potential. Due to the flow of charge, loss of energy also takes place in the form of heat.



Suppose there are two spherical conductors of radii r_1 and r_2 having charges Q_1 and Q_2 , potentials V_1 and V_2 , energies U_1 and U_2 , and capacitances C_1 and C_2 , respectively, as shown in figure. If these two spheres are connected through a conducting wire, then alteration of charge, potential, and energy takes place.

New charge: According to the conservation of charge $Q_1 + Q_2 = Q'_1 + Q'_2 = Q$ (say). Also

$$\frac{Q'_1}{Q'_2} = \frac{C_1V}{C_2V} = \frac{4\pi\epsilon_0 r_1}{4\pi\epsilon_0 r_2} \quad \text{or} \quad \frac{Q'_1}{Q'_2} = \frac{r_1}{r_2}$$

$$\text{or} \quad 1 + \frac{Q'_1}{Q'_2} = 1 + \frac{r_1}{r_2} \quad \text{or} \quad \frac{Q'_1 + Q'_2}{Q'_2} = \frac{r_1 + r_2}{r_2}$$

$$\text{or} \quad Q'_2 = Q \left[\frac{r_2}{r_1 + r_2} \right]$$

Similarly

$$Q'_1 = Q \left[\frac{r_1}{r_1 + r_2} \right]$$

Common potential:

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q'_1 + Q'_2}{C_1 + C_2}$$

$$= \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Energy loss: As the electrical energies stored in the system before and after connecting the spheres are

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$\text{and } U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2) \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2} \right)^2$$

$$\text{So energy loss is } \Delta U = U_i - U_f = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

Capacity of a conductor is a constant term; it does not depend upon the charge (Q), potential (V), and nature of the material of the conductor.

ILLUSTRATION 4.9

Two conducting spheres of radii 6 cm and 12 cm each, having the same charge 3×10^{-8} C, are kept very far apart. If the spheres are connected to each other by a conducting wire, find

- the direction and amount of charge transferred and
- final potential of each sphere

Sol.

- As equal charge is given to both the spheres, the potential $V (\propto q/R)$ of the smaller sphere will be higher and, hence, charge will flow from the smaller to the larger sphere when they are connected. Now, as charge is shared in proportion to capacity, and the capacity of the spherical conductor is proportional to its radius,

$$q'_1 = \frac{R_1}{R_1 + R_2} q = \frac{6}{6 + 12} (3 + 3) \times 10^{-8} = 2 \times 10^{-8} \text{ C}$$

$$\text{and } q'_2 = \frac{R_2}{R_1 + R_2} q = \frac{12}{6 + 12} (3 + 3) \times 10^{-8} = 4 \times 10^{-8} \text{ C}$$

So charge transferred is

$$(q_1 - q'_1) = (q'_2 - q_2) = 1 \times 10^{-8} \text{ C}$$

- After sharing the final potential of each sphere,

$$V'_1 = \frac{q'_1}{C_1} = \frac{q'_1}{4\pi\epsilon_0 R_1} = \frac{9 \times 10^9 \times 2 \times 10^{-8}}{6 \times 10^{-2}} = 3 \text{ kV}$$

$$\text{and } V'_2 = \frac{q'_2}{C_2} = \frac{q'_2}{4\pi\epsilon_0 R_2} = \frac{9 \times 10^9 \times 4 \times 10^{-8}}{12 \times 10^{-2}} = 3 \text{ kV}$$

So after sharing, both the conductors are at the same potential, i.e., $V'_1 = V'_2 = V = 3 \text{ kV}$

ILLUSTRATION 4.10

A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge $q = 20 \mu\text{C}$ is given to the inner sphere. Find the heat generated in the process. The inner sphere is connected to the shell by a conducting wire.

Sol. Before connection with the wire, the total electrical energy is $U_i = q^2/8\pi\epsilon_0 a$; after connection with the wire, all charges are transferred to the outer sphere. So

$$U_f = \frac{q^2}{8\pi\epsilon_0 b}$$

$$\therefore \Delta H = U_i - U_f = \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$= \frac{(20 \times 10^{-6})^2 \times 9 \times 10^9}{2} \times \left(\frac{1}{10 \times 10^{-2}} - \frac{1}{20 \times 10^{-2}} \right)$$

$$= 9 \text{ J}$$

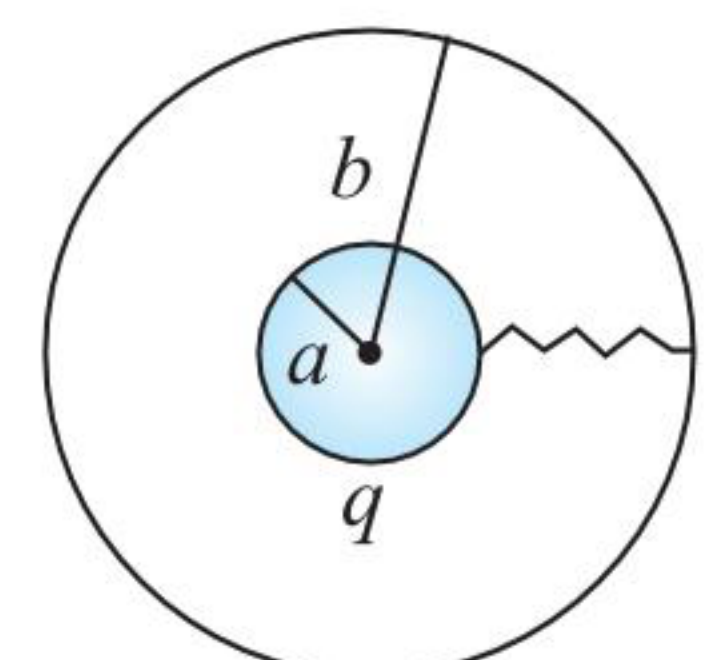


ILLUSTRATION 4.11

An isolated conductor, initially free from charge, is charged by repeated contacts with a plate, which after each contact has a charge Q due to some mechanism. If q is the charge on the conductor after the first operation, prove that the maximum charge that can be given to the conductor in this way is $Qq/(Q - q)$.

Sol. At each contact, the potential of both becomes same. At the first contact, charges on the conductor and the plate are $Q - q$ and q , respectively. So

$$V_1 = \frac{Q - q}{C_1} = \frac{q}{C_2} \quad \text{or} \quad \frac{C_1}{C_2} = \frac{Q - q}{q} \quad \dots(i)$$

In the second contact, the charge transferred to the plate is q_2 . Then

$$V_2 = \frac{Q - q_2}{C_1} = \frac{q + q_2}{C_2} \quad \text{or} \quad \frac{Q - q}{q} = \frac{Q - q_2}{q + q_2}$$

$$\text{or} \quad (q + q_2)(Q - q) = q(Q - q_2)$$

$$\text{or} \quad qQ - q^2 + q_2Q - q_2q = qQ - qq_2$$

$$\text{or} \quad q_2Q = q^2$$

$$\therefore q_2 = \frac{q^2}{Q}$$

The total charge transferred in a large number of contacts is

$$\begin{aligned} q_{\max} &= q + \frac{q^2}{Q} + \frac{q^3}{Q^2} + \frac{q^4}{Q^3} + \dots + \infty \\ &= \frac{q}{1 - \frac{q}{Q}} = \frac{qQ}{Q - q} \end{aligned}$$

DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS

Two capacitors C_1 and C_2 are connected as shown in figure.

Common potential: By charge conservation of plates A and C before and after connection,

$$Q_1 + Q_2 = C_1V + C_2V$$

So common potential is

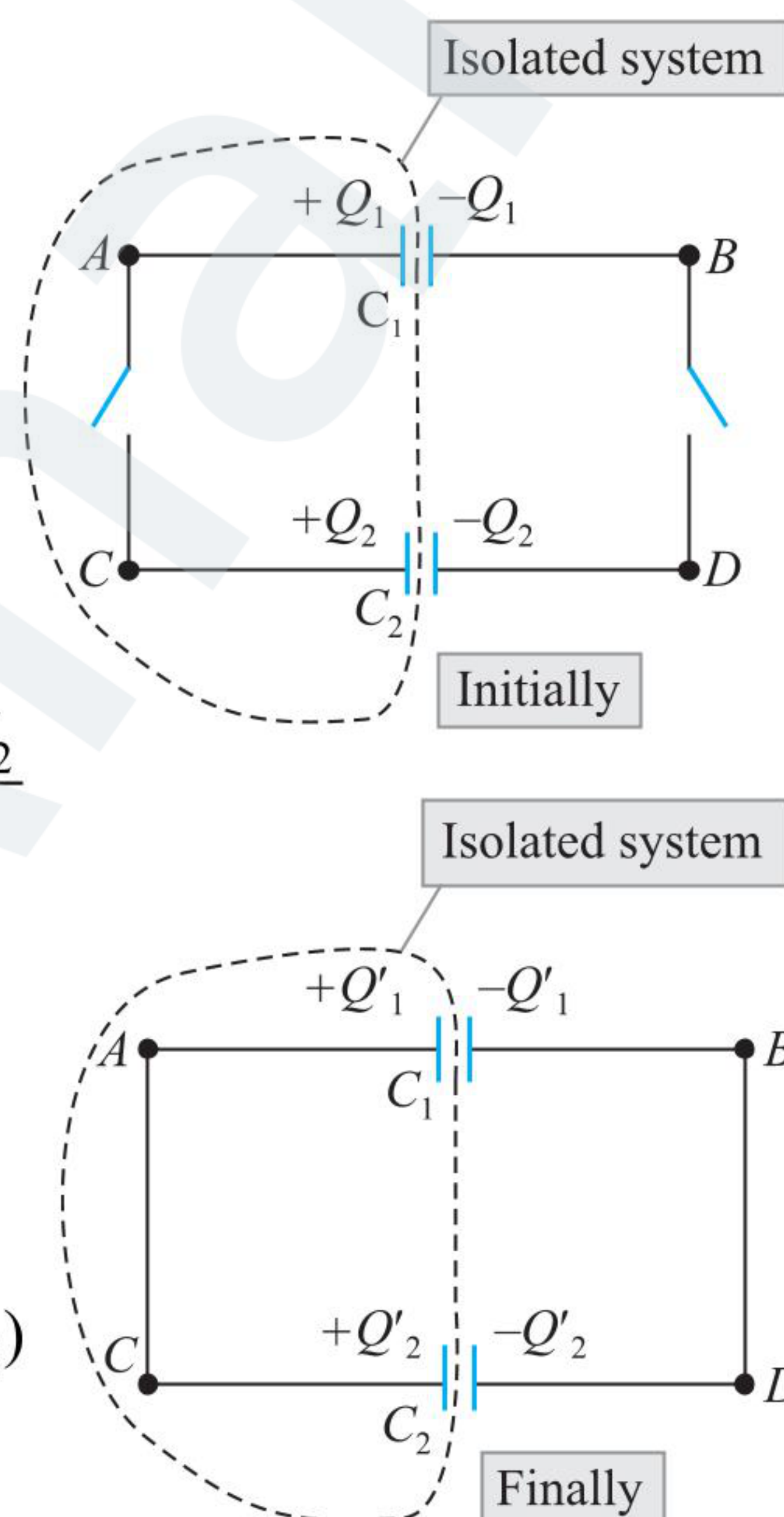
$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$= \frac{\text{Total charge}}{\text{Total capacitance}}$$

Also

$$Q'_1 = C_1V = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2)$$

$$Q'_2 = C_2V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

**Heat loss during redistribution:**

$$\Delta H = U_i - U_f = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

The loss of energy is in the form of heating in the wire.

Notes:

- When plates of similar charges are connected with each other (+ with + and - with -), put all values (Q_1 , Q_2 , V_1 , and V_2) with positive sign.
- When plates of opposite polarity are connected with each other (+ with -), take the charge and potential of one of the plates to be negative.

ILLUSTRATION 4.12

Two capacitors $C_1 = 2 \mu\text{F}$ and $C_2 = 3 \mu\text{F}$ are charged separately to potentials 20 V and 10 V, respectively. The terminals of capacitors C_1 and C_2 are marked as (A-B) and (C-D), respectively. A is connected with C and B is connected with D.

- Find the final potential difference across each capacitors.
- Find the final charge in both capacitors.
- How much heat is produced in the circuit

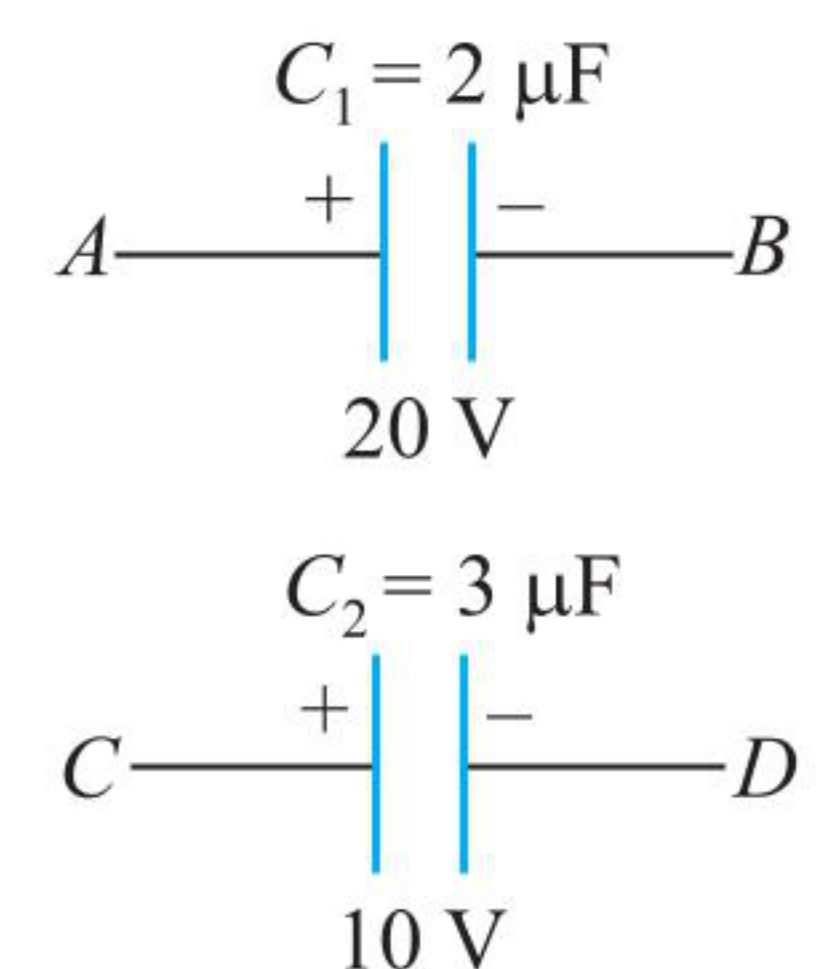
Sol.

- Initial charge in capacitor C_1 is

$$\begin{aligned} (Q_1)_{\text{initial}} &= C_1 V_1 \\ &= 2 \times 20 \\ &= 40 \mu\text{C} \end{aligned}$$

- Initial charge in capacitor C_2 is

$$\begin{aligned} (Q_2)_{\text{initial}} &= C_2 V_2 \\ &= 3 \times 10 \\ &= 30 \mu\text{C} \end{aligned}$$



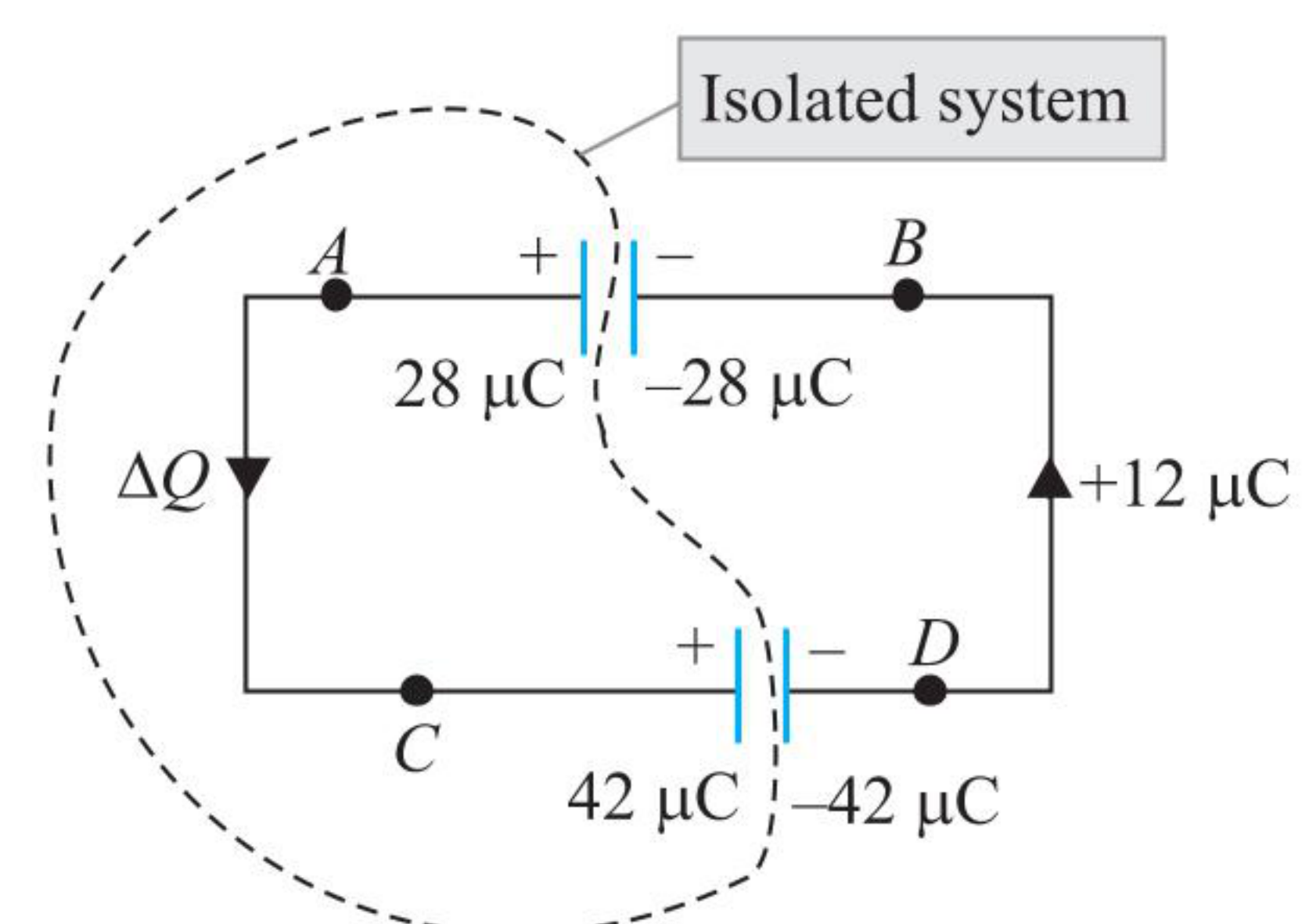
Initially capacitors C_1 and C_2 are separately charged

Let the potential of B and D be zero and the common potential difference across the capacitors be V , then the potentials at A and C will be V .

From figure, it is clear that the left plates of capacitors C_1 and C_2 are forming an isolated system, i.e., they are not connected from outside.

From charge conservation,

$$\begin{aligned} C_1 V + C_2 V &= 3V + 2V = 40 + 30 \\ 5V &= 70 \quad \text{or} \quad V = 14 \text{ V} \end{aligned}$$



Final charge in capacitor C_1 is

$$(Q_1)_{\text{final}} = 2 \times 14 = 28 \mu\text{C}$$

Final charge in capacitor C_2 is

$$(Q_2)_{\text{final}} = 3 \times 14 = 42 \mu\text{C}$$

The charge flowing in the circuit in the direction from A to C is

$$\Delta Q = 40 - 28 = 12 \mu\text{C}$$

Now final charges on each plate are shown in figure.

(ii) Heat produced in the circuit is

$$\begin{aligned} H &= \left[\frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \right] - \left[\frac{1}{2} (C_1 + C_2) V^2 \right] \\ &= \left[\frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2 \right] - \left[\frac{1}{2} \times 5 \times (14)^2 \right] \\ &= 400 + 150 - 490 = 550 - 490 = 60 \mu\text{J} \end{aligned}$$

ILLUSTRATION 4.13

If A is connected with D and B is connected with C , find the potential difference across each capacitor and the final charge in each capacitor.

Sol. Let the potential of B and C be zero and the common potential on the capacitors be V , then at A and D , it will be V . From charge conservation,

$$\begin{aligned} C_1 V + C_2 V &= 3V + 2V \\ &= 40 - 30 = 10 \end{aligned}$$

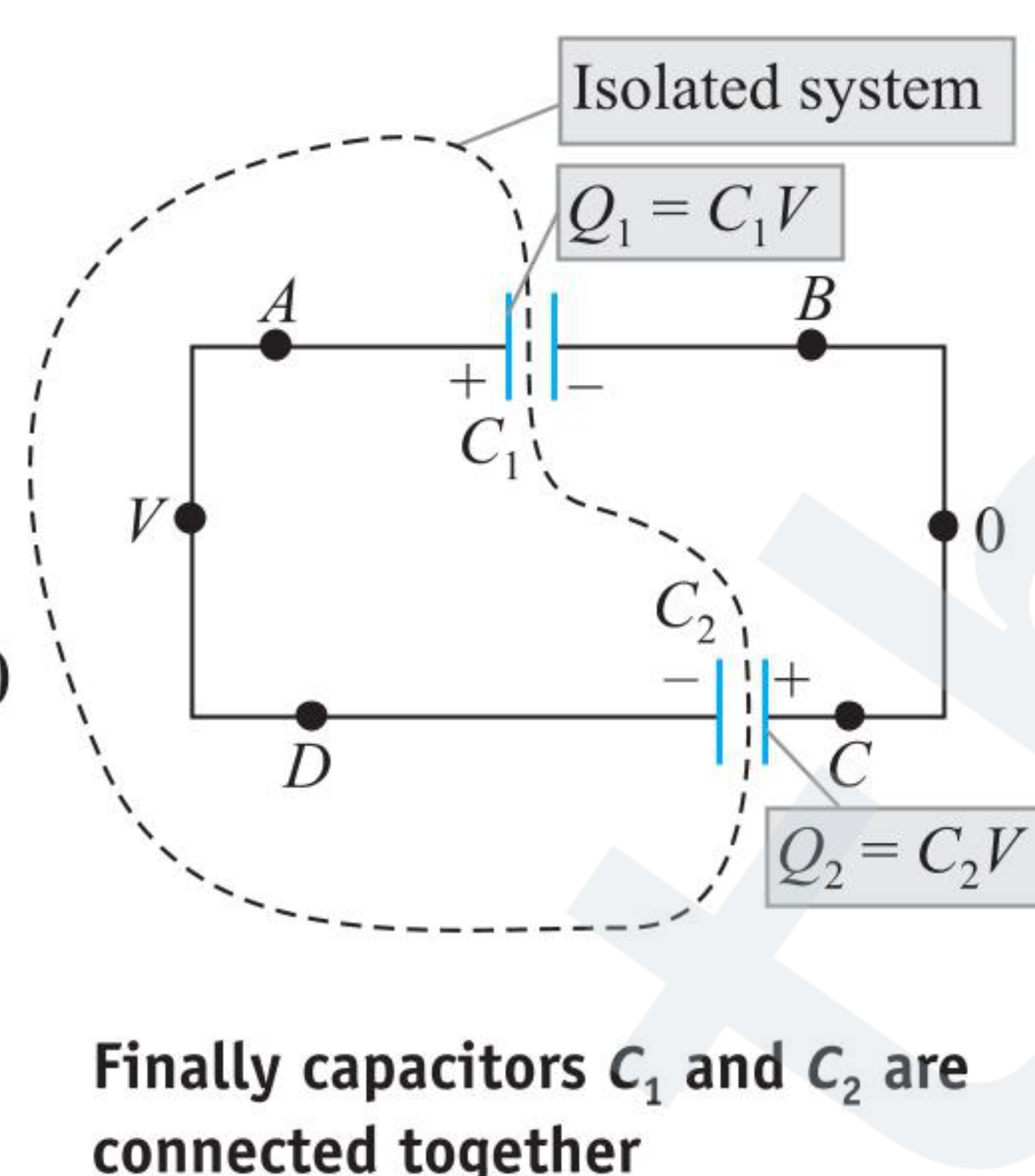
$$\text{or } V = 2 \text{ V}$$

Final charge in capacitor C_1 is

$$(Q_1)_{\text{final}} = 2 \times 2 = 4 \mu\text{C}$$

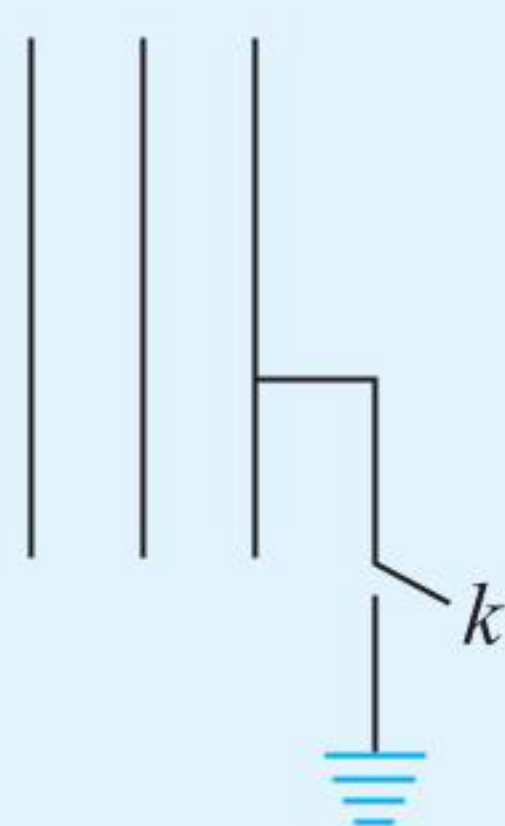
Final charge in capacitor C_2 is

$$(Q_2)_{\text{final}} = 3 \times 2 = 6 \mu\text{C}$$



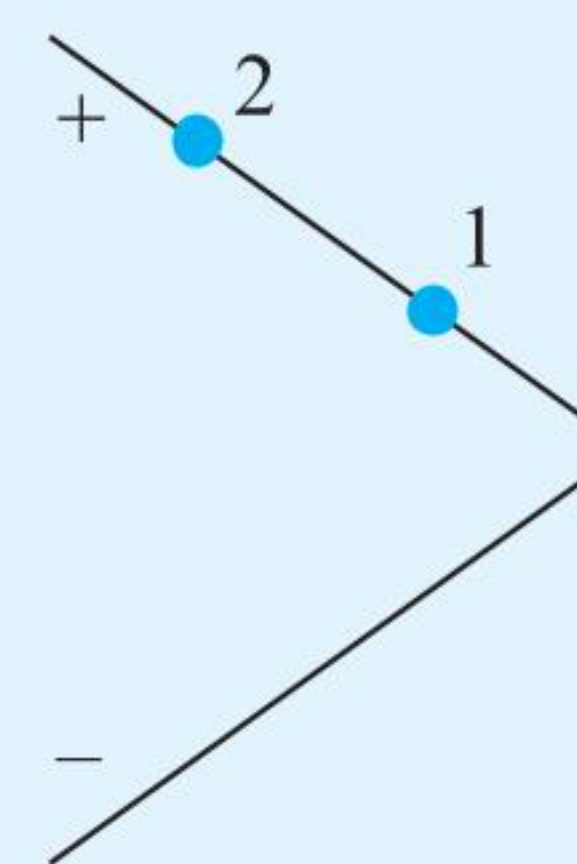
CONCEPT APPLICATION EXERCISE 4.1

- (a) How many excess electrons must be added to one plate and removed from the other to give a 5.00 nF parallel plate capacitor 25.0 J of stored energy?
(b) How could you modify the geometry of this capacitor so that it can store 50.0 J of energy without changing the charge on its plates?
- A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. Find the potential difference across the capacitor.
- Three identical large metallic plates are placed parallel to each other at a very small separation as shown in figure. The central plate is given a charge Q . What amount of charge will flow to earth when the key is pressed?
- The plates of a plane capacitor are drawn apart keeping them connected to a battery. Next, the same plates are drawn apart from the same initial condition keeping the battery disconnected. In which case is more work done?

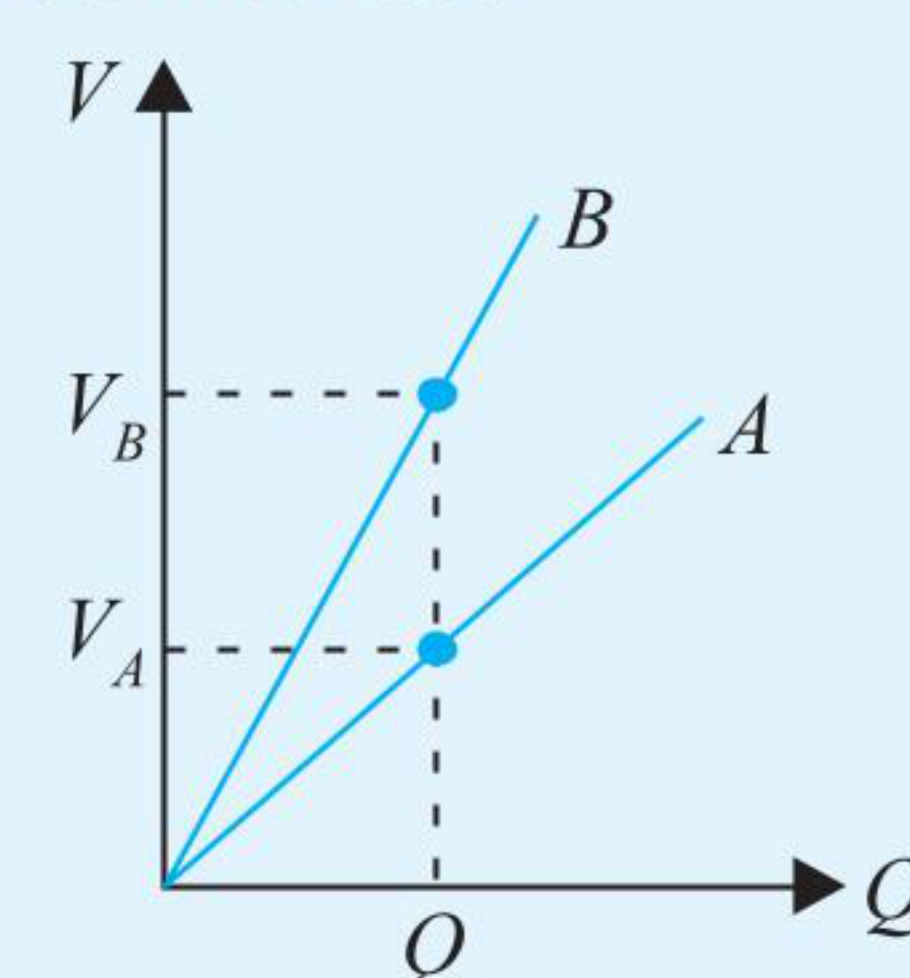


5. A parallel plate air capacitor is connected to a battery. If the plates of the capacitor are pulled farther apart, then state whether the following statements are true or false.

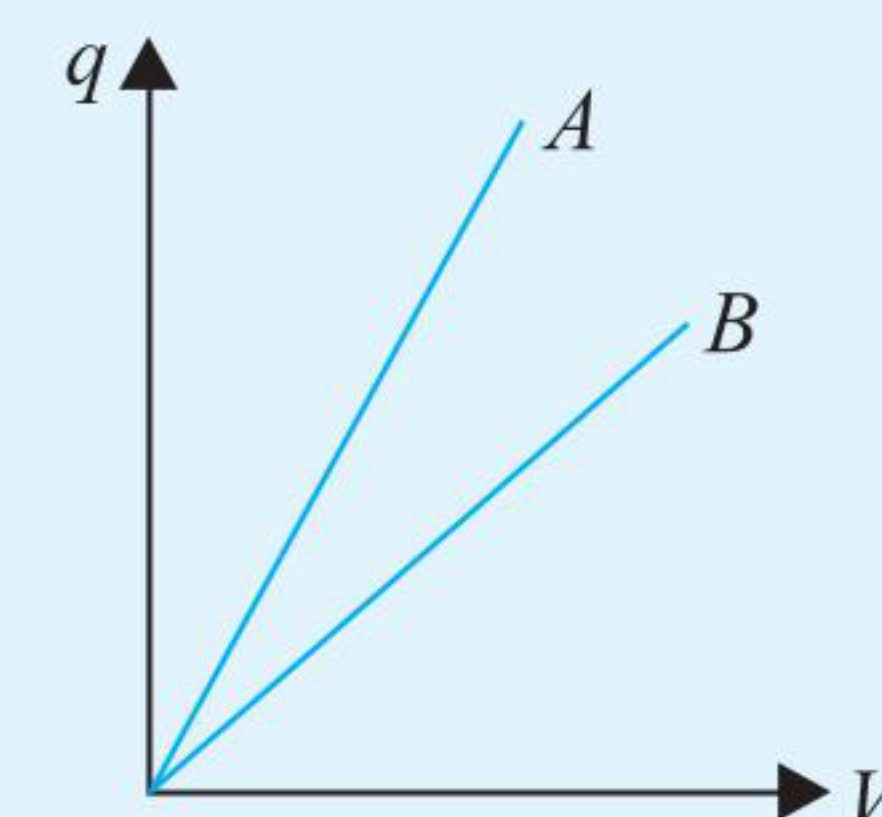
- Strength of the electric field inside the capacitor remains unchanged, if the battery is disconnected before pulling the plates.
- During the process, work is done by the external force applied to pull the plates irrespective of whether the battery is disconnected or not.
- Electrostatic potential energy in the capacitor decreases if the battery remains connected.



6. Figure shows the variation of voltage V across the plates of two capacitors A and B versus increase in charge Q stored in them. Which of the capacitors has higher capacitance? Give reason for your answer.



7. Figure shows the variation of charge q versus the potential difference V for two capacitors C_1 and C_2 . The two capacitors have the same plate separation, but the plate area of C_2 is double than that of C_1 . Which of the lines in the figure corresponds to C_1 and C_2 and why?



8. A capacitor of capacitance C is charged to a potential difference V_0 . The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $C/2$.

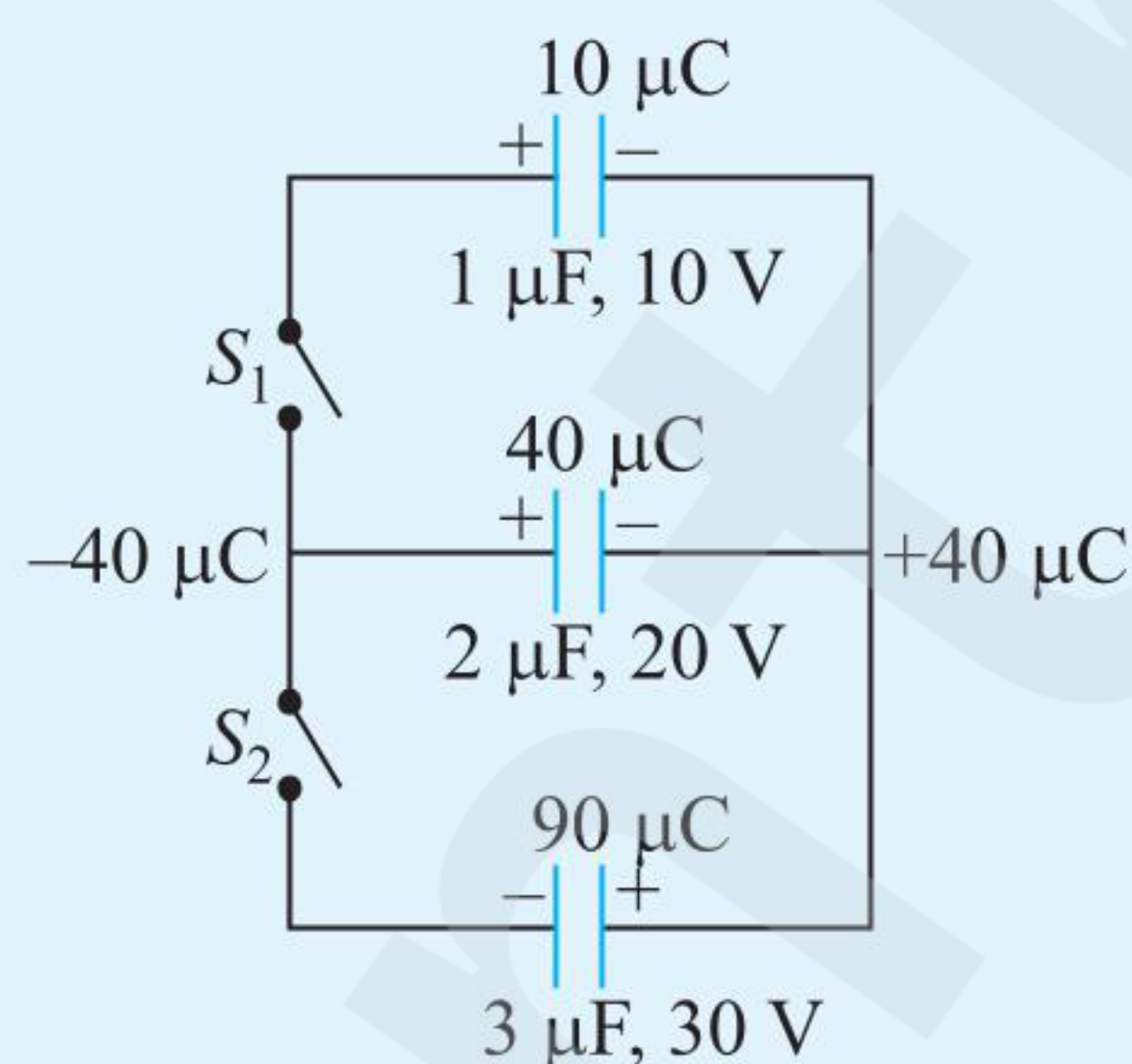
- Compute the original charge of the system.
- Find the final potential difference across each capacitor.
- Find the final energy of the system.
- Calculate the decrease in energy when the capacitors are connected.
- Where did the “lost” energy go?

9. A parallel plate vacuum capacitor with plate area A and separation x has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed.

- What is the total energy stored in the capacitor?
- The plates are pulled apart an additional distance dx . What is the change in the stored energy?
- If F is the force with which the plates attract each other, then the change in the stored energy must equal the work $dW = Fdx$ done in pulling the plates apart. Find an expression for F .
- Explain why F is not equal to QE , where E is the electric field between the plates?

10. A capacitor of capacitance C , which is initially charged up to a potential difference $\mathcal{E}$, is connected with a battery of emf $\mathcal{E}$ such that the positive terminal of the battery is connected with the positive plate of the capacitor. Find the heat loss in the circuit during the process of charging.

11. In the above question, if the positive terminal of the battery is connected with the negative plate of the capacitor, find the heat loss in the circuit during the process of charging.
12. A radioactive source in the form of a metal sphere of diameter 10^{-3} m emits β particles at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long will it take (in μ s) for its potential to rise by 1.0 volt, assuming that 80% of emitted β particles escape from the surface.
13. The plates of a capacitor are charged to a potential difference of 100 V and then connected across a resistor. The potential difference across the capacitor decays exponentially with respect to time. After one second the potential difference between the plates of the capacitor is 80 V. What is the fraction of the stored energy which has been dissipated?
14. A parallel plate capacitor of capacitance C_0 is charged using a battery of emf V_0 . Calculate the work done by external agent in reducing the separation between the plates to half its original value if
 - (a) The battery is disconnected before you start decreasing the plate separation.
 - (b) The battery remains connected while you are reducing the separation.
 (Assume that the plates were moved very slowly)
15. Three capacitors of capacities $1 \mu\text{F}$, $2 \mu\text{F}$ and $3 \mu\text{F}$ are charged by 10 V, 20 V and 30 V respectively. Now, positive plates of first two capacitors are connected with the negative plate of third capacitor on one side and negative plates of first two capacitors are connected with positive plate of third capacitor on the other side. Find
 - (a) common potential V
 - (b) final charges on different capacitors



ANSWERS

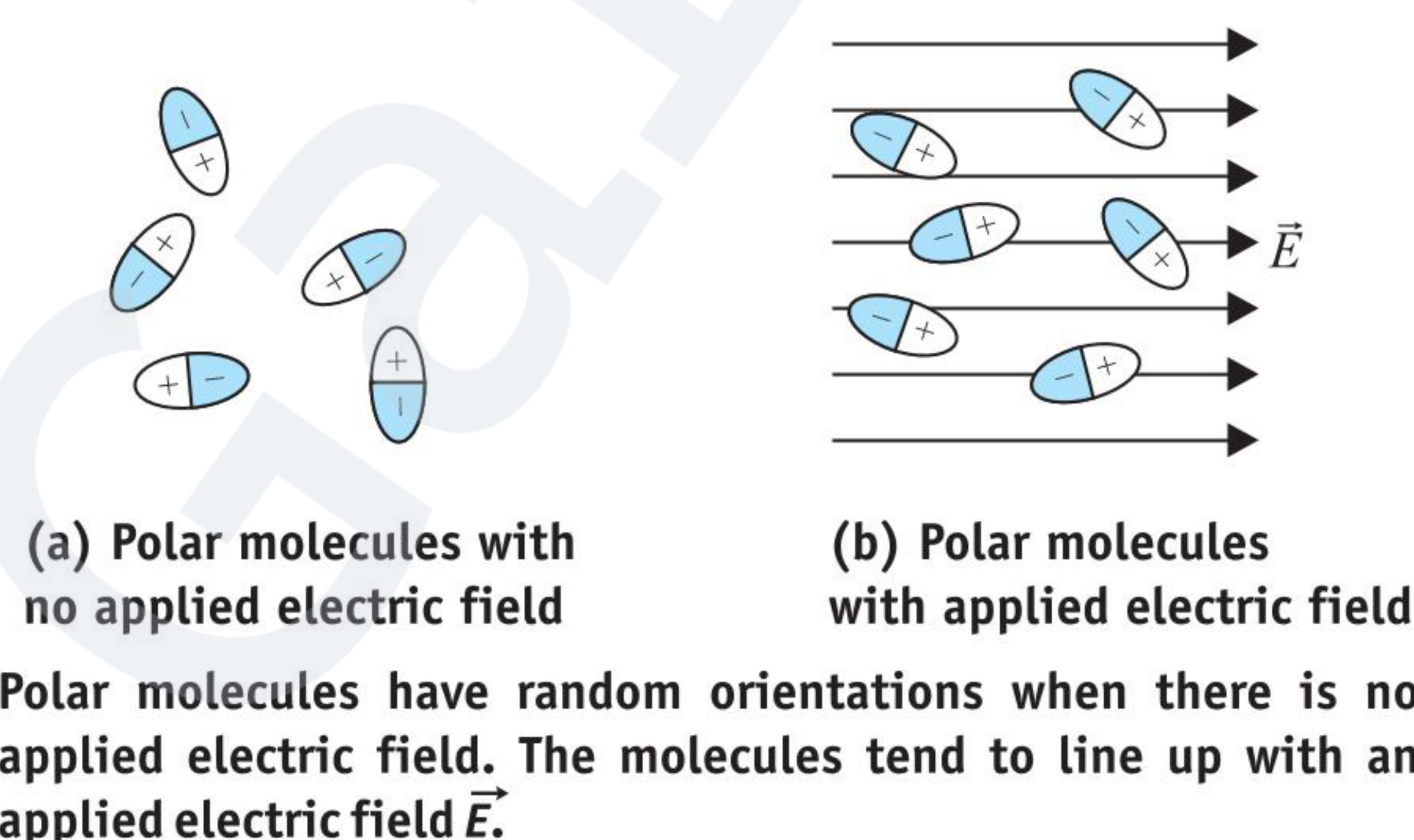
1. (a) 3.125×10^5 (b) Half the plate area or double the separation between the plates
2. $V + \frac{Q}{2C}$ 3. $+Q$ 4. When battery disconnected (second case)
5. (a) True (b) True (c) True 6. Capacitor 'A'
7. Line A: Capacitor C_2 , Line B: Capacitor C_1
8. (a) CV_0 (b) $\frac{2}{3}V_0$ (c) $\frac{1}{3}CV_0^2$ (d) $\frac{1}{6}CV_0^2$
9. (a) $\frac{xQ^2}{2\epsilon_0 A}$ (b) $\left(\frac{Q^2}{2\epsilon_0 A}\right)dx$ (c) $\frac{Q^2}{2\epsilon_0 A}$ 10. Zero
11. $2\epsilon^2 C$ 12. $6.95 \mu\text{s}$ 13. $\frac{9}{25}$ 14. (a) $-\frac{1}{4}C_0V_0^2$ (b) $-\frac{1}{2}C_0V_0^2$
15. (a) $\frac{20}{3}$ volt (b) $q_1 = \frac{20}{3}\mu\text{C}$, $q_2 = \frac{40}{3}\mu\text{C}$, $q_3 = 20 \mu\text{C}$

DIELECTRIC

A dielectric is a nonconductor up to a certain value of the field depending upon its nature. If the field exceeds the limiting value, called dielectric strength, a dielectric loses its insulating property and begins to conduct. Dielectrics are of two types:

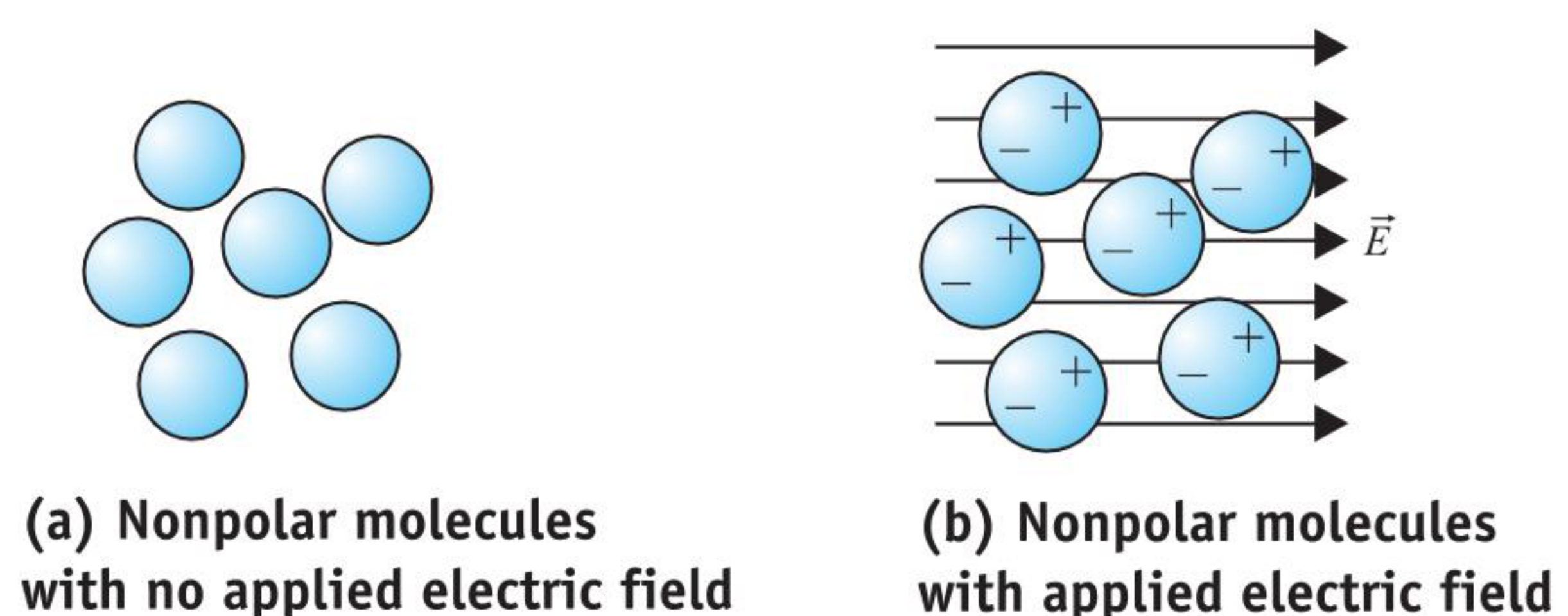
Polar dielectrics: In polar molecules when no electric field is applied, the center of positive charges does not coincide with the center of negative charges.

A polar molecule has a permanent electric dipole moment ($\vec{p}$) in the absence of an electric field. But a polar dielectric has a net dipole moment equal to zero in the absence of an electric field because polar molecules are randomly oriented as shown in figure.



In the presence of an electric field, polar molecules tend to line up in the direction of the electric field, and the substance has a finite dipole moment.

Nonpolar dielectrics: In nonpolar molecules, when no electric field is applied, the center of the positive charge coincides with the center of the negative charge in the molecule. Each molecule has zero dipole moment in its normal state.



Nonpolar molecules have their positive and negative charge centers at the same point. These centres become separated slightly by an applied electric field $\vec{E}$.

When an electric field is applied, the positive charge experiences a force in the direction of the electric field and negative charge experiences a force in the direction opposite to the field, i.e., molecules become induced electric dipole.

Note: In general, any nonconducting material can be called a dielectric, but nonconducting materials having nonpolar molecules are referred to as dielectric because an induced dipole moment is created in the nonpolar molecule.

POLARIZATION VECTOR

We know that under the action of an electric field, a net dipole moment appears in the dielectrics. Microscopically (for an atom or molecule) this effect may be very small but for the whole specimen this effect is significant inducing a considerable dipole

moment because millions of dipoles orient in same direction. As a result, equal positive and negative bound charges appear at the surface of the dielectric. This effect is called polarization which is given by a vector

$$\vec{P} = \frac{n\vec{p}}{V},$$

where n = total number of dipoles and p = dipole moment of each atom.

When a dielectric is subjected to an electric field a net dipole moment appears in it. As a result, an equal and opposite bound charge appears at the opposite surfaces of the dielectric which is called polarization. Polarization is defined as a vector

$\vec{P} = \frac{\text{net dipole moment}}{\text{volume of the dielectric}}$ that is, dipole moment per unit volume.

RELATION BETWEEN POLARISATION VECTOR AND SURFACE CHARGE DENSITY

When a rectangular dielectric slab is placed in a uniform electric field it gets polarized. In consequence, equal and opposite bound charges $-Q$ and Q appear on the faces lying perpendicular to the field $\vec{E}$.

The net dipole moment of the slab is given as $p_{\text{net}} = np$, where n = total number of dipoles and p = dipole moment of each dipole. Substituting $p_{\text{net}} = Ql$, we have $Ql = np$. Dividing both sides with the volume of the specimen, we have

$$\frac{Ql}{Al} = \frac{np}{Al} \quad \text{or} \quad \frac{Q}{A} = \left(\frac{np}{Al} \right)$$

$$\text{Here } \frac{np}{Al} = \frac{\text{total dipole moment}}{\text{total volume}} = P$$

and $\frac{Q}{A} = \sigma_b$ (surface bound charge density)

Then, $P = \sigma_b$

Hence the polarization vector $\vec{P}$ is defined as dipole moment per unit volume which is numerically equal to surface bound charge density.

Polarization of a dielectric slab: A dielectric slab can be polarized by inducing equal and opposite charges on the two faces of the dielectric by the application of an electric field. Suppose a dielectric slab is inserted between the plates of a capacitor as shown in figure.

Induced electric field inside the dielectric is E_i ; hence, this induced electric field decreases the main field E to $E - E_i$ i.e. new electric field between the plates will be $E' = E - E_i$.

Dielectric constant: After placing a dielectric slab in an electric field, the net field is decreased in that region. If E is the original electric field and E' is the reduced electric field, $E/E' = K$, where K is called dielectric constant. K is also known as relative permittivity (ϵ_r) of the material. The value of K is always greater than one. For vacuum, there is no polarization and hence $E = E'$ and $K = 1$.

Dielectric breakdown and dielectric strength: If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as dielectric breakdown.

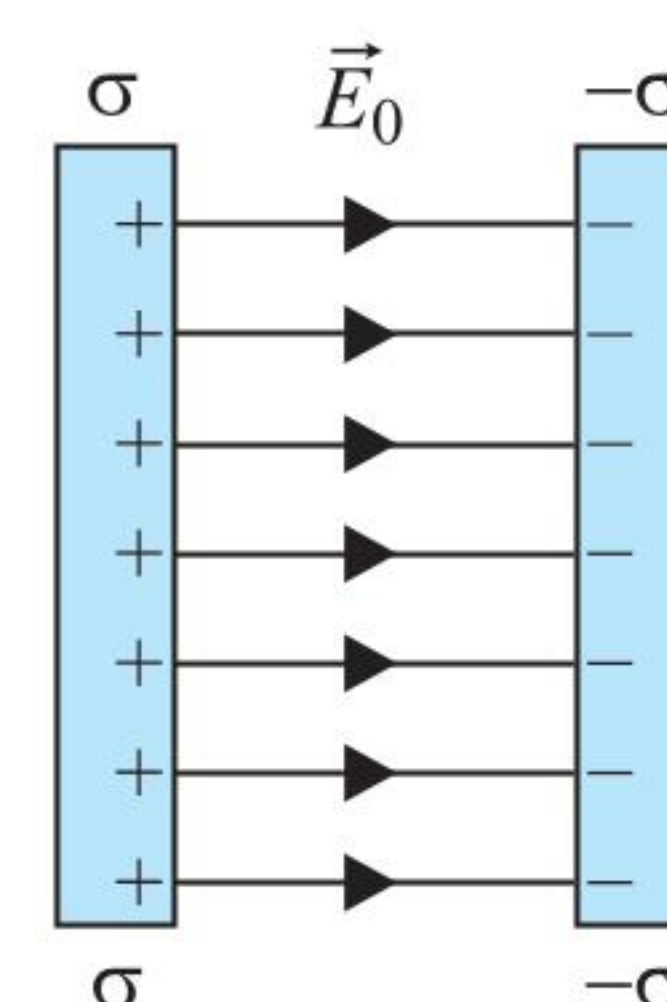
The maximum value of an electric field (or potential gradient) that a dielectric material can tolerate without electric breakdown is called its dielectric strength. SI unit of the dielectric strength of a material is Vm^{-1} .

INDUCED CHARGE ON THE SURFACE OF DIELECTRIC

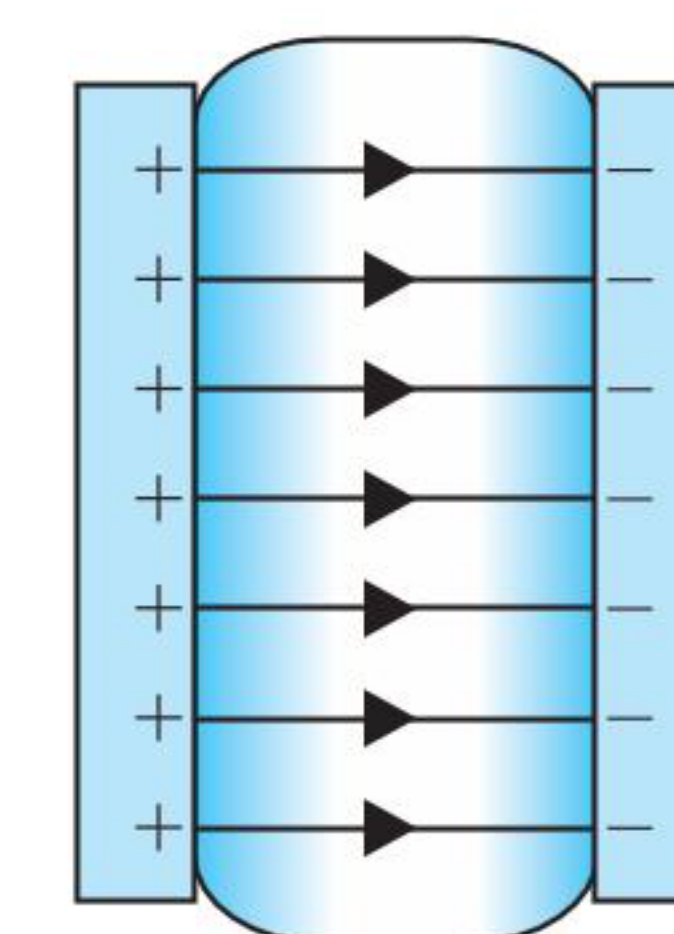
Consider an isolated, charged parallel plate capacitor with air or vacuum between its plates. Now we insert a dielectric slab, dielectric constant K , completely filling the space between the plates of the capacitor. The capacitor is not connected to the battery so charge cannot flow to or from the plates. This capacitor has a charge $+Q$ on one plate and $-Q$ on the other.

In the absence of electric field the dipole moments of polar molecules are randomly oriented and the net charge on all the dielectric's surfaces is zero. When the dielectric is placed in the electric field of the capacitor, the polar molecules experience a torque and tend to align with the electric field.

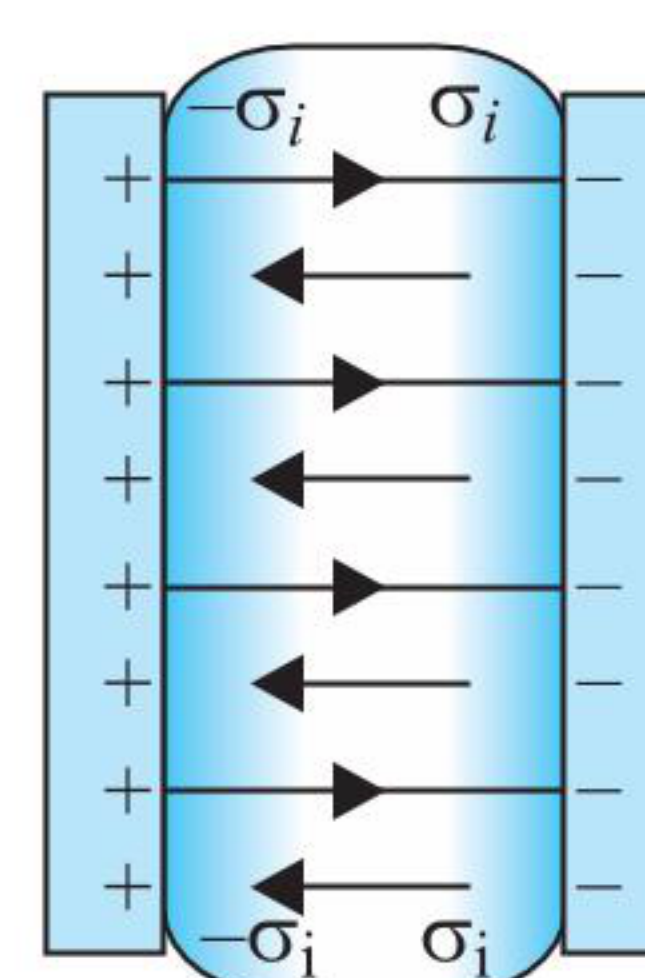
The overall effect of the dipole alignment is to make the left surface of the dielectric negative (dielectric surface facing the capacitor plate with charge $+Q$) and the right surface positive (dielectric surface facing the capacitor plate with charge $-Q$). These unbalanced charges are not free to move (charges are free in conductors only) they are tied to the molecule. These charges on the surface of the dielectric are called **bound charges**, **induced surface charges**. This bound charge density, σ_i creates an electric field of magnitude σ_i/ϵ_0 , while that due to the free charge on the capacitor plates creates a field of magnitude σ/ϵ_0 . Note that free charge is that which we place on the conducting capacitor plates by some external source (Battery) whereas bound charges appear due to polarization effects.



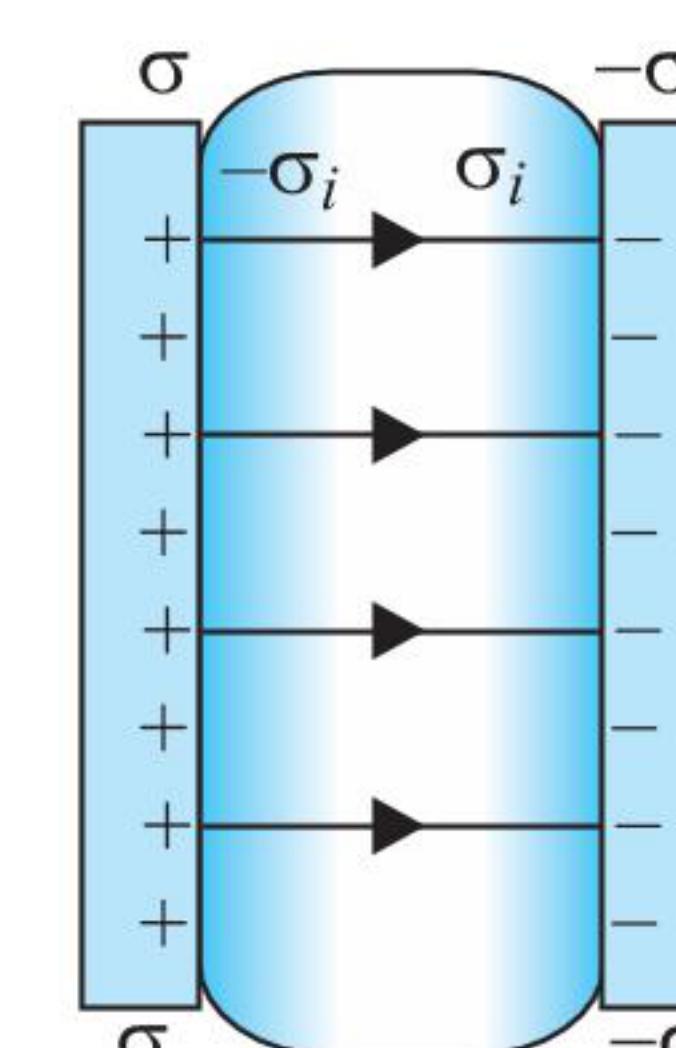
(a) Electric field of magnitude E_0 between two charged plates.



(b) Introduction of a dielectric constant K .



(c) Induced surface charges and their field (thinner lines).



(d) Resultant field of magnitude E_0/K when a dielectric is between charged plates.

The field produced by the bound charges is opposite to the original field between the plates. The total field is the superposition of the field from the charges on the conducting plates and the field caused by the bound charge on the surface of the dielectric.

Let K be the dielectric constant, E_0 be the original field in vacuum if the dielectric slab was not there, E_i be the electric field induced in the dielectric slab, and E be the net electric field in the dielectric slab. Therefore,

$$E = E_0 - E_i \quad \dots(i)$$

$$\text{and } \frac{E_0}{E} = K \quad (\text{by definition of } K \text{ of } \epsilon_r)$$

$$\text{or } E = \frac{E_0}{K} \quad \dots(ii)$$

From (i) and (ii),

$$E_0 - E_i = \frac{E_0}{K}$$

$$\text{or } E_0 K - E_i K = E_0 \quad \text{or } E_0 K - E_0 = E_i K$$

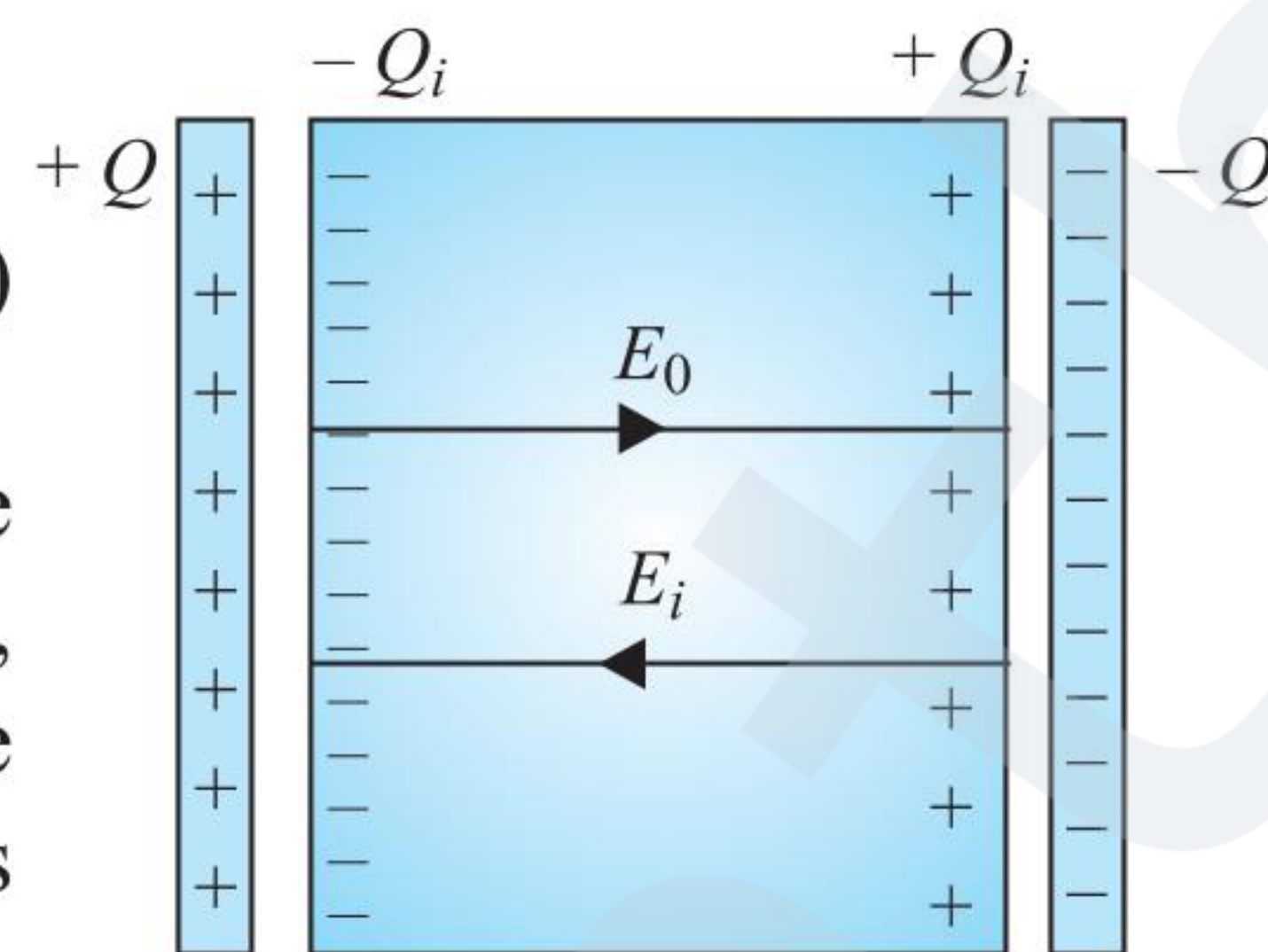
$$\text{or } E_i = \frac{K-1}{K} E_0 \quad \dots(iii)$$

$$\text{or } \frac{\sigma_i}{\epsilon_0} = \frac{K-1}{K} \frac{\sigma}{\epsilon_0} \quad \text{or } \sigma_i = \frac{K-1}{K} \sigma$$

$$\text{or } \frac{Q_i}{A} = \frac{K-1}{K} \frac{Q}{A}$$

$$\text{or } Q_i = Q \left(1 - \frac{1}{K}\right) \quad \dots(iv)$$

This is irrespective of the thickness of the dielectric slab, i.e., whether it fills up the entire space between the charged plates or any part of it.



CAPACITY OF PARALLEL PLATE CAPACITOR WITH DIELECTRIC

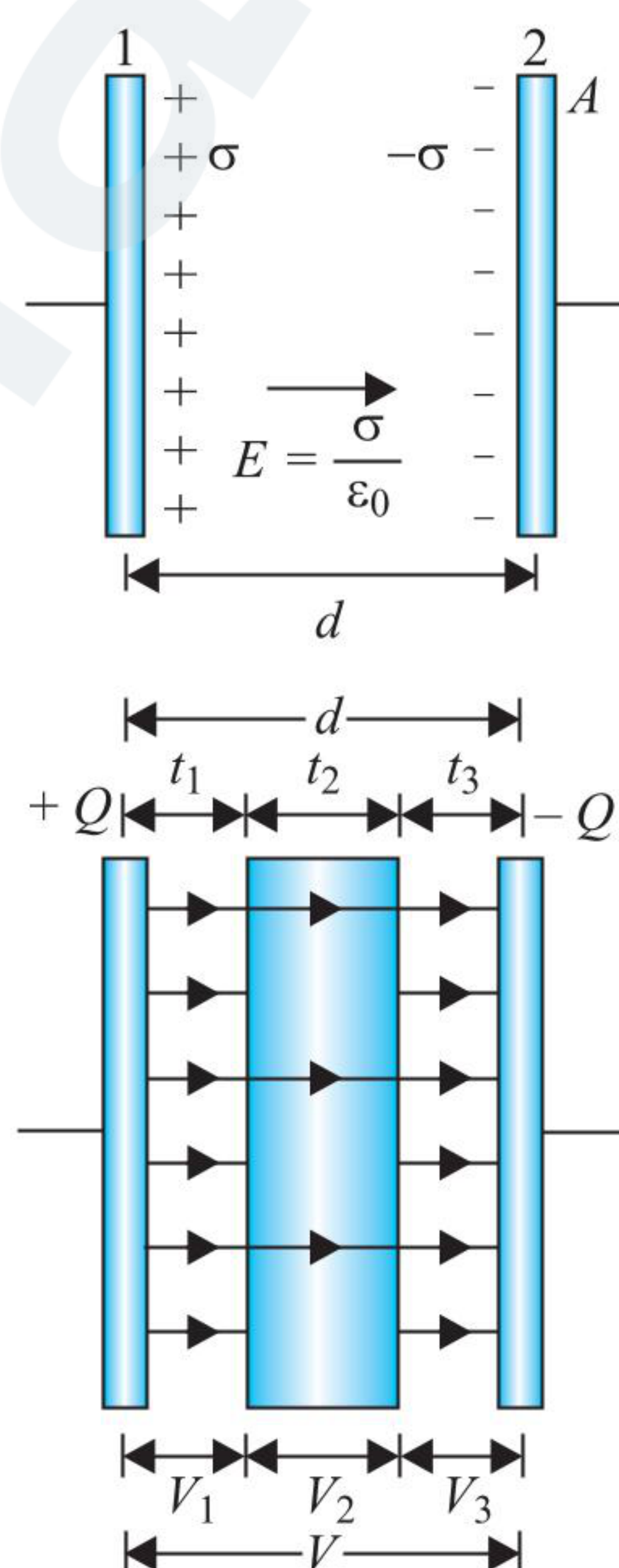
Suppose that a parallel plate capacitor has a plate area A and a separation d , and a dielectric slab of thickness t and area A is inserted between the plates. Let Q be the charge given to the capacitor plates. The electric field between the plates of the capacitor is given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

The electric field in the region of the dielectric slab is

$$E' = \frac{E}{K} = \frac{Q}{K\epsilon_0 A}$$

We know that if the electric field is constant, then the potential difference between two points separated by distance d along the field line is Ed . The potential difference between the two plates is, therefore,



$$V = V_1 + V_2 + V_3 = Et_1 + E't_2 + Et_3$$

$$= \frac{Q}{\epsilon_0 A} t_1 + \frac{Q}{K\epsilon_0 A} t_2 + \frac{Q}{\epsilon_0 A} t_3$$

$$= \frac{Q}{\epsilon_0 A} (t_1 + t_3) + \frac{Q}{K\epsilon_0 A} t_2 = \frac{Q}{\epsilon_0 A} (d - t) + \frac{Q}{K\epsilon_0 A} t$$

$$\therefore C = \frac{Q}{V_+ - V_-} = \frac{1}{\frac{(d-t)}{\epsilon_0 A} + \frac{t}{K\epsilon_0 A}} = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$$

Hence capacitance of the system will be

$$C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$$

Note:

- The capacitance in the above situation is independent of the position of the dielectric slab with respect to the plates. The capacitance depends upon the thickness of the dielectric slab and the dielectric constant.
- The dielectric constant of the conducting slab (metal plate) is infinity; therefore, the term t/K reduces to zero. If we insert a metal plate of thickness t between the plates of the capacitor having area A and separation d , the capacitance will become $C = \epsilon_0 A / (d - t)$. Also the capacitance will be independent of the position of the metal plate between the plates of the capacitor.
- If $t \ll d$, then $C = \epsilon_0 A / d$. Hence if we place a thin metal plate parallel to the plate of a capacitor, the capacitance of the capacitor remains unchanged.
- If the slab completely fills the space between the plates, then $t = d$, and therefore,

$$C = \frac{\epsilon_0 A}{d/K} = \frac{K\epsilon_0 A}{d}$$

- If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

If we place many dielectric slabs parallel to the plate of a capacitor as shown in figure, then the capacitance is given by

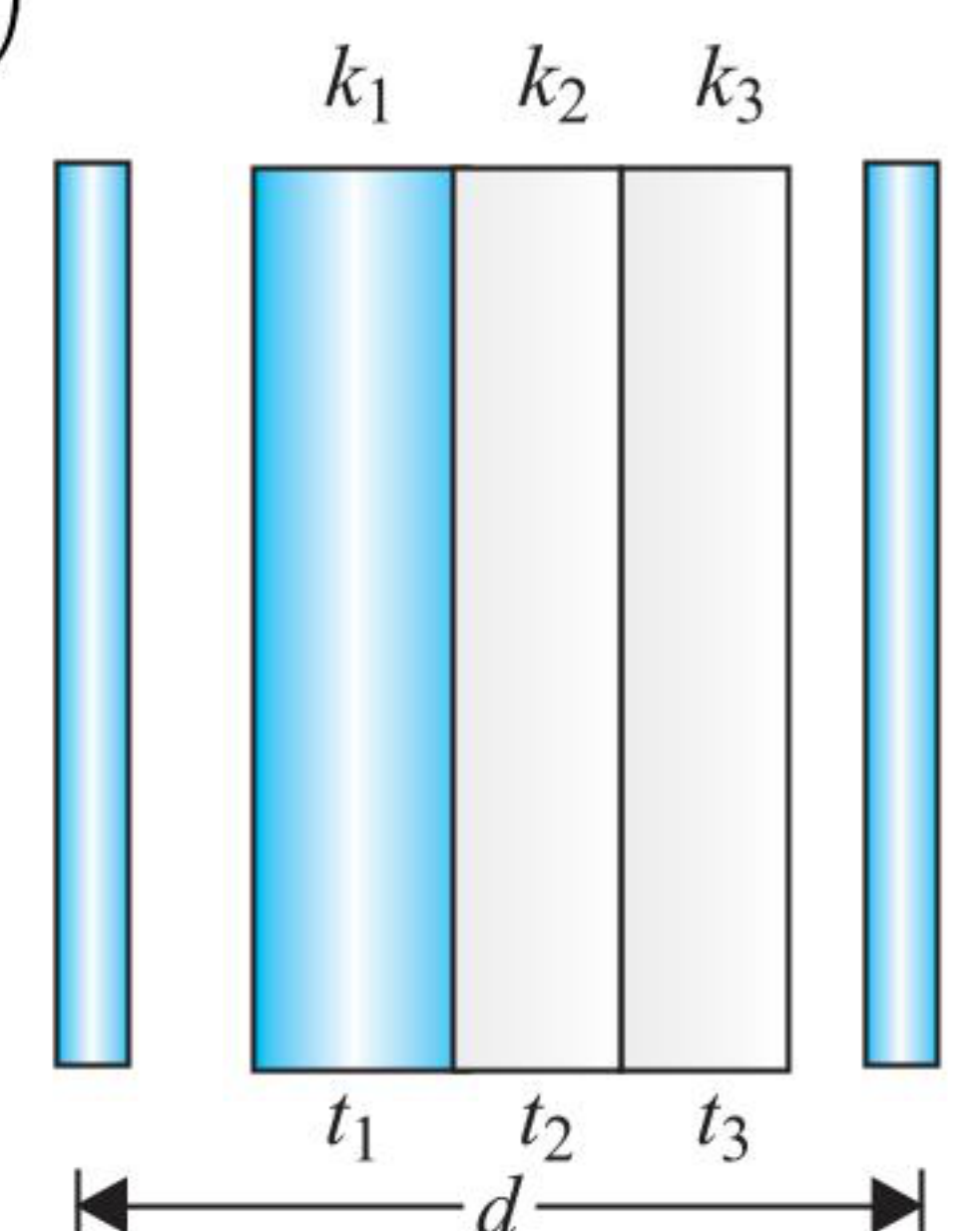
$$C = \frac{\epsilon_0 A}{d - (t_1 + t_2 + \dots) + \left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \dots \right)}$$

If we introduce a number of dielectric slabs, which completely fill the space between the plates, then

$$d = t_1 + t_2 + t_3 + \dots t_n$$

Therefore, the capacity of the capacitor will be

$$C = \frac{\epsilon_0 A}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \dots \right)}$$



EFFECT OF DIELECTRIC ON DIFFERENT PARAMETERS

Let the entire space between the plates of a capacitor be filled with a dielectric of dielectric constant K under two conditions:

(i) when the battery remains connected and (ii) after the battery is disconnected.

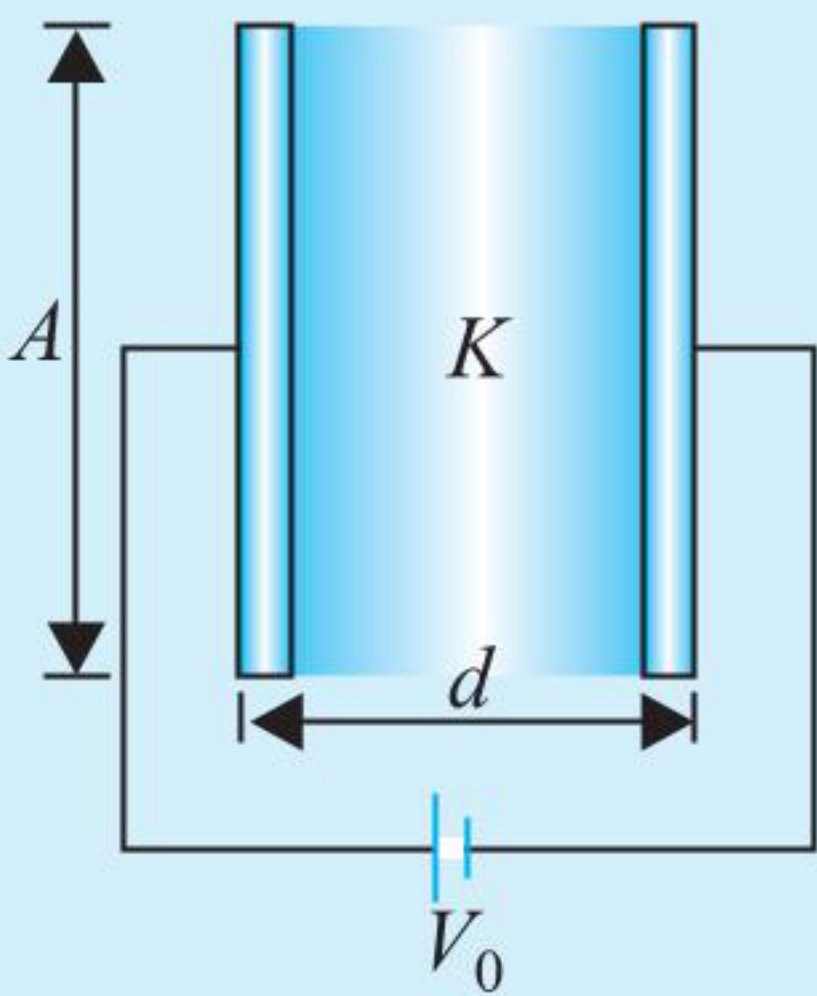
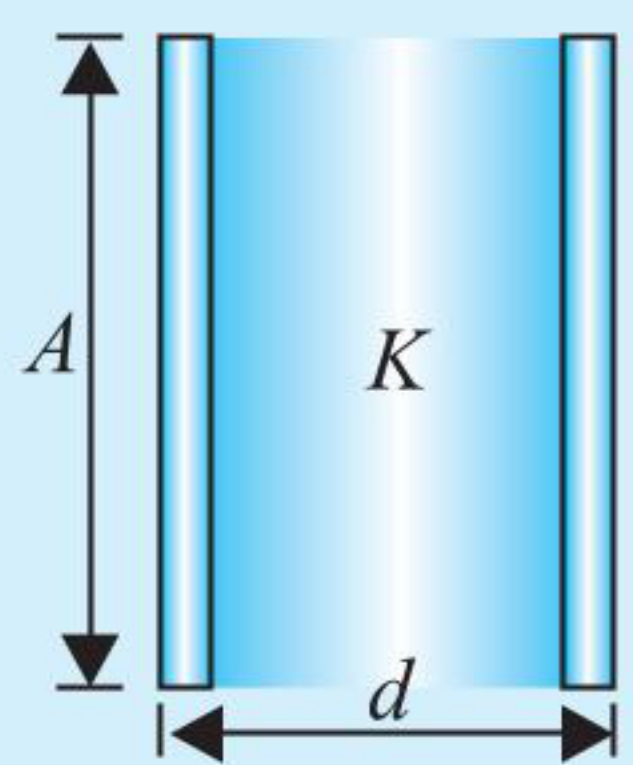
If battery remains connected	When battery is disconnected
In this case, the potential difference across the plates will remain same.  - Capacitance increases, i.e., $C = KC_0$, since capacitance depends upon geometrical factors only. - Charge on capacitor is $q = CV = KC_0V_0 = Kq_0$ (Since initially $q_0 = C_0V_0$) Thus, charge increases and becomes K times of previous charge. - Electric field is $E = \left[\frac{V}{d} \right] = \frac{V_0}{d} = E_0$ (as $V = V_0$ and $\frac{V_0}{d} = E_0$) Thus, electric field remains same. - Energy stored in the capacitor is $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2$ $= K \frac{1}{2}C_0V_0^2 = KU_0$ (as $C = KC_0$ and $U_0 = \frac{1}{2}C_0V_0^2$) Thus, energy increases and becomes K times of previous energy.	In this case, the charge on the plates will remain same, i.e., $q = q_0$, since in an isolated system, charge is conserved.  - Capacitance increases, i.e., $C = KC_0$, since capacitance depends upon geometrical factors only. - Potential difference between the plates is $V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K}$ So the potential difference decreases. - Field between the plates is $E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K}$ (as $V = \frac{V_0}{K}$ and $E_0 = \frac{V_0}{d}$) So, the electric field decreases. - Energy stored in the capacitor is $U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K}$ (as $q = q_0$ and $C = KC_0$) Thus, energy decreases and becomes $1/K$ times of previous energy.

ILLUSTRATION 4.14

The electric field between the plates of a parallel plate capacitor is E_0 . The space between the plates is filled completely with a dielectric. There are n molecules in unit volume of the dielectric and each molecule is like a dumb-bell of length L with its ends carrying charge $+q$ and $-q$. Assume that all molecular dipoles get aligned along the field between the plates. Find the electric field between the plates after insertion of the dielectric.

Sol. Due to electric field between the plates the molecules of the dielectric get polarized, the dipole moment per unit volume (after perfect alignment) in the dielectric $= qLn$

Net dipole moment induced in dielectric slab $= qLn.(Ad)$

$\therefore$ Charge induced on walls of the dielectric

$$Q_{in} = \pm \frac{qLnAd}{d} = \pm qLnA$$

$\therefore$ The electric field due to induced charge $E_{in} = \frac{Q_{in}}{\epsilon_0 A} = \frac{qLn}{\epsilon_0}$

The electric field between the plates, $E = E_0 - E_{in} = E_0 - \frac{qLn}{\epsilon_0}$

ILLUSTRATION 4.15

You have been given a parallel plate air capacitor having capacitance C_0 , a battery of emf V and three dielectric blocks having dielectric constants K_1 , K_2 and K_3 such that $K_1 > K_2 > K_3$. Each dielectric block can fill completely the space between the plates. Describe a sequence of steps such as connecting or disconnecting the capacitor to the battery, inserting or taking out of one of the dielectrics etc-so that the capacitor ends up having maximum possible energy stored in it.

Sol. The potential energy of a capacitor is given by, $U = \frac{Q^2}{2C}$

For maximum value of U , we should have maximum value of charge (Q) and minimum value of capacitance (C). Q will be maximum when battery is connected with capacitance at maximum possible value. The capacitance of parallel plate air capacitor with completely filled with dielectric is given by $C = KC_0$

For maximum capacitance K should be maximum. Hence dielectric with constant K_1 should be inserted.

Hence, $C_{max} = K_1C_0$

If battery is connected to this capacitor, charge on it will be maximum

$$Q_{max} = VC_{max} = K_1VC_0$$

Now battery is disconnected and dielectric is removed so that the capacitance of the capacitor becomes minimum.

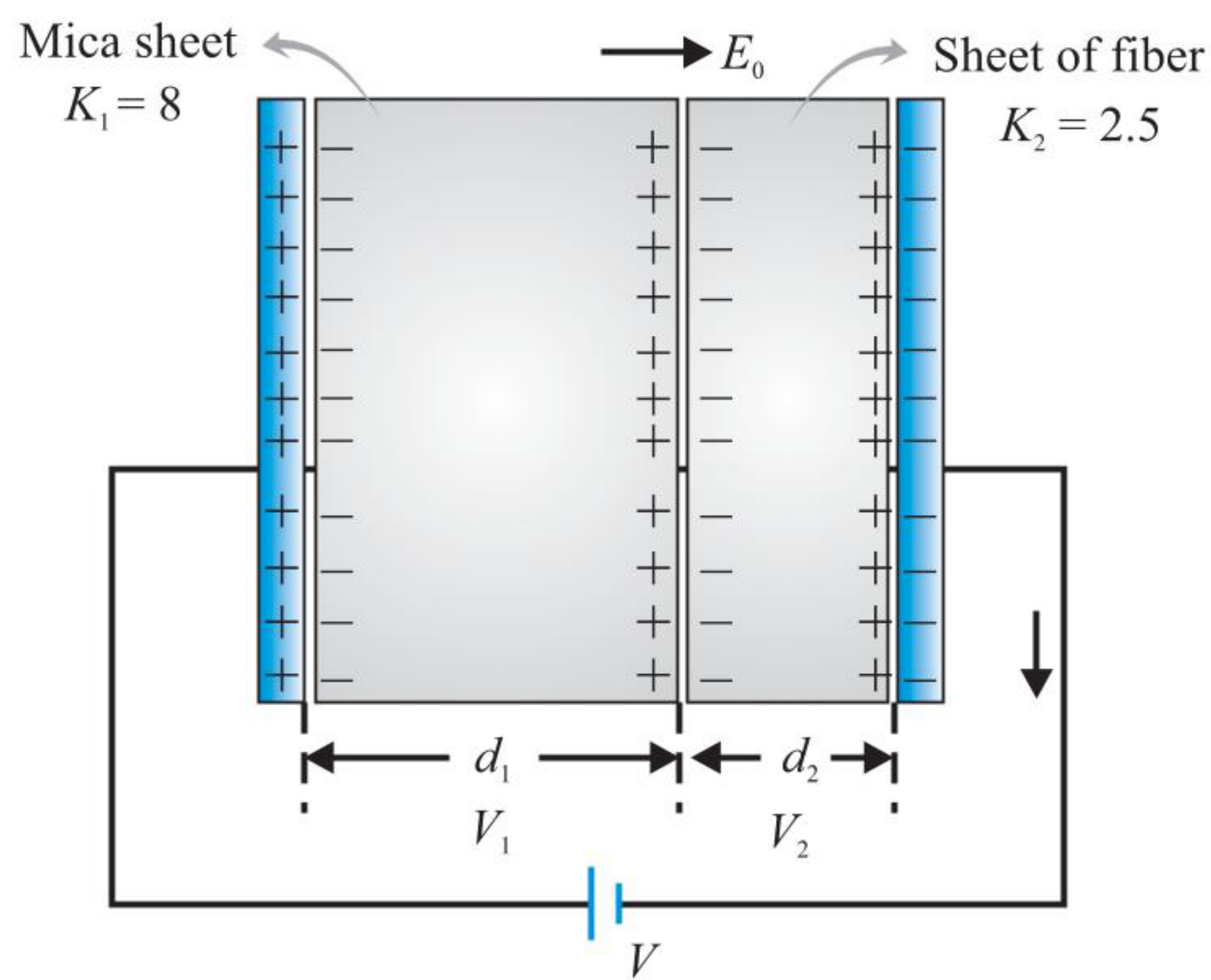
Hence the maximum possible energy stored in capacitor,

$$U_{max} = \frac{Q_{max}^2}{2C} = \frac{K_1^2V^2C_0^2}{2C_0} = \frac{1}{2}K_1^2V^2C_0$$

ILLUSTRATION 4.16

A parallel plate capacitor contains a mica-sheet (thickness 10^{-3} m) and a sheet of fibre (thickness 0.5×10^{-3} m). The dielectric constant of mica is 8 and that of fibre is 2.5. Assuming that the fibre breaks down when subjected to an electric field of 6.4×10^6 V/m. Find the maximum safe voltage that can be applied to the capacitor.

Sol. Let E_0 be the electric field due to charges on plates (not due to induced charges appearing on the surface of the dielectric sheet).



Given maximum electric field the fiber sheet can resist,

$$(E_2)_{\max} = 6.4 \times 10^6 \text{ V/m}$$

Hence maximum possible potential difference that can be applied across the fiber sheet,

$$\Rightarrow (V_2)_{\max} = (E_2)_{\max} \times d_2$$

$$(V_2)_{\max} = 6.4 \times 10^6 \times 0.5 \times 10^{-3} = 3.2 \times 10^3 \text{ V} = 3.2 \text{ kV}$$

Let maximum possible electric field due to charge on the plates be $(E_0)_{\max}$.

$$\text{then, } (E_2)_{\max} = \frac{(E_0)_{\max}}{K_2} \Rightarrow (E_0)_{\max} = (E_2)_{\max} \times K_2$$

$$(E_0)_{\max} = 6.4 \times 10^6 \times 2.5 = 16 \times 10^6 \text{ V/m}$$

The maximum electric field in the mica sheet corresponding to $(E_0)_{\max}$.

$$E_1 = \frac{(E_0)_{\max}}{K_1} = \frac{16 \times 10^6}{8} \text{ V/m} = 2.0 \times 10^6 \text{ V/m}$$

The potential difference across the mica sheet,

$$V_1 = E_1 \times d_1 = 2.0 \times 10^6 \times 10^{-3} = 2.0 \text{ kV}$$

The potential difference across the plates of the capacitor

$$V = V_1 + (V_2)_{\max} = 2.0 + 3.2 = 5.2 \text{ kV}$$

ILLUSTRATION 4.17

A parallel plate capacitor completely filled with a dielectric slab, dielectric constant $K = 3.4$, is fully charged with a 100 V battery. When the capacitor is fully charged, the battery is removed. The area of plates $A = 4.0 \text{ m}^2$ and separation between plates $d = 4.0 \text{ mm}$. (a) Find the capacitance, the electric field strength and the energy stored in the capacitor. (b) The dielectric slab is slowly removed without changing the plate separation and any charge transfer from capacitor. Find the new capacitance, electric field strength, voltage between plates and the energy stored in the capacitor.

Sol.

(a) In presence of dielectric slab the capacitance of a capacitor will increase K times, $C = KC_0$

$$C = \frac{K\epsilon_0 A}{d} = \frac{(3.4)(8.85 \times 10^{-12})(4)}{4 \times 10^{-3}} = 3.0 \times 10^{-8} \text{ F}$$

The electric field between the plates,

$$E = \frac{V}{d} = \frac{100}{4.0 \times 10^{-3}} = 25 \text{ kV/m}$$

The total energy stored in the capacitor is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (3 \times 10^{-8})(100)^2 = 1.5 \times 10^{-4} \text{ J}$$

(b) The capacitance without dielectric is

$$C_0 = \frac{C}{K} = \frac{3 \times 10^{-8}}{3.4} = 8.8 \times 10^{-9} \text{ F}$$

There is no transfer of charge, hence

$$CV = C_0 V_0 \Rightarrow (KC_0)V = C_0 V_0 \Rightarrow V_0 = KV$$

So potential difference between plates increases by a factor $K = 3.4$

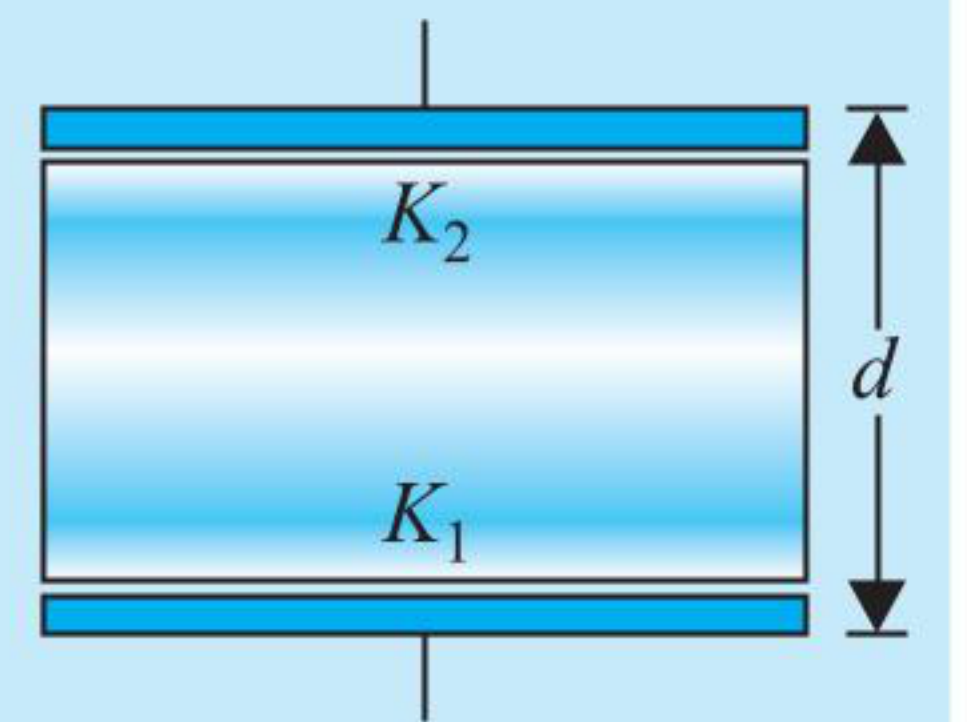
$$\text{The electric field is } E = \frac{V}{d} = \frac{340}{4.0 \times 10^{-3}} = 85 \text{ kV/m}$$

The energy of the capacitor is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (8.8 \times 10^{-9})(340)^2 = 5.1 \times 10^{-4} \text{ J}$$

ILLUSTRATION 4.18

A dielectric slab fills the entire space of a parallel plate capacitor. The dielectric constant of the slab varies linearly with distance, varies from K_1 to K_2 , from one plate to other. Find the equivalent capacity of the system. Separation between plates is d and area of plates A .



Sol. It is given that the dielectric constant varies linearly, hence we can write the expression for dielectric constant as:

$$K(y) = ay + b \quad \dots(1)$$

where a and b are constants and y is measured from lower plate, these values can be determined from boundary conditions

$$K = K_1 \text{ at } y = 0 \text{ hence } K_1 = b \quad \dots(ii)$$

$$K = K_2 \text{ at } y = d \text{ hence } K_2 = ad + b \quad \dots(iii)$$

$$\text{From (ii) and (iii) } a = \frac{K_2 - K_1}{d}, \quad b = K_1$$

Let charge Q be given to the system, electric field at a distance y from lower plate is

$$E(y) = \frac{Q}{K(y)\epsilon_0 A}$$

So the potential difference across a differential element dy at a distance y is

$$dV = -E(y)dy$$

$$\text{Thus } -\int_{V_1}^{V_2} dV = \int_0^d \frac{Q}{K(y)\epsilon_0 A} dy$$

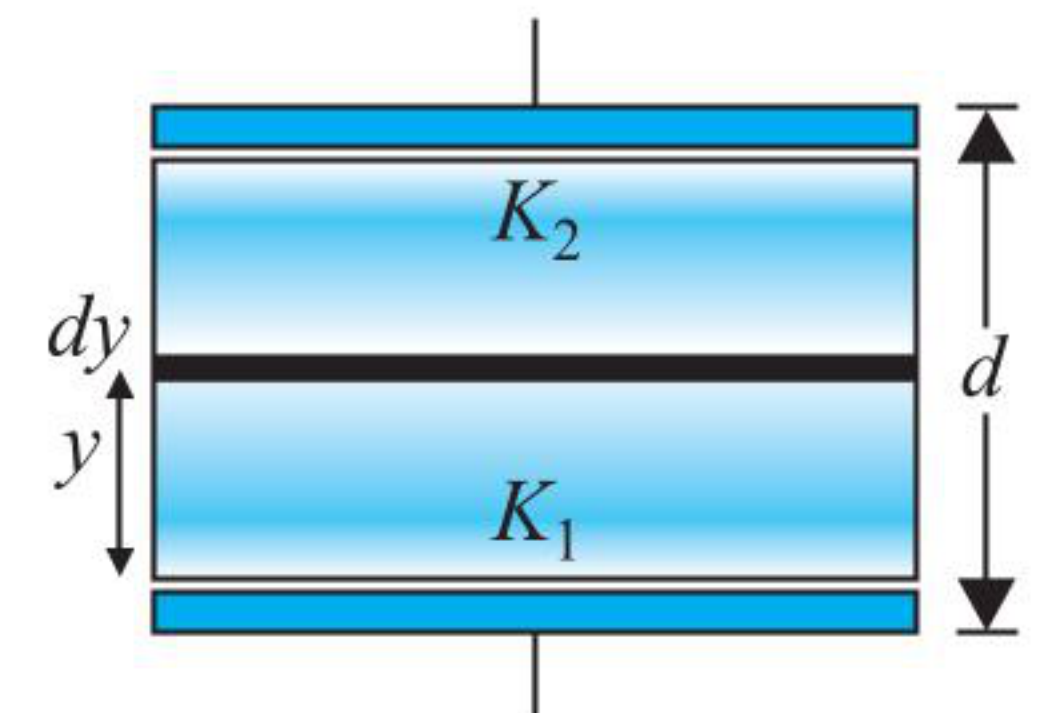
$$= \frac{Q}{\epsilon_0 A} \int_0^d \frac{dy}{ay + b} = \frac{Q}{\epsilon_0 A} \left[\frac{1}{a} \ln(ay + b) \right]_0^d$$

$$V_1 - V_2 = \frac{Q}{\epsilon_0 Aa} \ln\left(\frac{ad + b}{b}\right)$$

$$\text{Thus } C = \frac{Q}{V_1 - V_2} = \frac{\epsilon_0 Aa}{\ln\left(\frac{ad + b}{b}\right)}$$

On substituting values of a and b , we get,

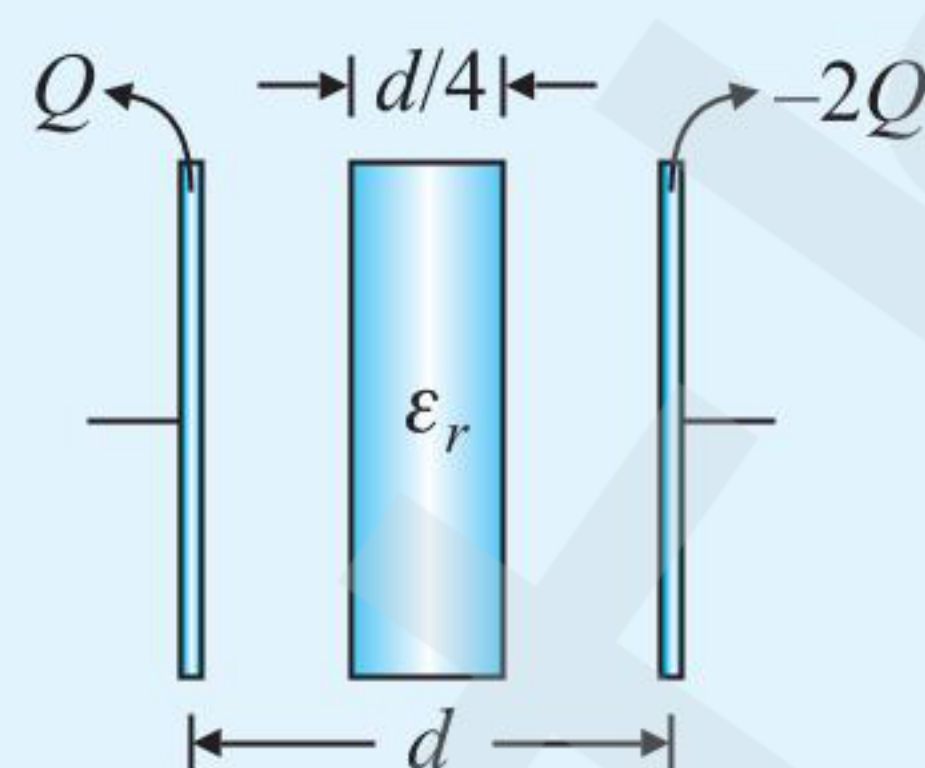
$$C = \frac{\epsilon_0 A}{d} \frac{K_2 - K_1}{\ln(K_2/K_1)}$$



CONCEPT APPLICATION EXERCISE 4.2

1. A large thin conducting plate is kept in an external uniform electric field E_0 . Find the induced surface charge density.
2. An isolated parallel plate capacitor stores an energy $U_0 = 10^{-3}$ J in the absence of a dielectric. If the dielectric of $\epsilon_r = 100$ is introduced,
 - (i) find the energy in the capacitor.
 - (ii) where does the extra energy go?
3. A charged capacitor has energy $U_0 = 2 \times 10^{-4}$ J and $C_0 = 4 \times 10^{-8}$ F, in the absence of a dielectric. When the potential difference is kept constant by keeping the capacitor connected to the supply, assuming $\epsilon_r = 150$,
 - (i) find the new energy stored when the dielectric slab is completely filled with the capacitor. Account for this change in energy.
 - (ii) how much excess charge flows from the supply
 - (iii) find the ratio of the charge on the plate of the capacitor and the charge induced on the dielectric;
4. A parallel plate capacitor with plate area 100 cm^2 , separated by a distance of 1 mm . A dielectric of dielectric constant 5.0 and dielectric strength $1.9 \times 10^7 \text{ V/m}$ is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

5. A dielectric slab of thickness $d/4$ and relative permittivity $\epsilon_r = 2$ is inserted between two parallel conducting plates each of area A kept at a separation d . Find the:

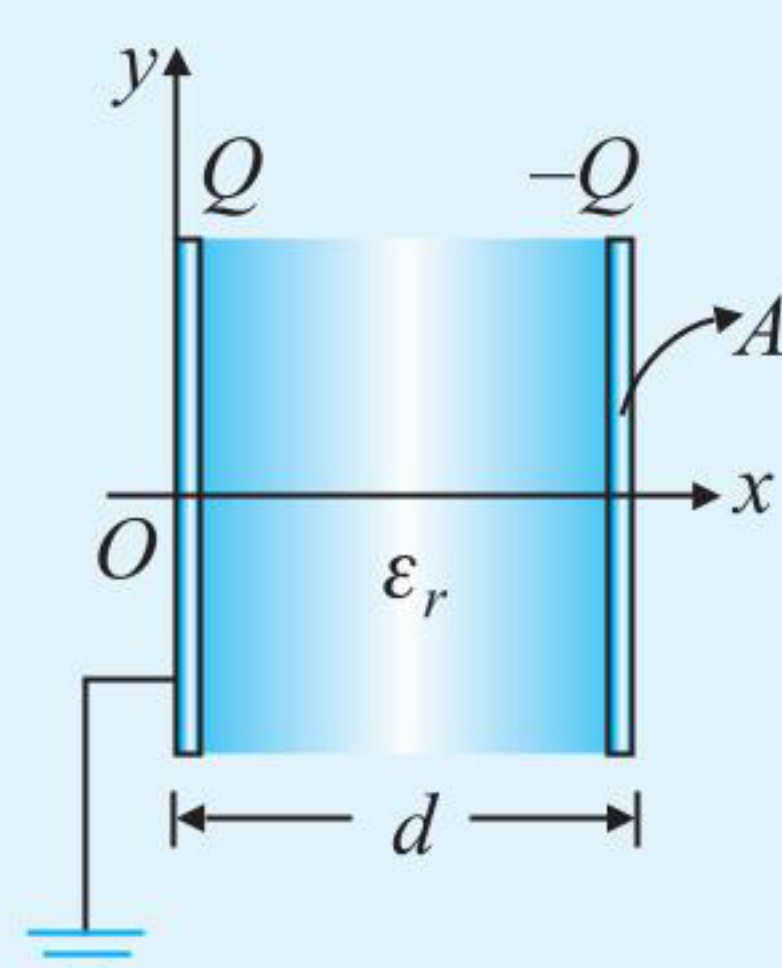


- (a) The charge density in the inner side of conductors
 - (b) The charge density on the dielectric slab
 - (c) The electric field in the dielectric
 - (d) The potential difference between the conductor
 - (e) The capacitance
6. The separation between the plates of a parallel-plate capacitor is 0.05 m . A field of $3 \times 10^4 \text{ V/m}$ is established between the plates. It is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted between the plates of the capacitor.
 - (a) What would be the potential difference across the capacitor, (i) before the introduction of the metal plate and (ii) after its introduction.
 - (b) What would be the potential difference if a plate of dielectric constant $K = 2$ is introduced in place of metal plate?

7. A dielectric slab is filled in the space between the parallel plate capacitor whose relative permittivity varies as

$$\epsilon_r = a + bx$$

Find the (a) capacitance of the system
(b) potential at a point situated at a distance x from the origin O .



ANSWERS

1. $\sigma = \epsilon_0 E_0$
2. (i) 10^{-5} J
3. (i) 0.03 J (ii) $5.96 \times 10^{-6} \text{ C}$ (iii) $\frac{149}{150}$
4. $8.4 \times 10^{-6} \text{ C}$
5. (a) $\frac{3Q}{2A}$ (b) $\frac{3Q}{4A}$ (c) $\frac{3Q}{4\epsilon_0 A}$ (d) $\frac{21Qd}{16\epsilon_0 A}$ (e) $\frac{8\epsilon_0 A}{7d}$
6. (a) (i) 1.5 kV (ii) 1.20 kV (b) 1.35 kV
7. (a) $\frac{\epsilon_0 Ab}{\ln\left(\frac{a+bd}{a}\right)}$ (b) $\frac{Q}{\epsilon_0 Ab} \ln\left(\frac{a+bx}{a}\right)$

COMBINATION OF CAPACITORS

Sometimes to obtain a desired value of capacitance, we group two or more than two capacitors together. In this section we will learn how to find the equivalent capacitance when two or more than two capacitors are grouped together?

The equivalent capacitance of a group of capacitors is that value of capacitance of a single capacitor that will allow to flow (or store) the same amount of charge for a given potential difference as done by the combination. In other words, that single capacitor will perform the same task as done by the combination in all respects. Capacitors can be grouped in many ways, but the two common types of combinations are (i) series combination and (ii) parallel combination.

CAPACITORS CONNECTED IN SERIES

Suppose n capacitors are connected in series as shown in figure. C_{eq} is the equivalent capacitance of this system.

Let a battery V be applied across the combination, then

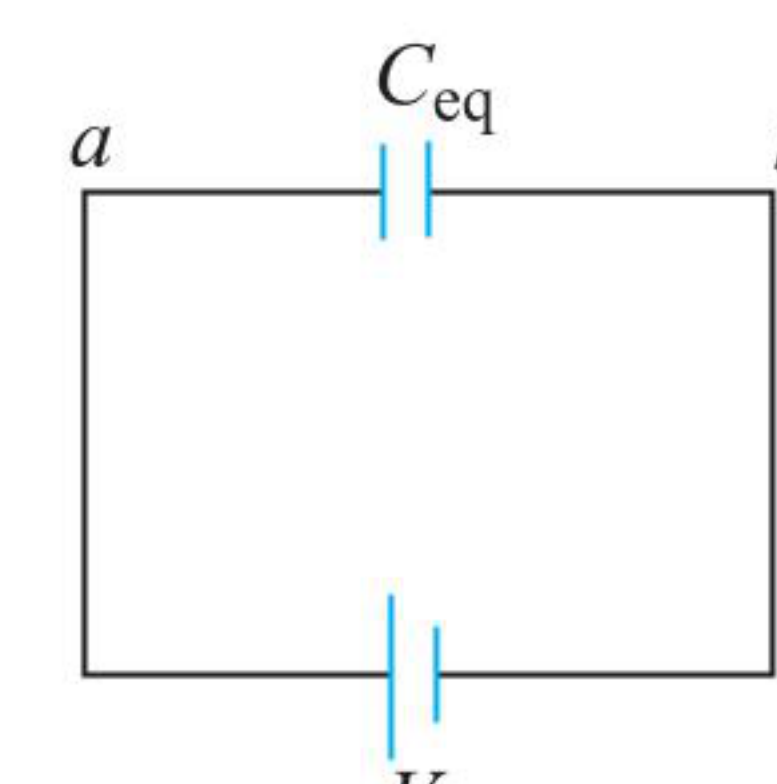
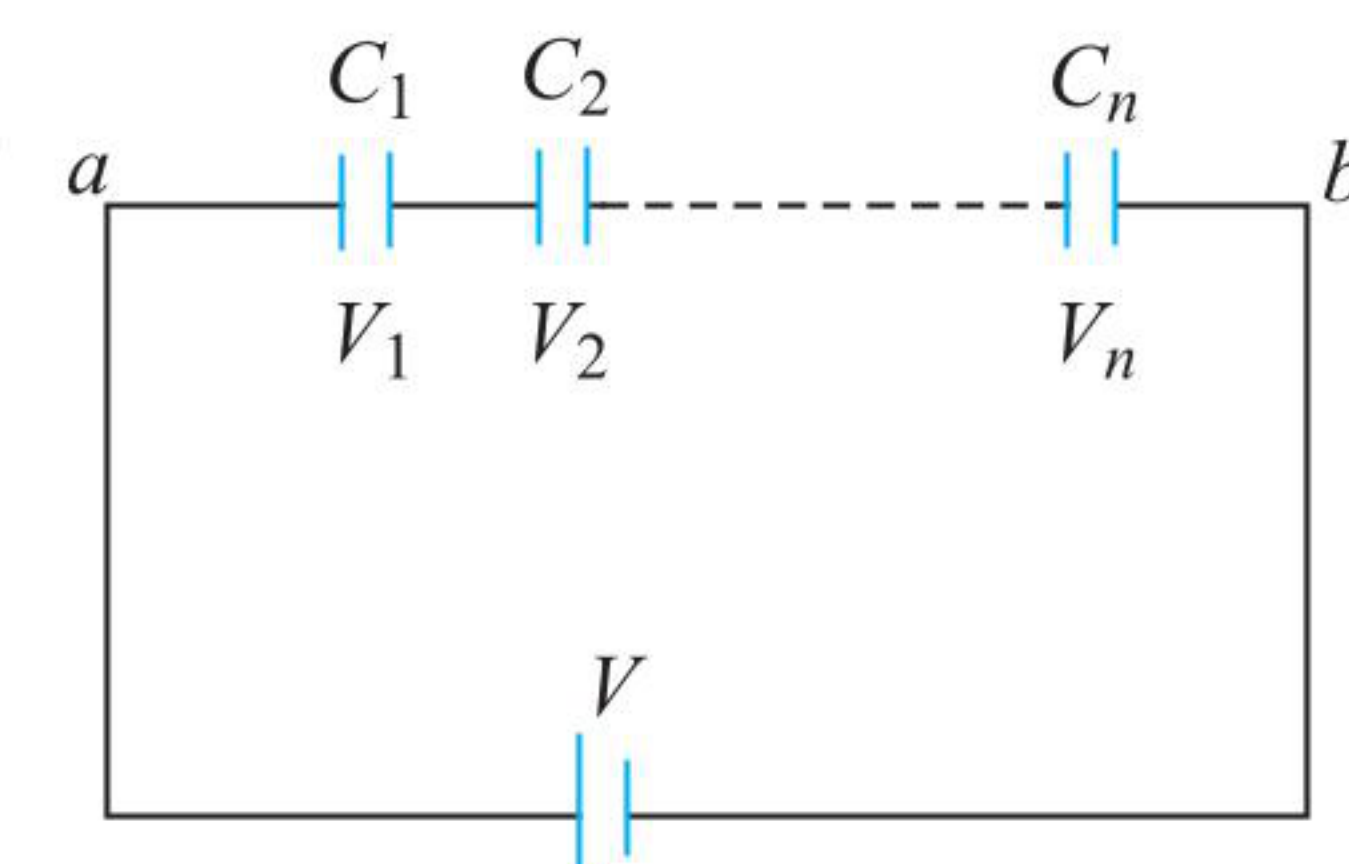
$$V = V_1 + V_2 + \dots + V_n$$

$$\text{or } \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots + \frac{Q}{C_n}$$

$$\text{or } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

In general

$$\frac{1}{C_{eq}} = \sum_{n=1}^n \frac{1}{C_n}$$

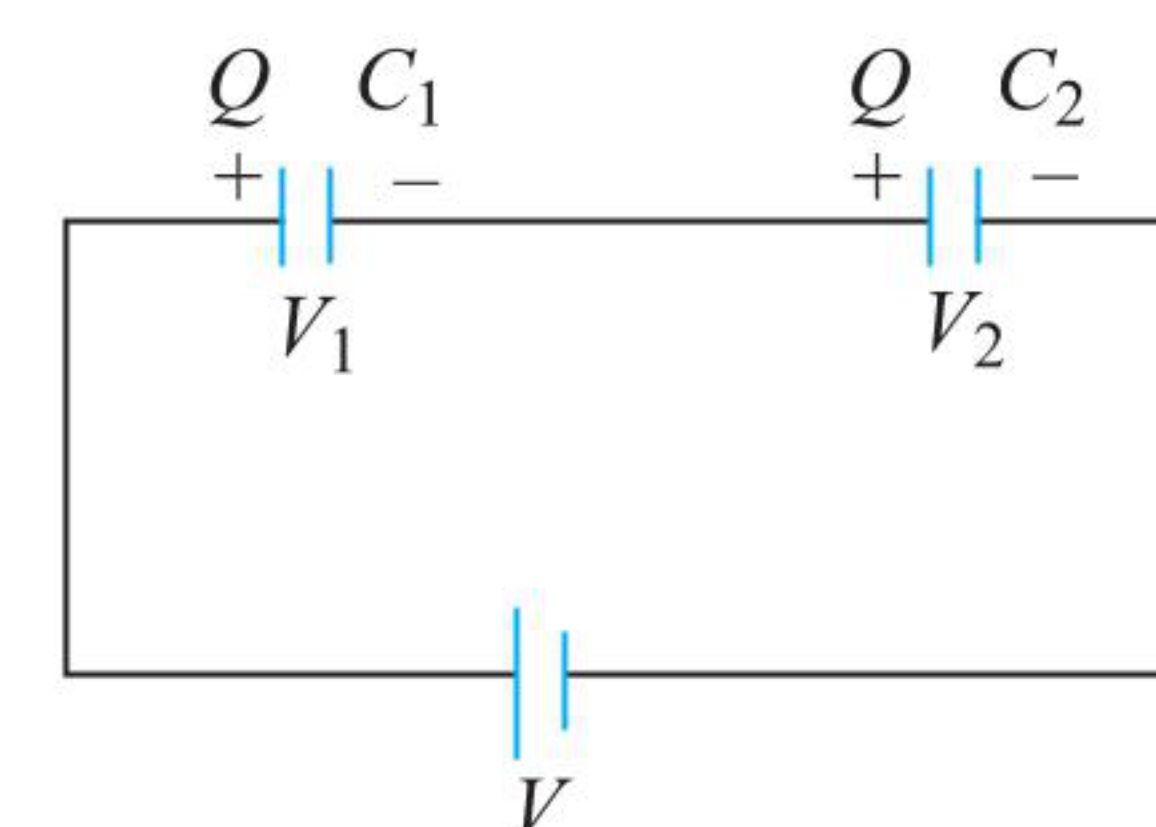


Thus, the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances.

FINDING POTENTIAL DIFFERENCE ACROSS THE CAPACITORS CONNECTED IN SERIES

Suppose two capacitors C_1 and C_2 are connected in series and a potential difference V is applied across them as in figure.

Potentials appearing on the capacitors are V_1 and V_2 , respectively. Q is the net charge that flows in circuit, then



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$V_1 + V_2 = V \quad \dots(i)$$

$$Q = C_1 V_1 = C_2 V_2 \quad \text{or} \quad \frac{V_1}{V_2} = \frac{C_2}{C_1} \quad \dots(ii)$$

It means potentials on the capacitors will be divided in the inverse ratio of their capacitances.

From (i) and (ii),

$$V_1 = \frac{C_2 V}{C_1 + C_2}, \quad V_2 = \frac{C_1 V}{C_1 + C_2}$$

Note:

- In a series combination, the equivalent capacitance is always less than any of the individual capacitance.
- In a series combination, charge on each capacitor is same but potential is different. From $V = q/C$, it can be said that larger is the capacitance, lesser is the potential. We can say that the potential difference across the capacitor is inversely proportional to its capacitance in series combination.

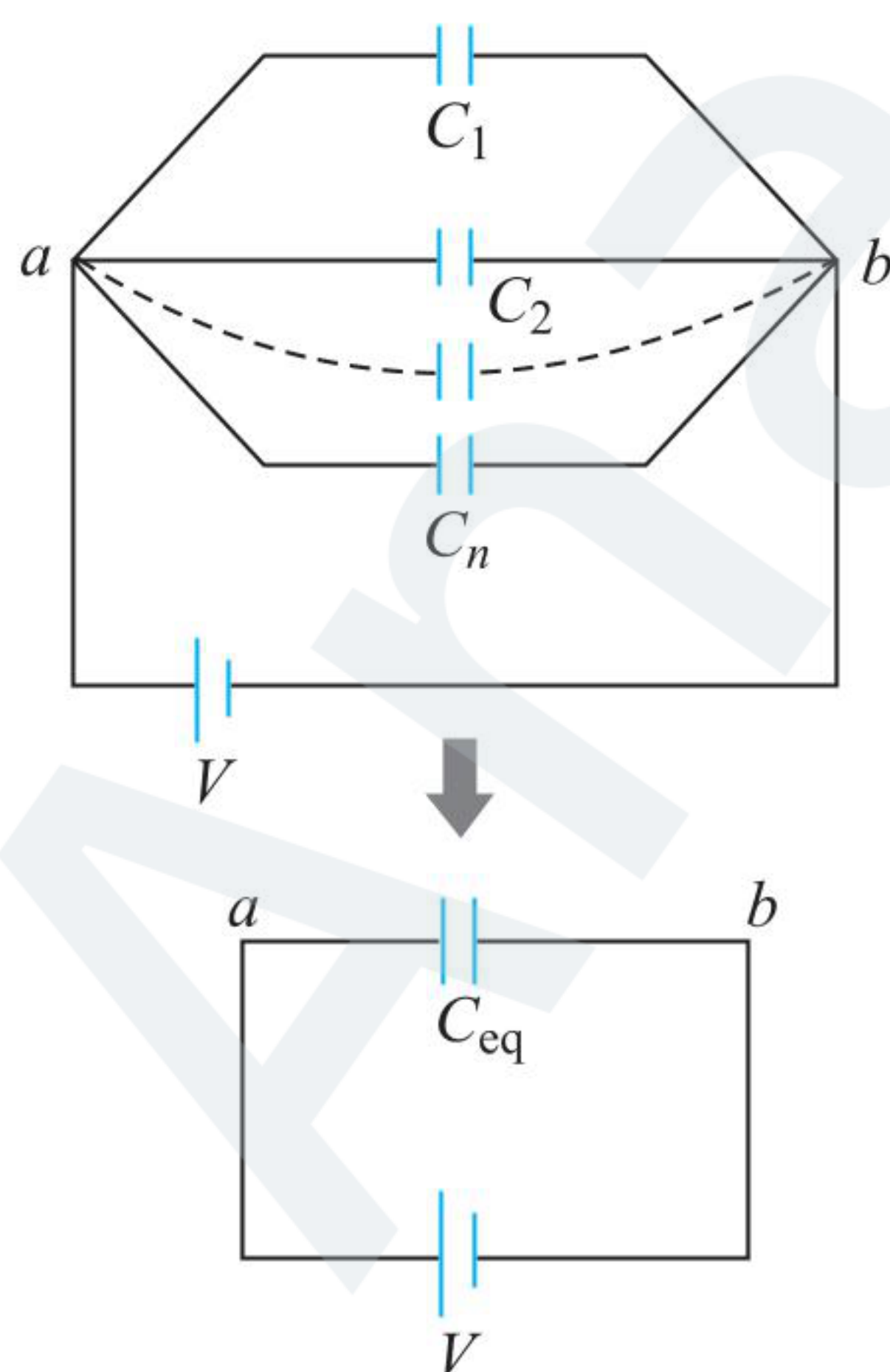
$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

$$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots} V; \quad V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots} V$$

where $V = V_1 + V_2 + V_3$

CAPACITORS CONNECTED IN PARALLEL

Let n capacitors be connected in parallel as shown in figure. C_{eq} is the equivalent capacitance of this system.



Let the total charge flowing through the battery be q , then

$$q = q_1 + q_2 + \dots + q_n$$

$$\text{or} \quad C_{\text{eq}} V = C_1 V + C_2 V + \dots + C_n V$$

$$\text{or} \quad C_{\text{eq}} = C_1 + C_2 + \dots + C_n$$

In general

$$C_{\text{eq}} = \sum_{n=1}^n C_n$$

That is, the equivalent capacitance of a parallel combination equals the sum of the individual capacitances.

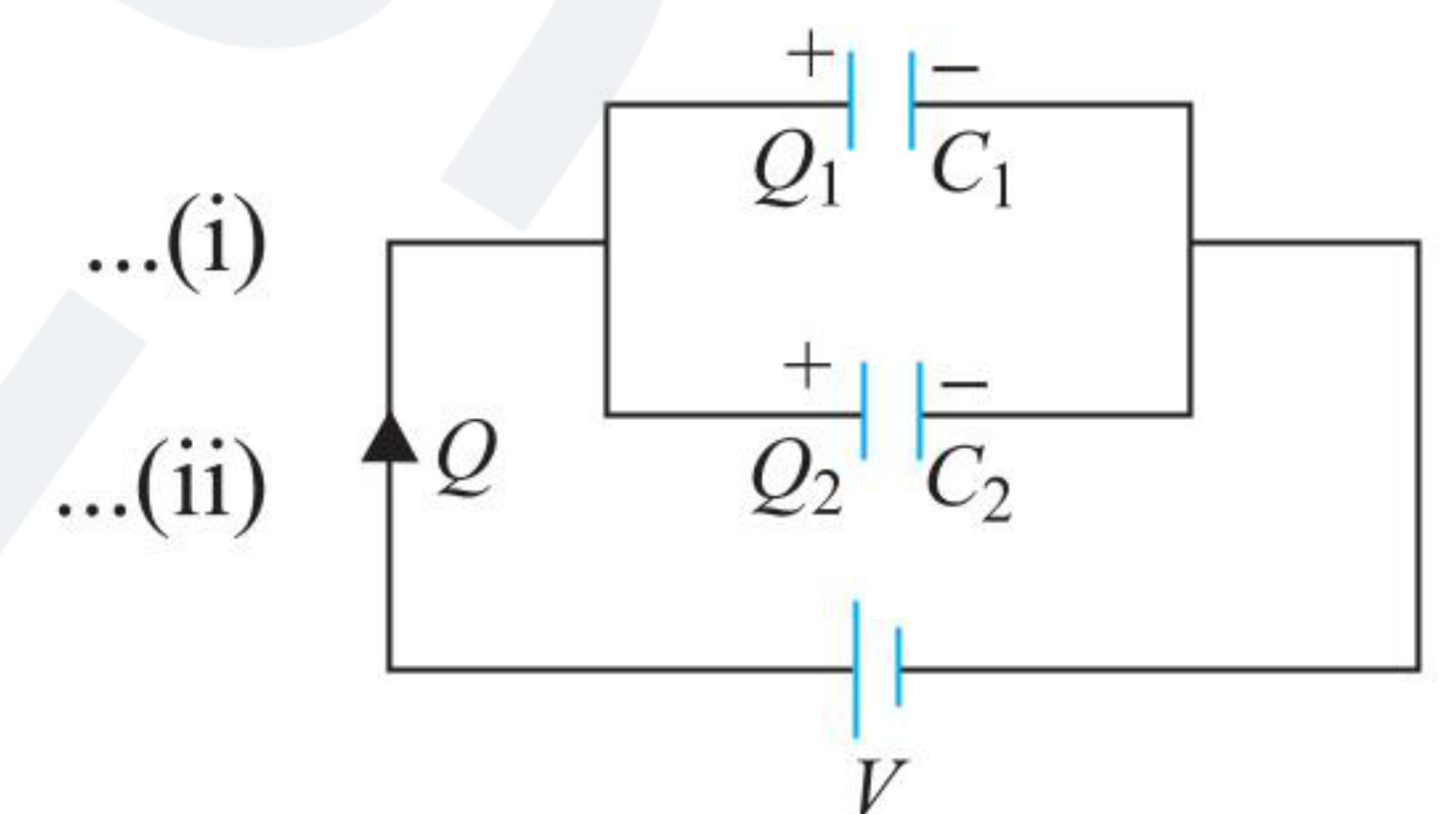
FINDING POTENTIAL DIFFERENCE ACROSS THE CAPACITORS CONNECTED IN PARALLEL

Let two capacitors C_1 and C_2 be connected in parallel and a potential difference V is applied across them. Then

$$C_{\text{eq}} = C_1 + C_2$$

$$Q_1 + Q_2 = Q \quad \dots(i)$$

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad \dots(ii)$$



From (ii)

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} \quad \dots(iii)$$

It means charge will be divided in the direct ratio of the capacitances. From (i) and (iii),

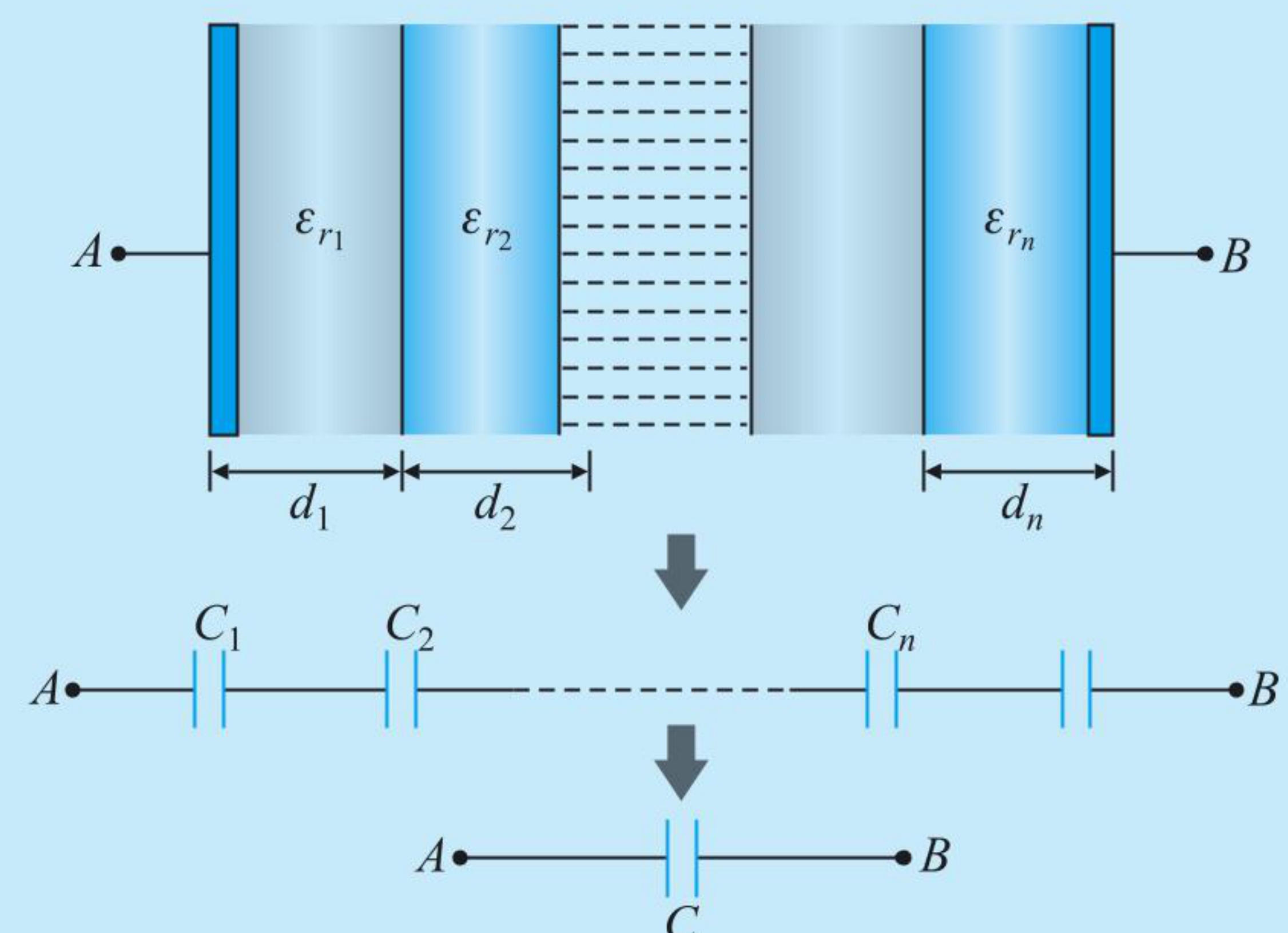
$$Q_1 = \frac{C_1 Q}{C_1 + C_2}, \quad Q_2 = \frac{C_2 Q}{C_1 + C_2}$$

Note:

- In a parallel combination, the equivalent capacitance is always greater than any individual capacitance.
- In a parallel combination, the potential on each capacitor is same but charge may be different. From $q = CV$, it can be said that greater is the capacitance, greater is the charge.
- Charge is distributed on the capacitors in the ratio of their capacitances, i.e., $q_1 : q_2 : \dots : q_n = C_1 : C_2 : \dots : C_n$

Important Points:

- When the dielectric are placed so that the field intensity is perpendicular to their interfaces, we can treat each dielectric forming a parallel plate capacitor with two thin conducting plates identical with that of the given capacitor. As a result, the given combination can be reduced to a series combination of n number of capacitors having the effective capacitance C given as $\frac{1}{C} = \sum \frac{1}{C_i}$, where $C_i = \frac{\epsilon_0 \epsilon_{r_i} A}{x_i}$.



- When the dielectrics are connected so that their interfaces are parallel to the applied field, we can treat each dielectric forming a parallel plate capacitor corresponding to the area of its contact with the conducting plates. Then, taking all such capacitors in parallel, the equivalent capacitance of the combination can be given as $C = \Sigma C_i$, where $C_i = \frac{\epsilon_0 \epsilon_{r_i} A_i}{d}$

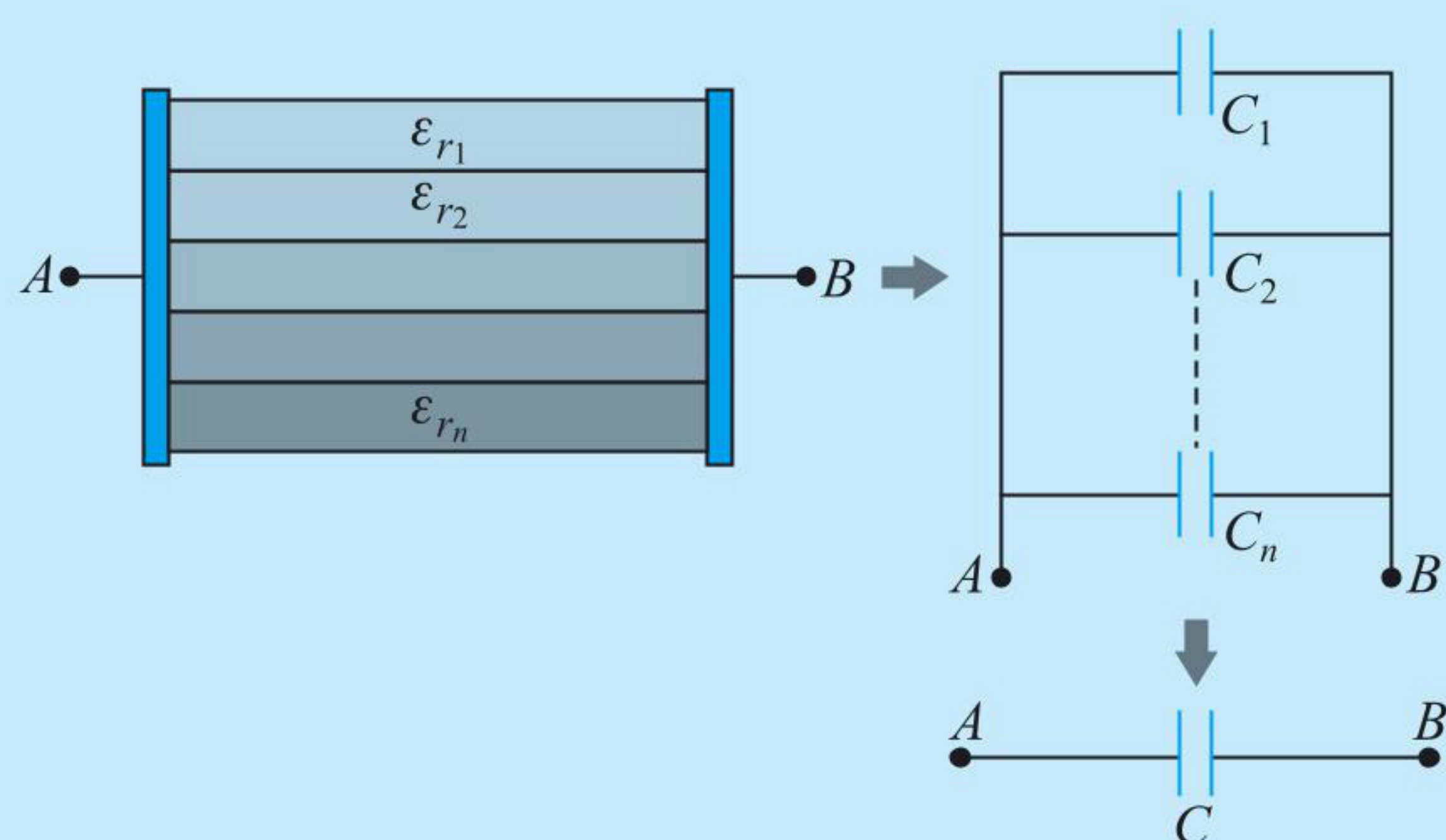
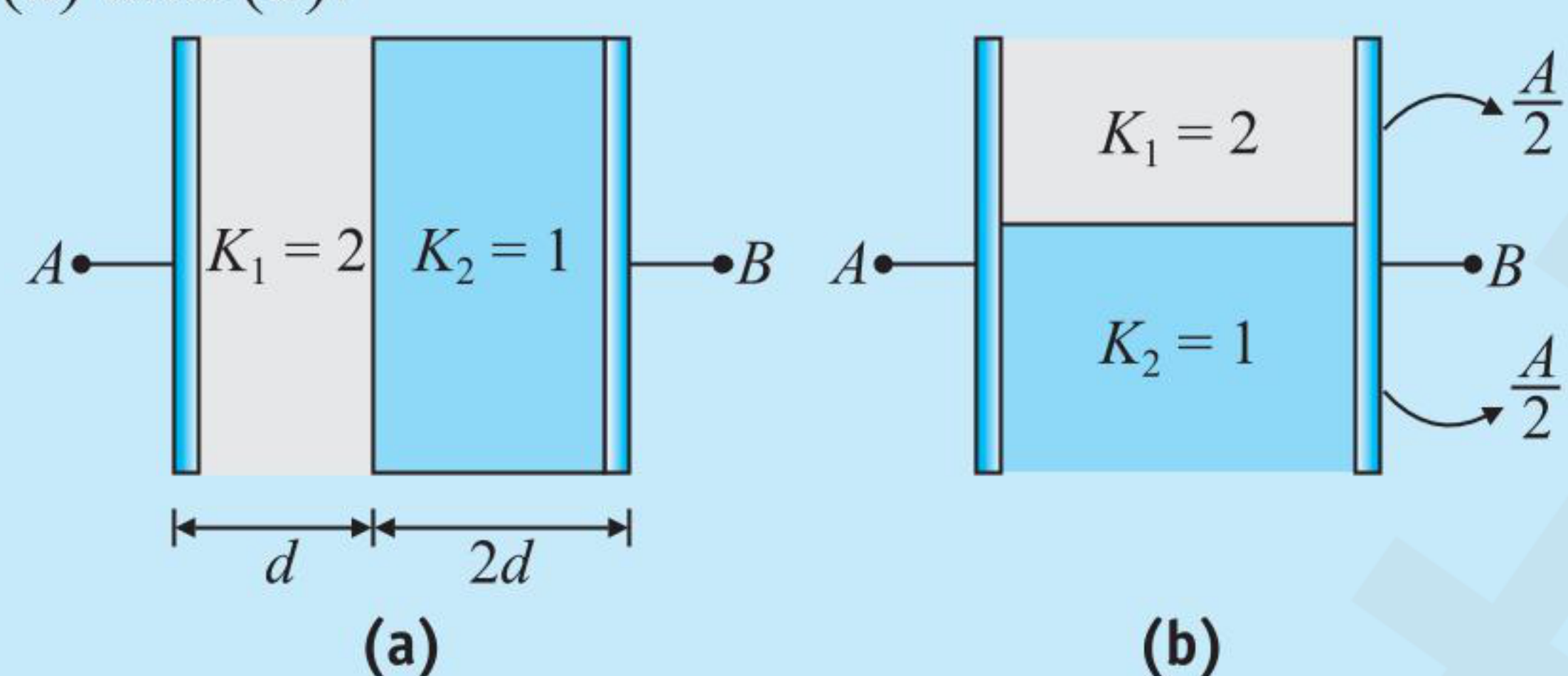


ILLUSTRATION 4.19

Find the equivalent capacitance of an air capacitor of capacitance C_0 when the two dielectric slabs of dielectric constants $K_1 = 2$ and $K_2 = 1$ are introduced as shown in Figs. (a) and (b).



Sol.

- (i) The given system can be made equivalent to two capacitors C_1 and C_2 in series.

$$\text{For left part } C_1 = \frac{\epsilon_0 K_1 A}{d} = \frac{2\epsilon_0 A}{d}$$

$$\text{For right part } C_2 = \frac{\epsilon_0 K_2 A}{2d} = \frac{\epsilon_0 A}{2d}$$



As both capacitors are connected in series, hence the equivalent capacitance

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2\epsilon_0 A} + \frac{2d}{\epsilon_0 A} = \frac{5d}{2\epsilon_0 A}$$

$$\text{As } C_0 = \frac{\epsilon_0 A}{3d} \text{ hence } C = \frac{5C_0}{5}$$

2nd approach:

$$C = \frac{\epsilon_0 A}{\frac{x_1}{K_1} + \frac{x_2}{K_2}}, \text{ where } x_1 = d, x_2 = 2d, K_1 = 2 \text{ \& } K_2 = 1$$

$$C = \frac{\epsilon_0 A}{\frac{d}{2} + \frac{2d}{1}} = \frac{\epsilon_0 A}{5d/2} = \frac{2\epsilon_0 A}{5d}$$

$$\text{As } C_0 = \frac{\epsilon_0 A}{3d} \text{ hence } C = \frac{6C_0}{5}$$

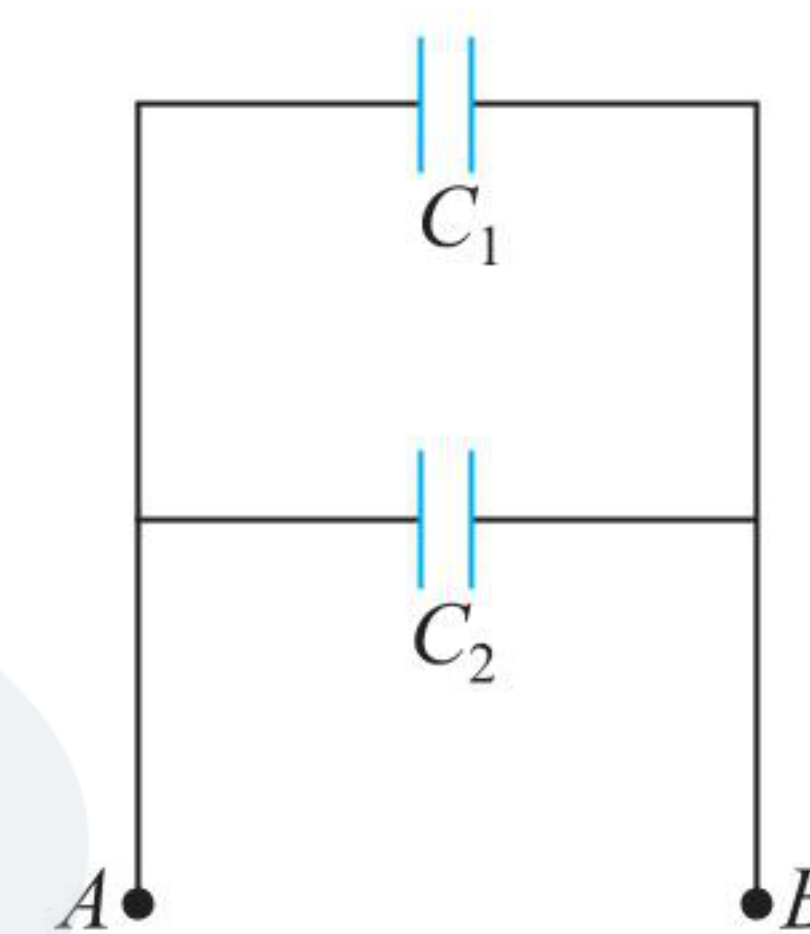
- (ii) The given system can be made equivalent to two capacitors C_1 and C_2 in parallel.

$$\text{For upper part } C_1 = \frac{\epsilon_0 K_1 \left(\frac{A}{2}\right)}{d} = \frac{\epsilon_0 A}{d}$$

$$\text{For lower part } C_2 = \frac{\epsilon_0 K_2 \left(\frac{A}{2}\right)}{d} = \frac{\epsilon_0 A}{2d}$$

Then, $C_{AB} = C_1 + C_2$, (because C_1 and C_2 are in parallel).

$$= \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{2d} = \frac{3\epsilon_0 A}{2d} = \frac{3}{2} C_0$$

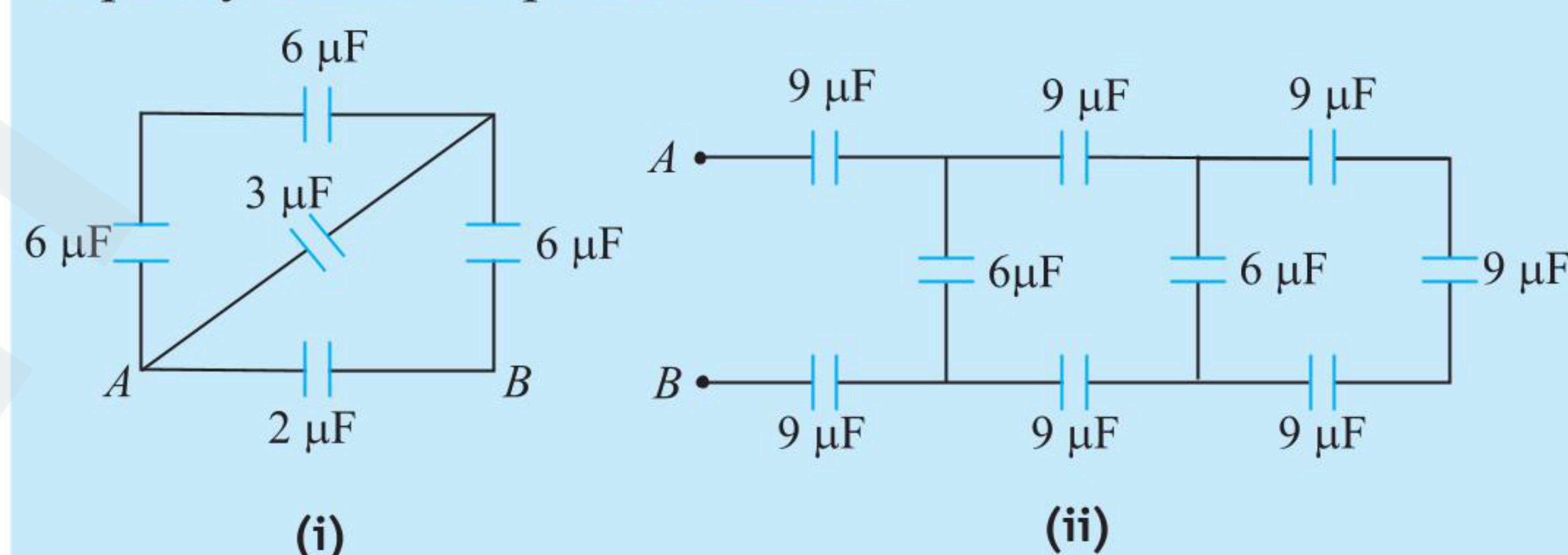


FINDING EQUIVALENT CAPACITANCE

Equivalent capacitance can be found by using the successive reduction method. Let us learn this method through the following illustrations.

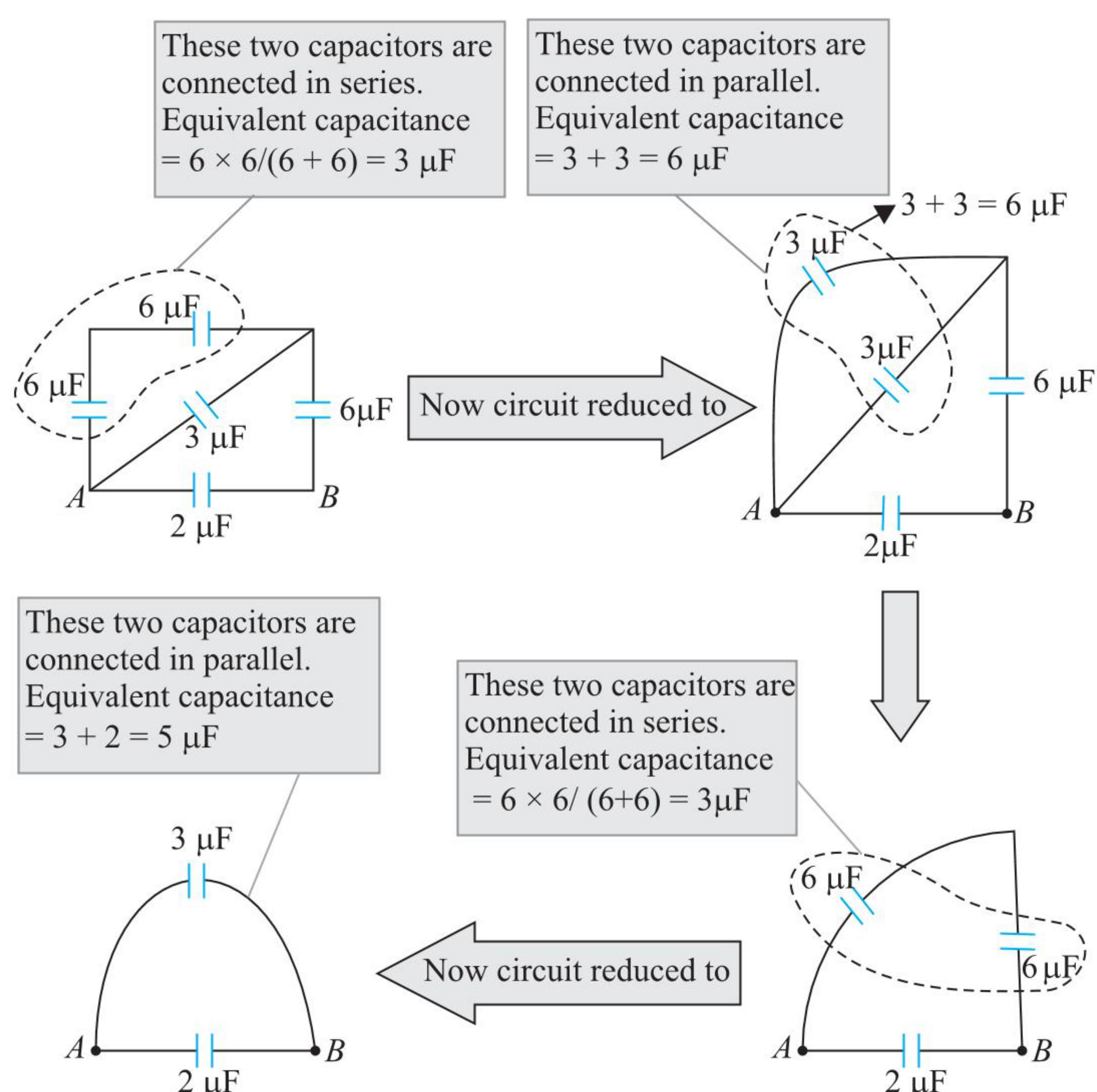
ILLUSTRATION 4.20

In figure, different capacitors are arranged. Find the equivalent capacity across the points A and B .

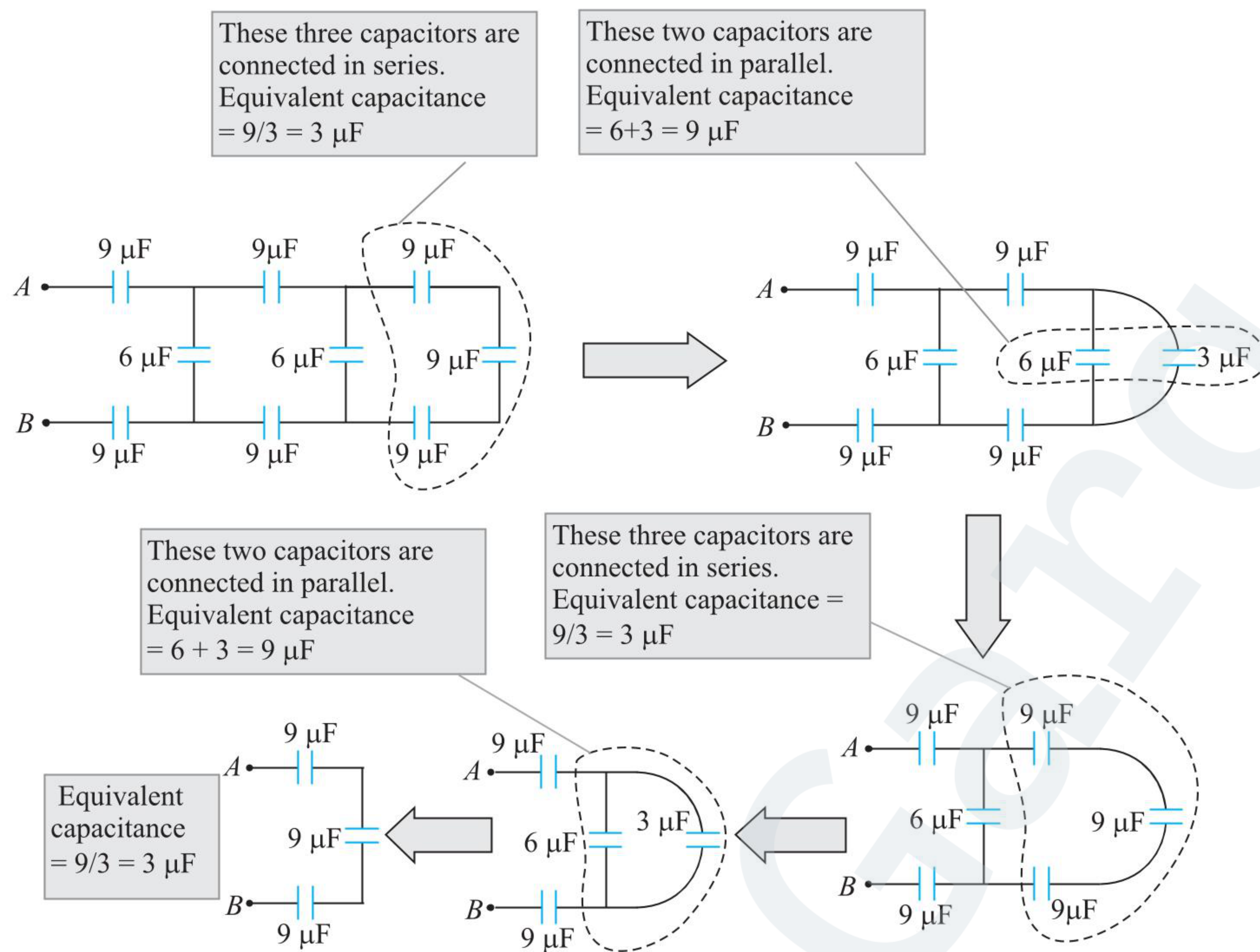


Sol.

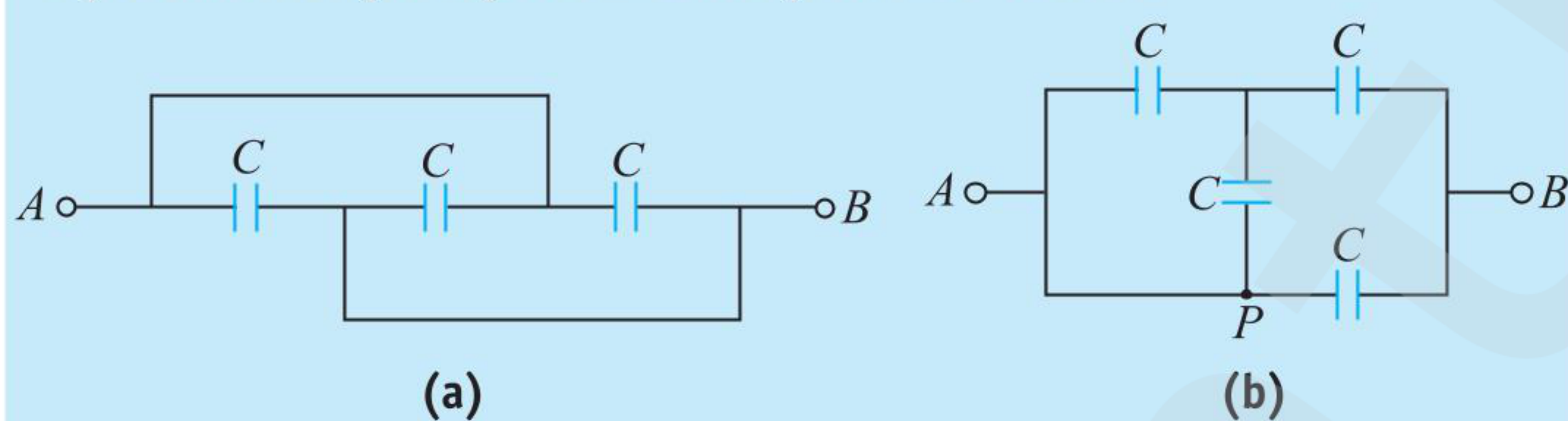
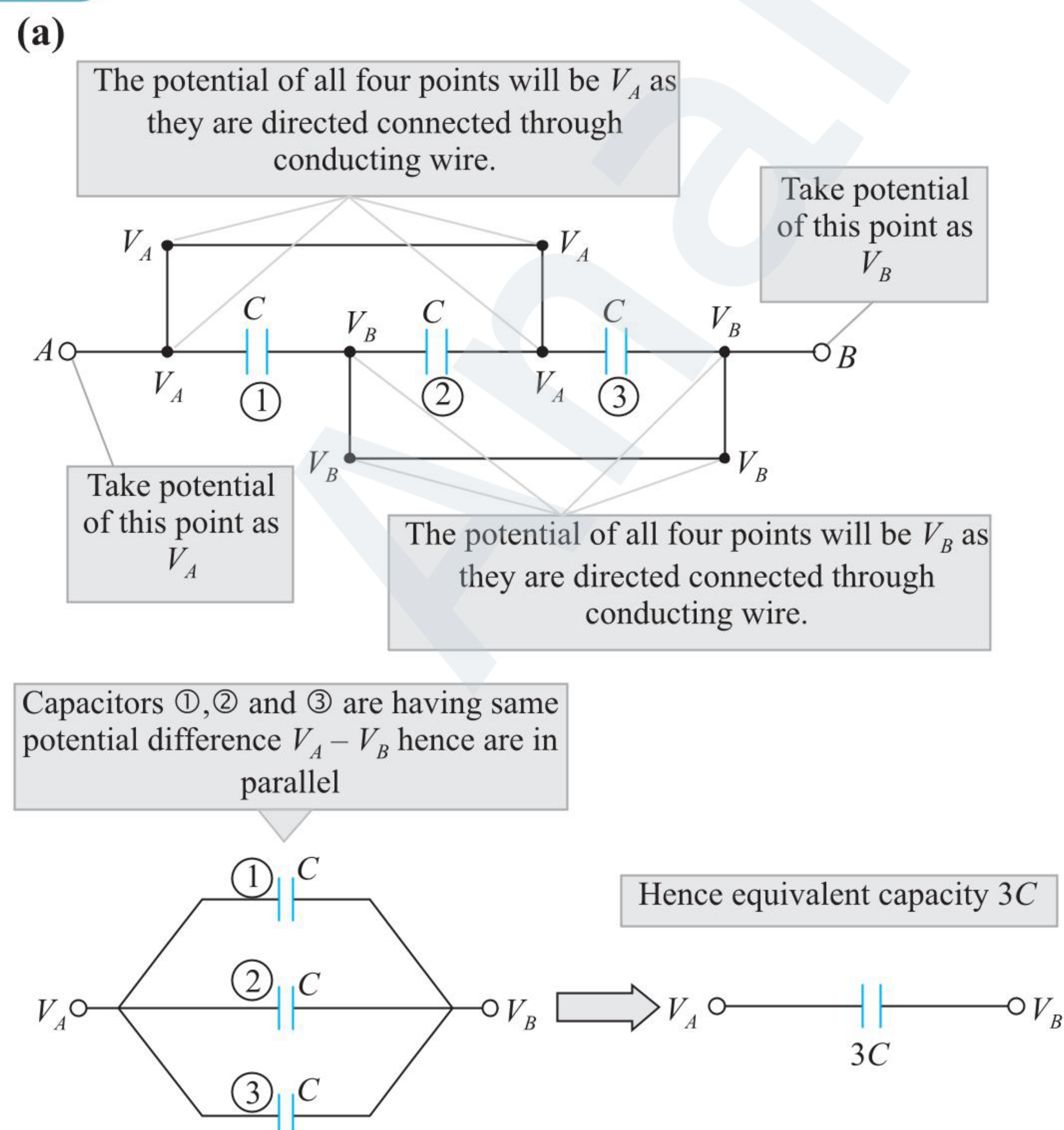
(i)



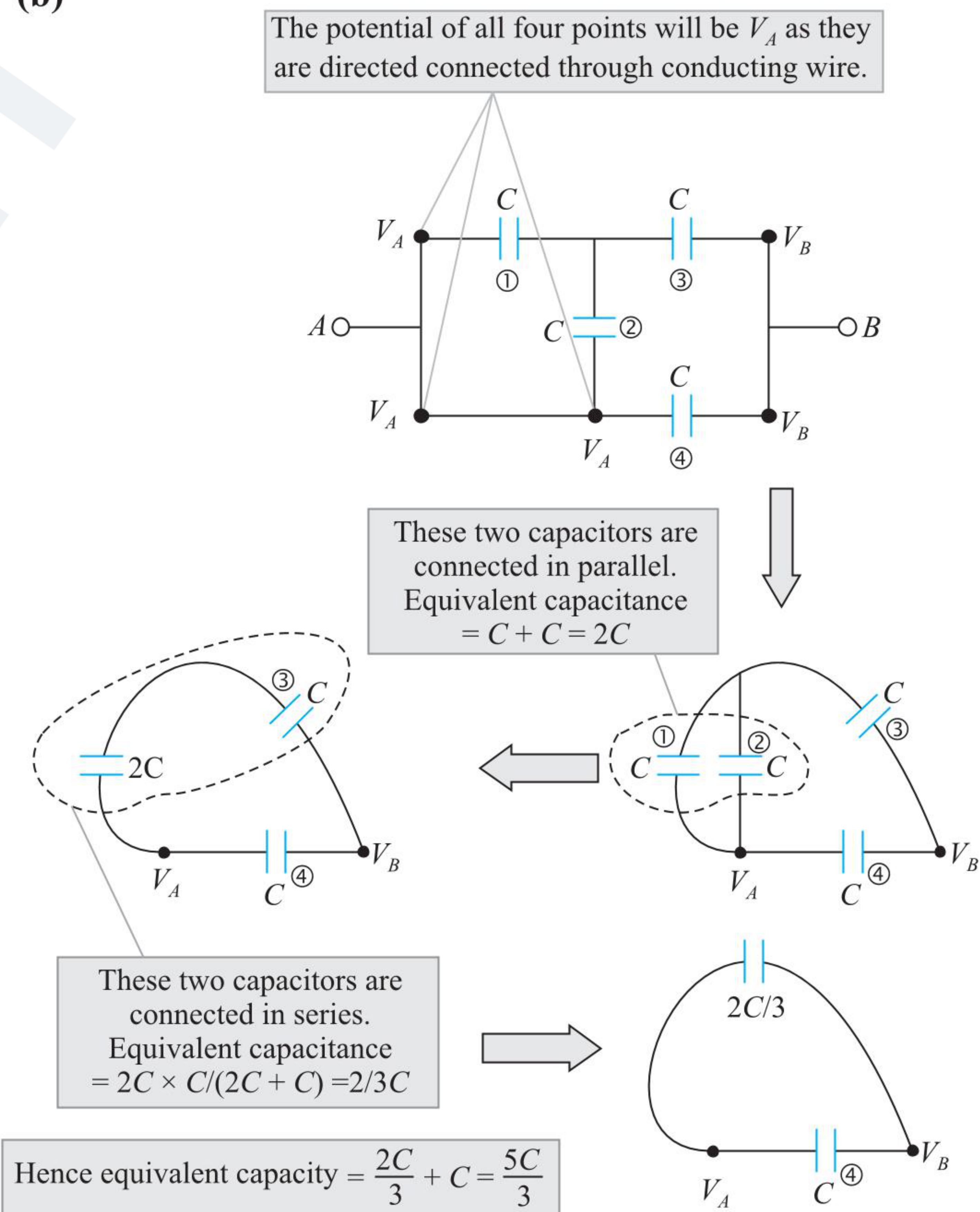
(ii)

**ILLUSTRATION 4.21**

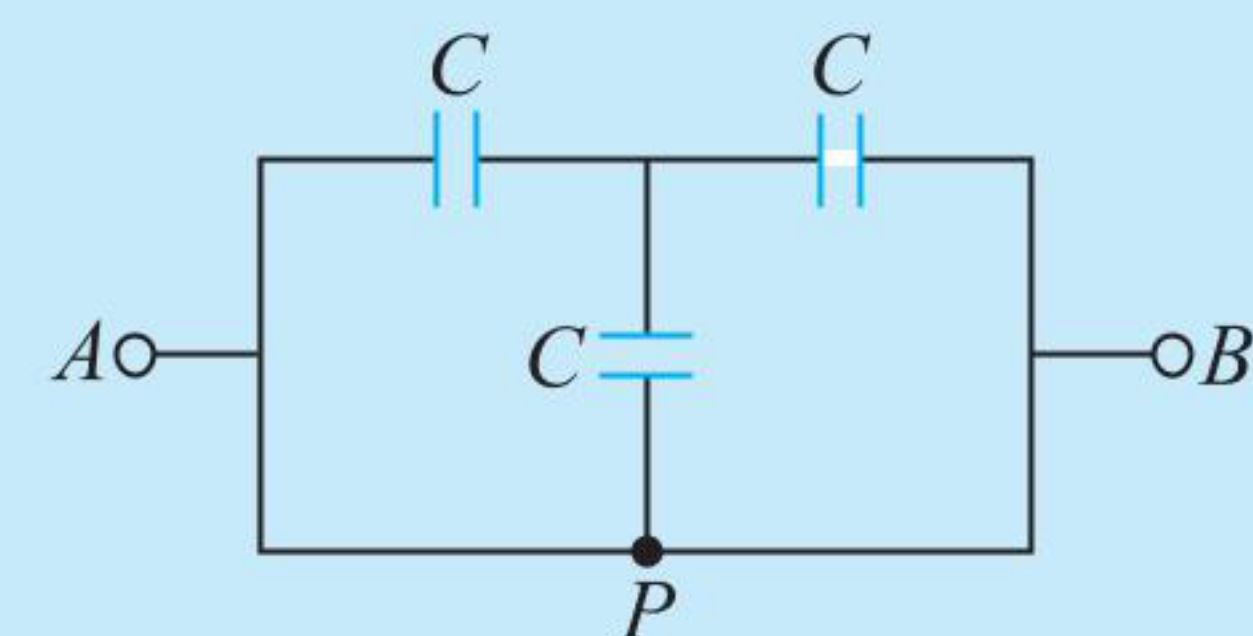
Different capacitors are arranged as shown in figure. Find the equivalent capacity across the points A and B .

**Sol.**

(b)

**ILLUSTRATION 4.22**

Three capacitors are arranged as shown in figure. Find the equivalent capacity across the points A and B .

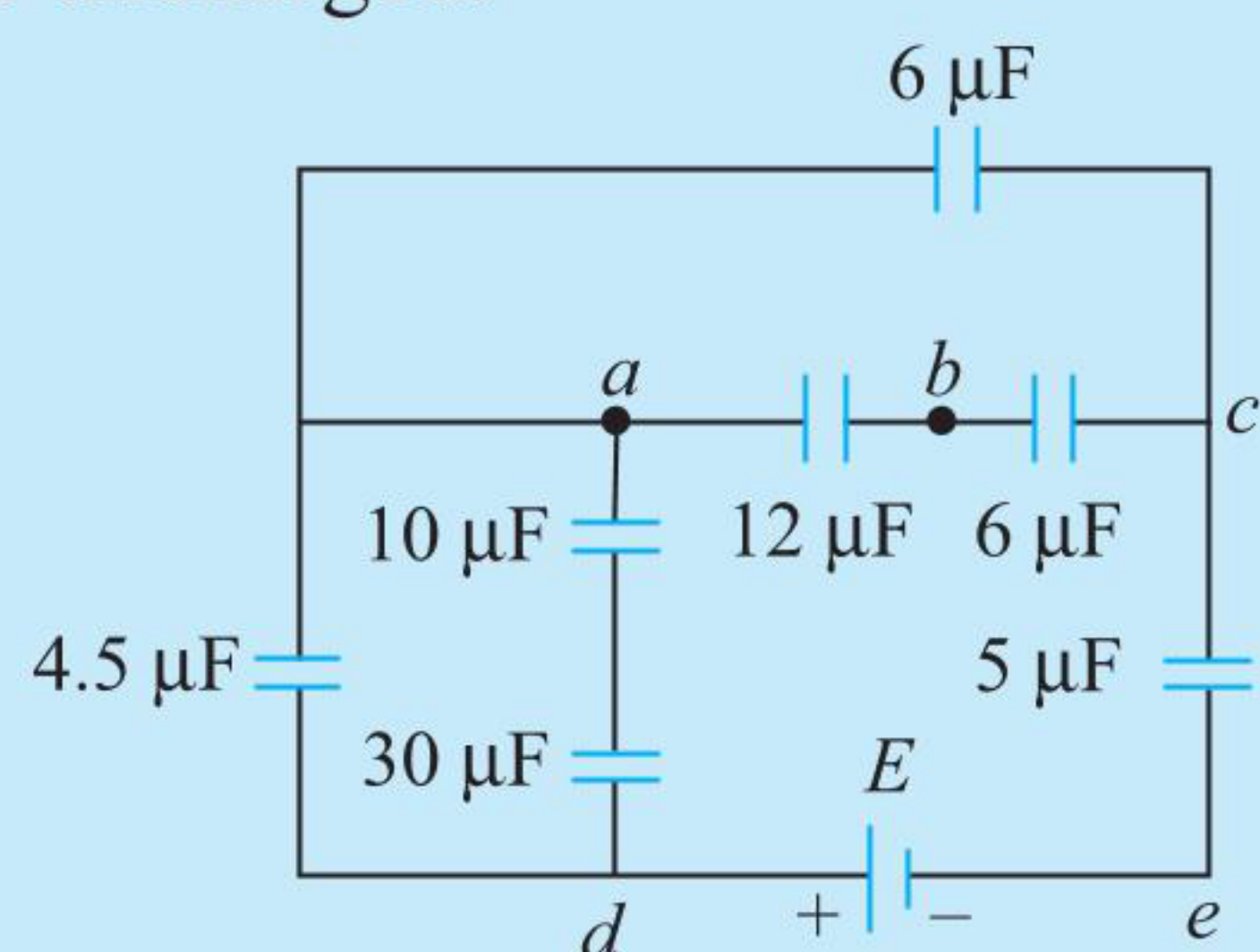


Sol. Since there is no capacitor in the path APB , the points A , P , and B are electrically same, i.e., the input and output points are directly connected (short circuited).

Thus, the entire charge will prefer to flow along the path APB . It means that the capacitors connected in the circuit will not receive any charge for storing. Thus, the equivalent capacitance of this circuit is zero.

ILLUSTRATION 4.23

In the circuit shown in figure, the potential difference between the points a and b is 4 V. Find the emf $\mathcal{E}$ of the battery. Assume that before connecting the battery in the circuit, all the capacitors were uncharged.

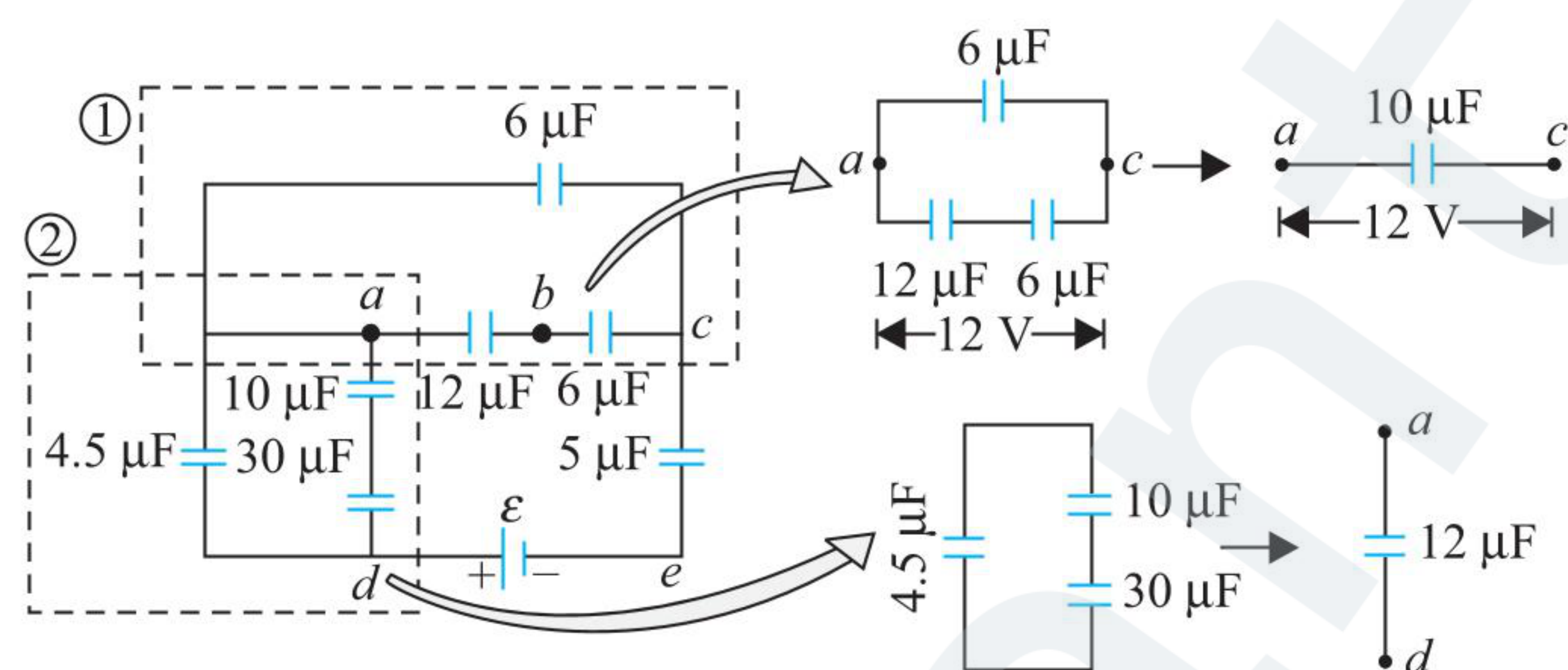


Sol. The capacitors $12 \mu\text{F}$ and $6 \mu\text{F}$ are in series arrangement. The charge in each will be equal.

$$q_1 = CV = 12 \times 4 = 48 \mu\text{C}$$

The potential difference across the $6 \mu\text{F}$ capacitors is

$$V_{bc} = \frac{12 \times 4}{6} \text{ V} = 8 \text{ V}$$



The potential difference between points a and c is $V_a - V_c = 4 + 8 = 12 \text{ V}$. The equivalent capacity in portion (1) is $10 \mu\text{F}$. The charge on it is $(10 \times 12) \mu\text{C} = 120 \mu\text{C}$. Portions (1) and (2) of the circuit are in series combination.

Hence, $12 \times (V_d - V_a) = 120 \mu\text{C}$ or $V_d - V_a = 10 \text{ V}$. The capacitor $5 \mu\text{F}$ is also in series with $10 \mu\text{F}$ (see figure). Hence, charge on it is $120 \mu\text{C}$. Thus, $V_c - V_e = 120/5 = 24 \text{ V}$. Now $E = (12 + 10 + 24) \text{ V} = 46 \text{ V}$.

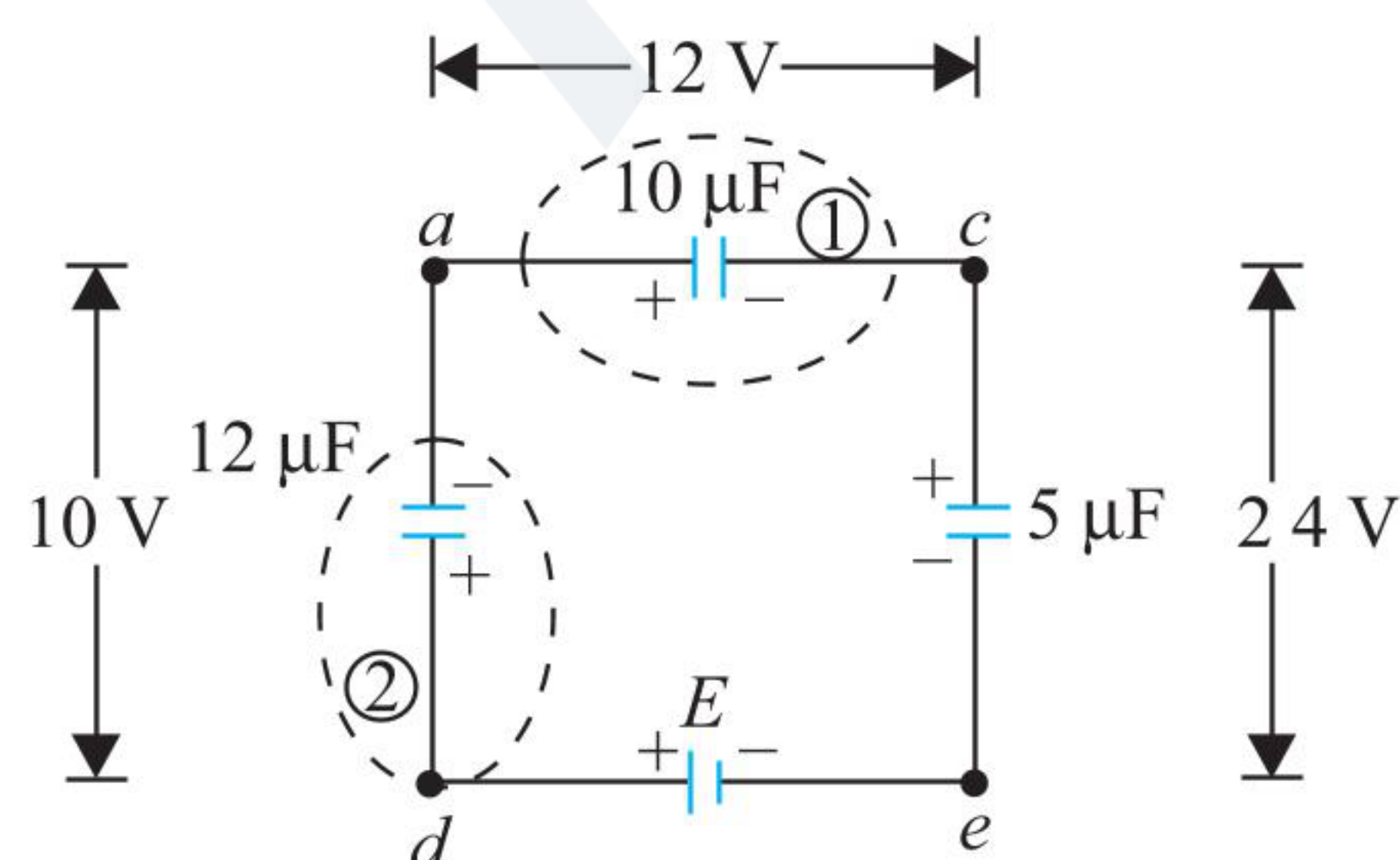
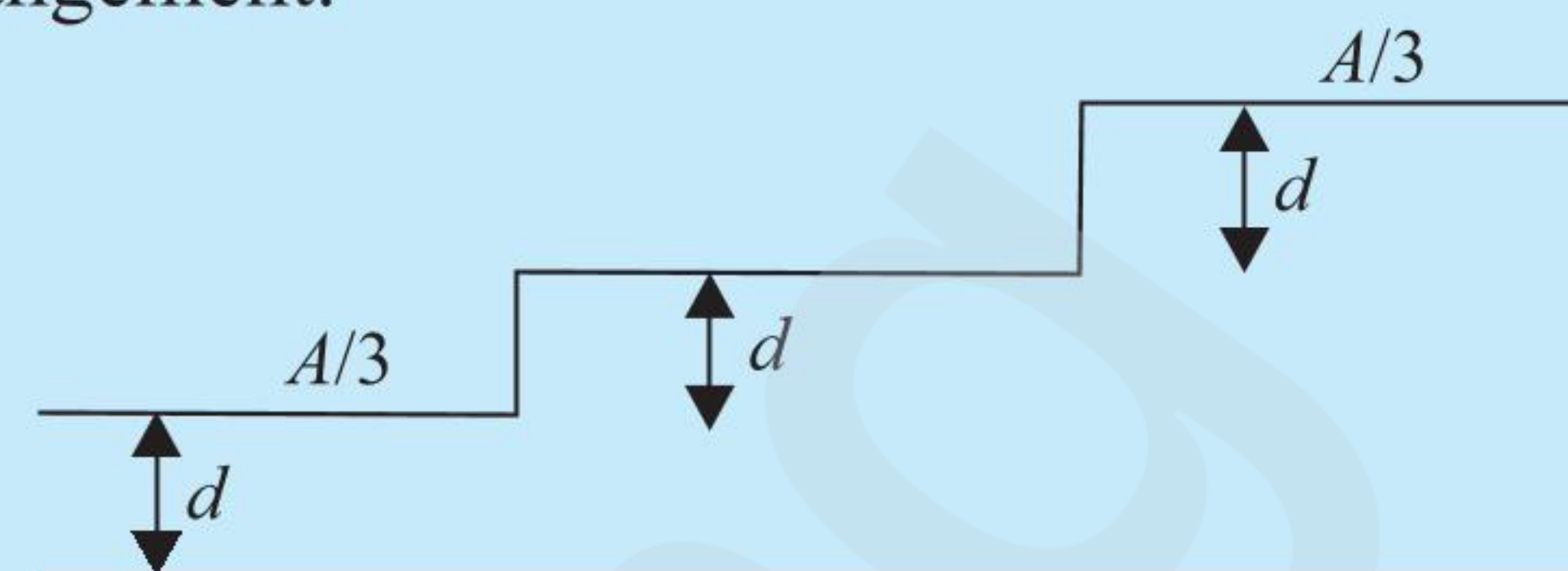


ILLUSTRATION 4.24

A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. The area of each stair is $A/3$, and the height is d . Find the capacitance of this arrangement.

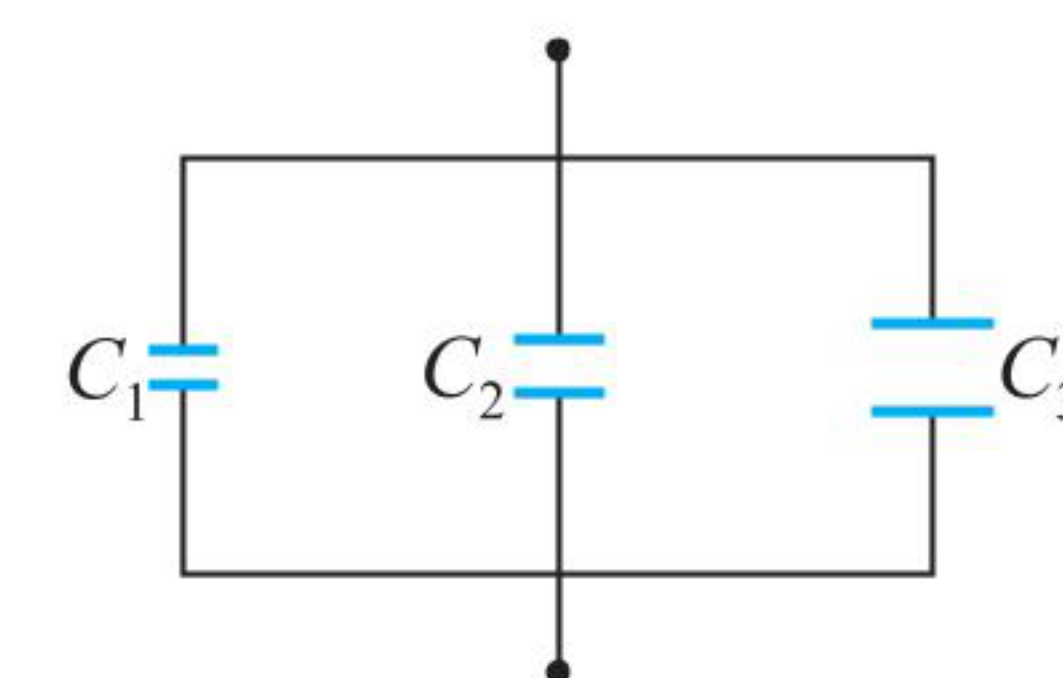


Sol. The arrangement is a parallel combination of three capacitors. Each capacitor has a plate area $A/3$, and the separations between the plates are d , $2d$, and $3d$, respectively. So

$$C_1 = \frac{\epsilon_0 A/3}{d} = \frac{\epsilon_0 A}{3d}$$

$$C_2 = \frac{\epsilon_0 A/3}{2d} = \frac{\epsilon_0 A}{6d}$$

$$C_3 = \frac{\epsilon_0 A/3}{3d} = \frac{\epsilon_0 A}{9d}$$

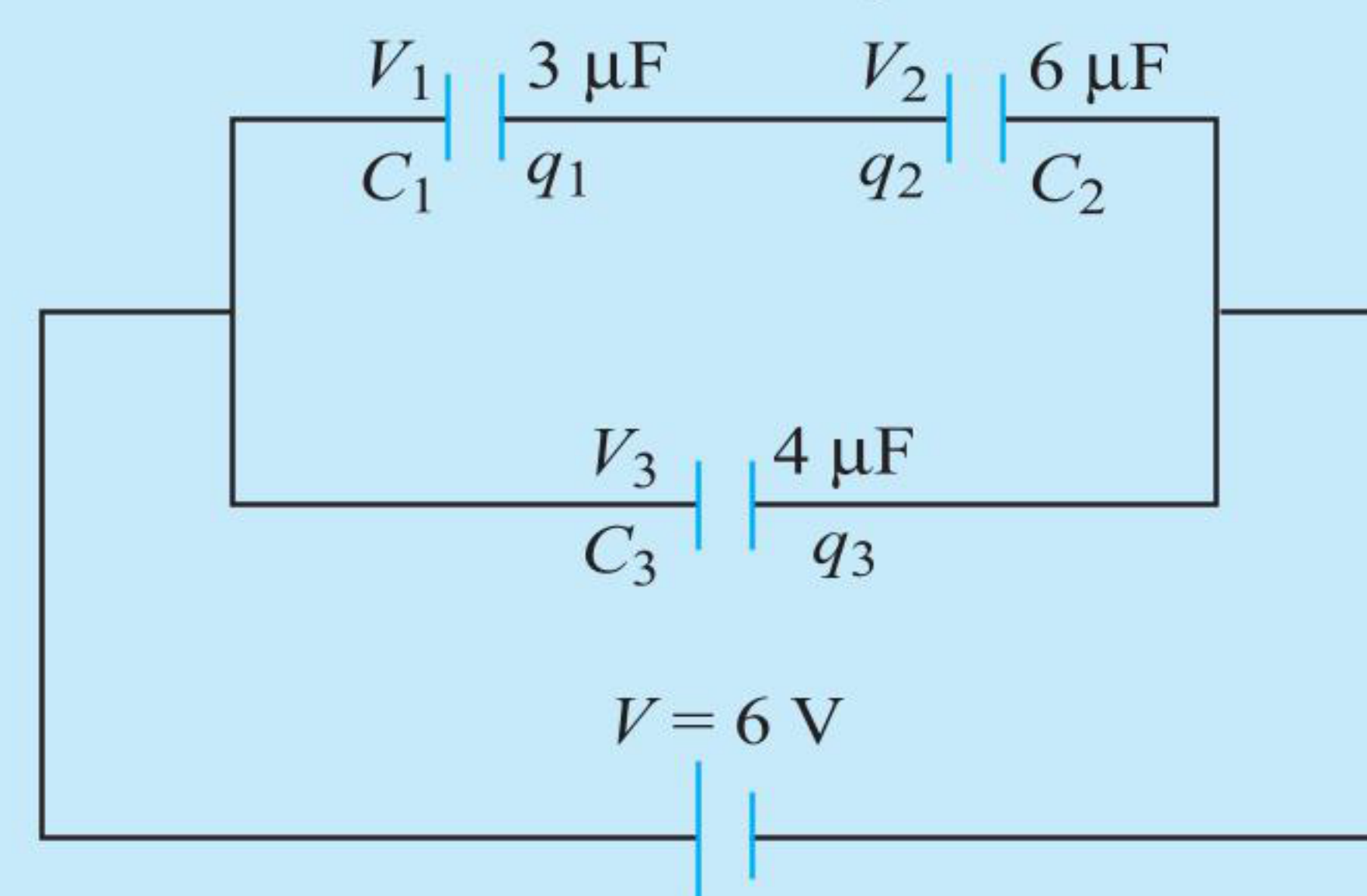


As these three capacitances are in parallel, their equivalent capacitance is given by

$$C = C_1 + C_2 + C_3 = \frac{\epsilon_0 A}{3d} + \frac{\epsilon_0 A}{6d} + \frac{\epsilon_0 A}{9d} = \frac{11\epsilon_0 A}{18d}$$

ILLUSTRATION 4.25

Three capacitors of capacitances $3 \mu\text{F}$, $6 \mu\text{F}$, and $4 \mu\text{F}$ are connected as shown across a battery of emf 6 V.



- Find the equivalent capacitance.
- Find the potential difference and charge on each capacitor.
- Find the energy stored in each capacitor and the total energy stored in the system of capacitors.

Sol.

- C_1 and C_2 are in series, so their equivalent capacitance is

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2 \mu\text{F}$$

This C' will be in parallel with C_3 , so

$$C_{\text{eq}} = C' + C_3 = 2 + 4 = 6 \mu\text{F}$$

- 6 V will be divided across C_1 and C_2 in the inverse ratio of the capacitances. So potential difference across C_1 is

$$V_1 = \frac{C_2 V}{C_1 + C_2} = \frac{6 \times 6}{3 + 6} = 4 \text{ V}$$

and potential difference across C_2 is

$$V_2 = \frac{C_1 V}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2 \text{ V}$$

Because C_3 is connected directly across the battery without any other capacitor in between, so potential difference across C_3 is $V_3 = V = 6 \text{ V}$. Also charge on C_1 is

$$q_1 = C_1 V_1 = 3 \times 4 = 12 \text{ } \mu\text{C}$$

and charge on C_2 is

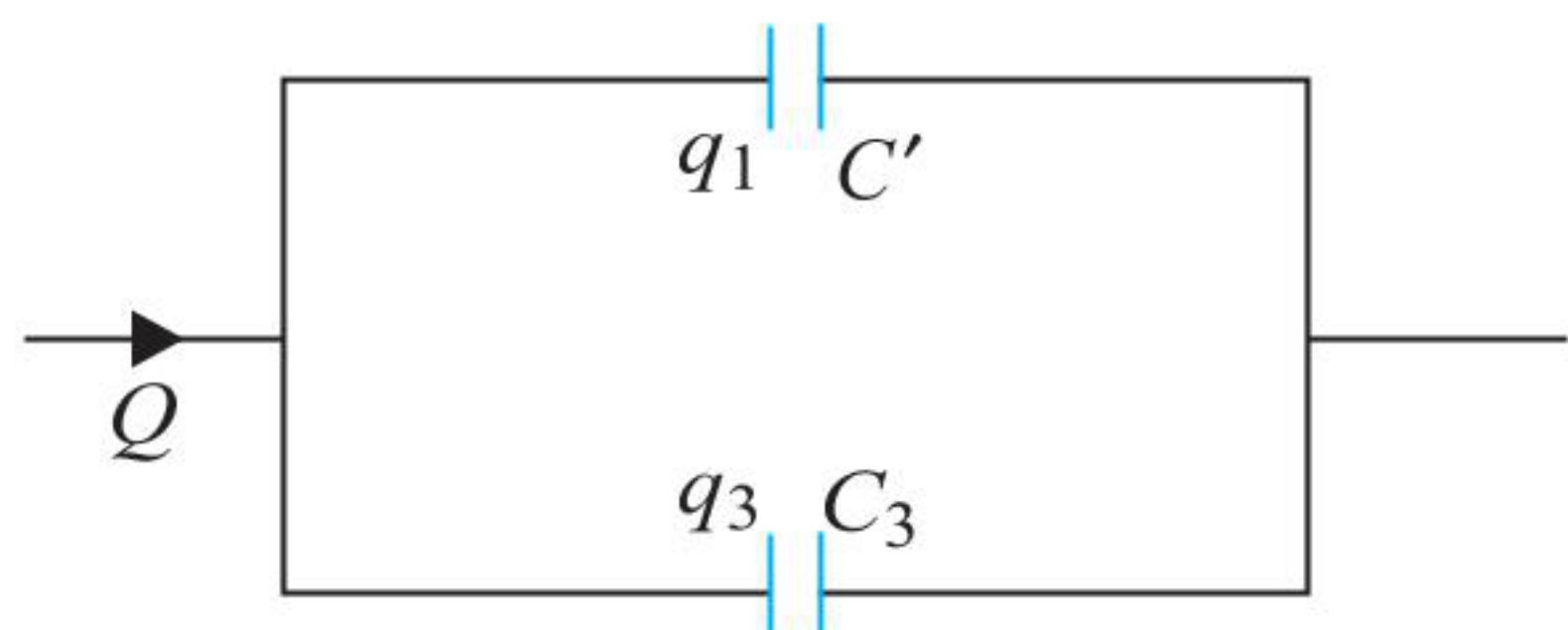
$$q_2 = C_2 V_2 = 6 \times 2 = 12 \text{ } \mu\text{C}$$

We see that $q_1 = q_2$, because in series, charge is the same.

Charge on C_3 is

$$q_3 = C_3 V_3 = 4 \times 6 = 24 \text{ } \mu\text{C}$$

Alternative method to find charge:



Total charge that will flow through the battery is

$$Q = C_{\text{eq}} V = 6 \times 6 = 36 \text{ } \mu\text{C}$$

It will be divided in the direct ratio of C' and C_3 , so

$$q_1 = \frac{C' Q}{C' + C_3} = \frac{2 \times 36}{2 + 4} = 12 \text{ } \mu\text{C}$$

$$q_3 = \frac{C_3 Q}{C' + C_3} = \frac{4 \times 36}{2 + 4} = 24 \text{ } \mu\text{C}$$

(iii) Energy in C_1 is

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 3 \times 4^2 = 24 \text{ } \mu\text{J}$$

Energy in C_2 is

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \times 6 \times 2^2 = 12 \text{ } \mu\text{J}$$

Energy in C_3 is

$$U_3 = \frac{1}{2} C_3 V^2 = \frac{1}{2} \times 4 \times 6^2 = 72 \text{ } \mu\text{J}$$

Total energy is

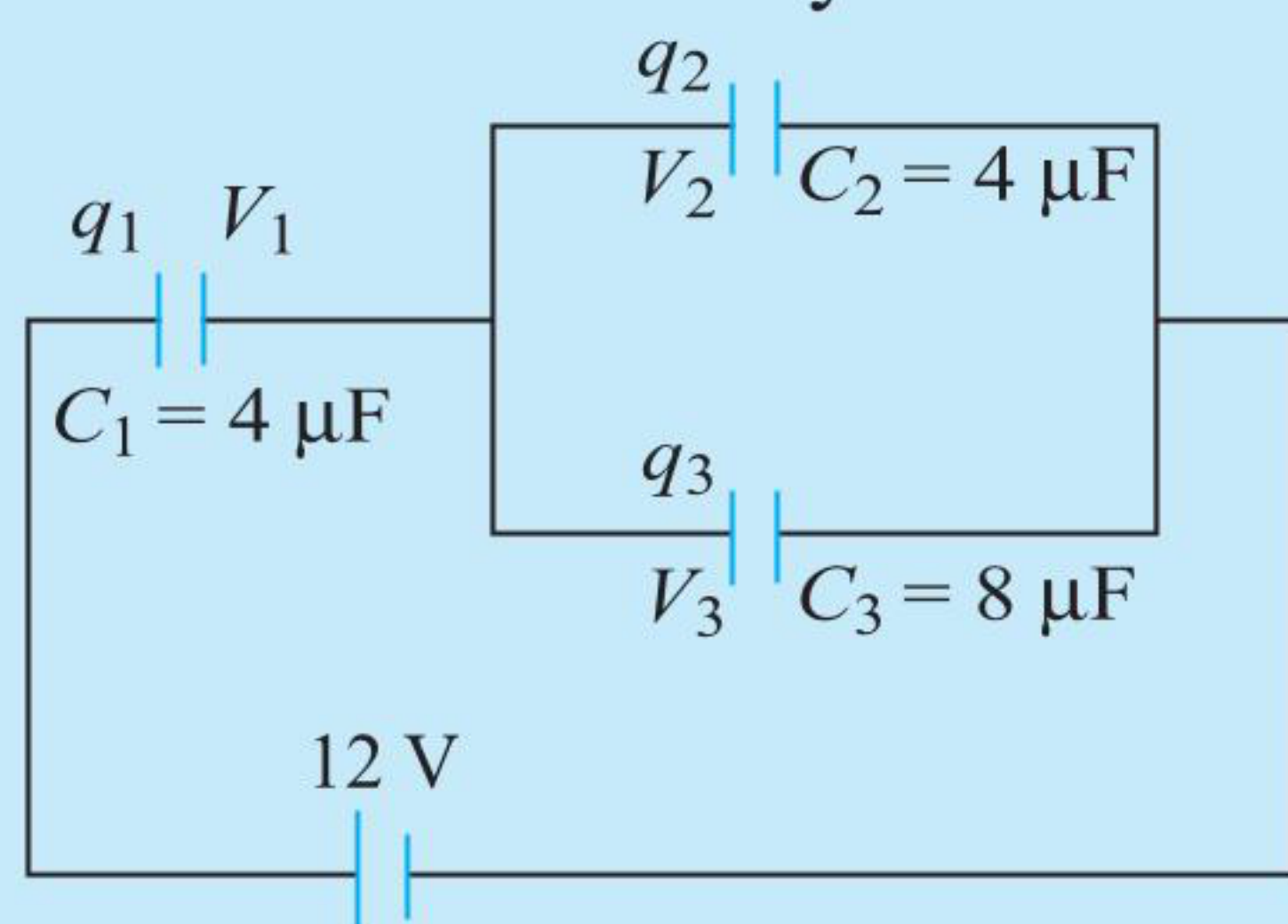
$$U = U_1 + U_2 + U_3 = 24 + 12 + 72 = 108 \text{ } \mu\text{J}$$

Alternatively

$$U = \frac{1}{2} C_{\text{eq}} V^2 = \frac{1}{2} \times 6 \times 6^2 = 108 \text{ } \mu\text{J}$$

ILLUSTRATION 4.26

Three capacitors of capacitances $4 \text{ } \mu\text{F}$, $4 \text{ } \mu\text{F}$, and $8 \text{ } \mu\text{F}$ are connected as shown across a battery of emf 12 V .



(i) Find the equivalent capacitance.

(ii) Find the potential difference and charge on each capacitor.

(iii) Find the energy stored in each capacitor and the total energy stored in the system of capacitors.

Sol.

(i) C_2 and C_3 are in parallel, so their equivalent capacitance is

$$C' = C_2 + C_3 = 4 + 8 = 12 \text{ } \mu\text{F}$$

This C' will be in series with C_1 , so the net equivalent capacitance is

$$C_{\text{eq}} = \frac{C_1 C'}{C_1 + C'} = \frac{4 \times 12}{4 + 12} = 3 \text{ } \mu\text{F}$$

(ii) $q_1 = C_{\text{eq}} V = 3 \times 12 = 36 \text{ } \mu\text{C}$.

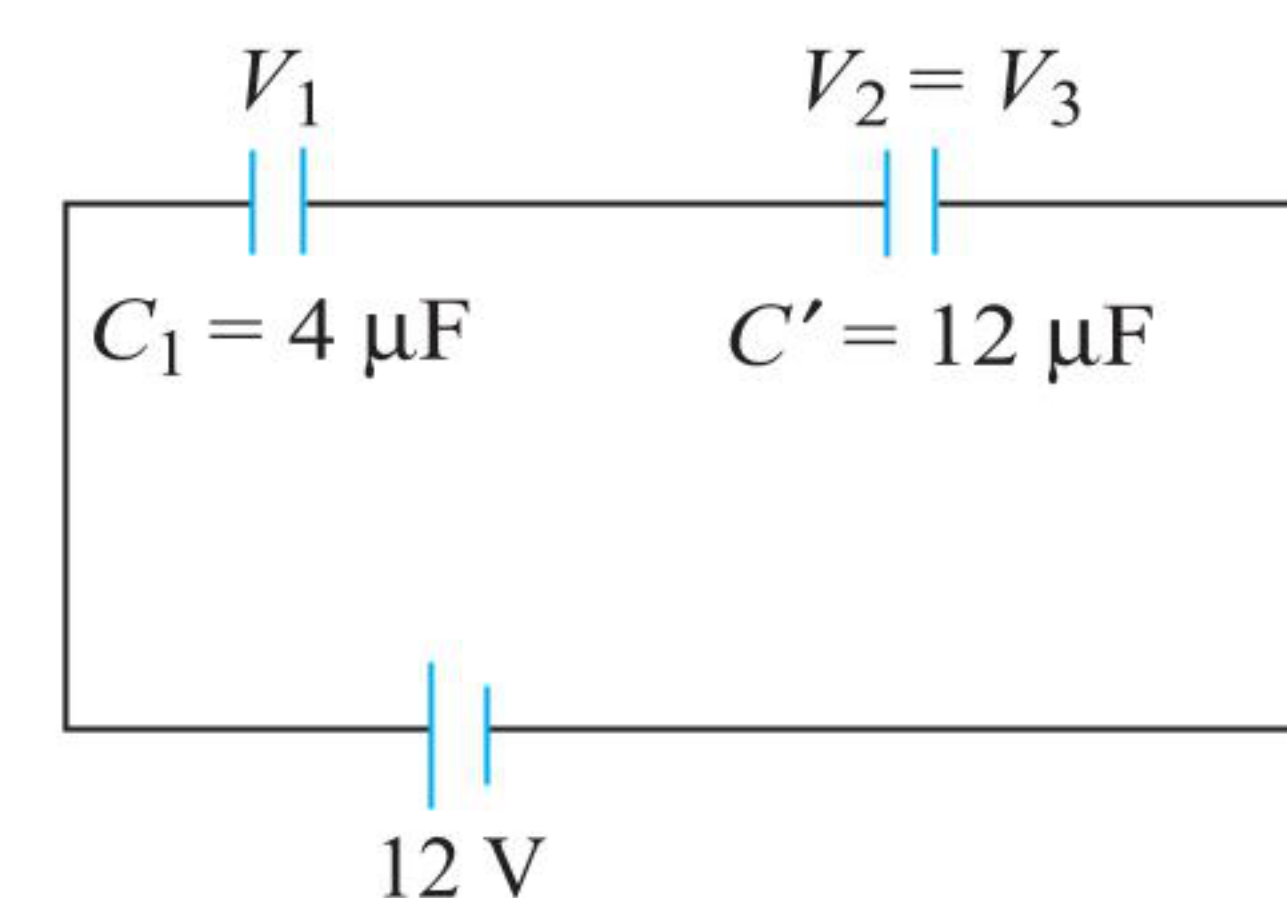
q_1 will be divided into q_2 and q_3 , so

$$q_2 = \frac{C_2 q_1}{C_2 + C_3} = \frac{4 \times 36}{4 + 8} = 12 \text{ } \mu\text{C}$$

$$q_3 = \frac{C_3 q_1}{C_2 + C_3} = \frac{8 \times 36}{4 + 8} = 24 \text{ } \mu\text{C}$$

$$V_1 = \frac{q_1}{C_1} = \frac{36}{4} = 9 \text{ V}$$

$$V_2 = V_3 = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \frac{36}{4} = 3 \text{ V}$$



Alternatively, potentials can also be calculated by dividing 12 V in the inverse ratio of C_1 and C' .

$$V_1 = \frac{C' V}{C_1 + C'} = \frac{12 \times 12}{4 + 12} = 9 \text{ V}$$

$$V_2 = V_3 = \frac{C_1 V}{C_1 + C'} = \frac{4 \times 12}{4 + 12} = 3 \text{ V}$$

(iii) $U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 4 \times 9^2 = 162 \text{ } \mu\text{J}$

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \times 4 \times 3^2 = 18 \text{ } \mu\text{J}$$

$$U_3 = \frac{1}{2} C_3 V_3^2 = \frac{1}{2} \times 8 \times 3^2 = 36 \text{ } \mu\text{J}$$

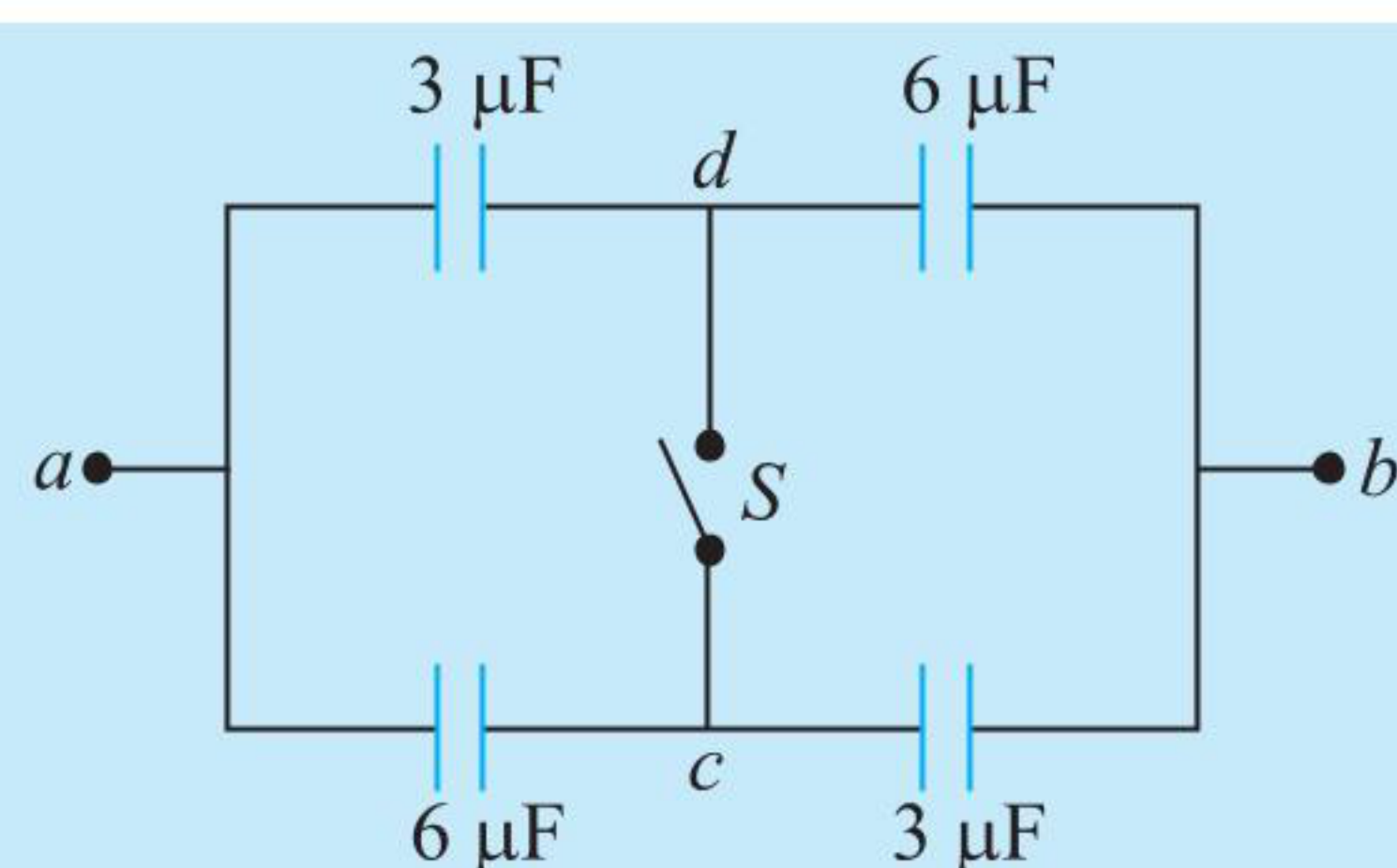
Total energy is $U_1 + U_2 + U_3 = 216 \text{ } \mu\text{J}$

Alternatively

$$U = \frac{1}{2} C_{\text{eq}} V^2 = \frac{1}{2} \times 3 \times (12)^2 = 216 \text{ } \mu\text{J}$$

ILLUSTRATION 4.27

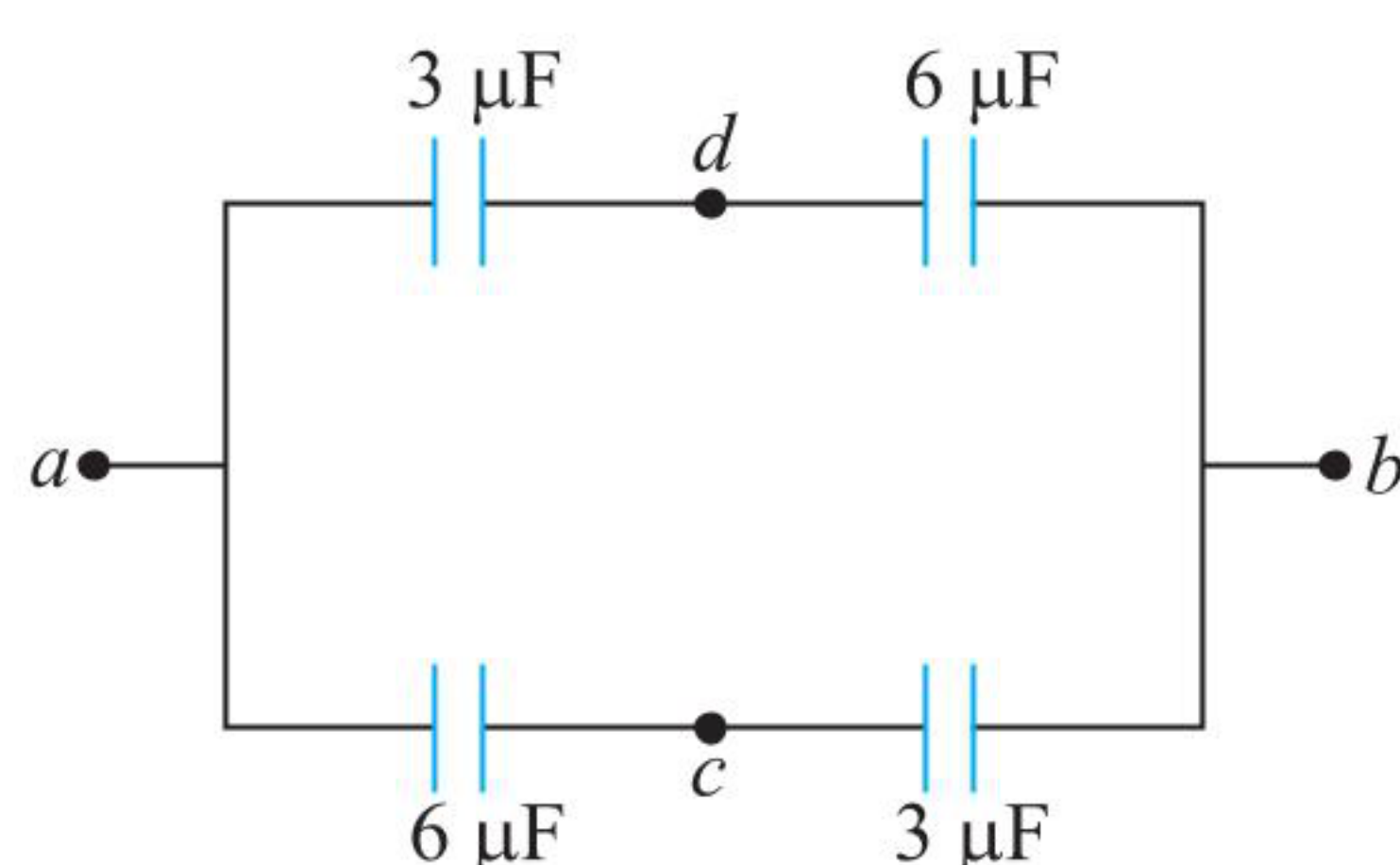
The capacitors in figure are initially uncharged and are connected as in the diagram with switch S open. The applied potential difference is $V_{ab} = +360 \text{ V}$.



- (a) What is the potential difference V_{cd} ?
 (b) What is the potential difference across each capacitor after switch S is closed?
 (c) How much charge flowed through the switch when it was closed?

Sol.

- (a) When switch is open, the potential difference across the upper and lower branch should be equal as both are connected in parallel.



The potential difference across a and d ,

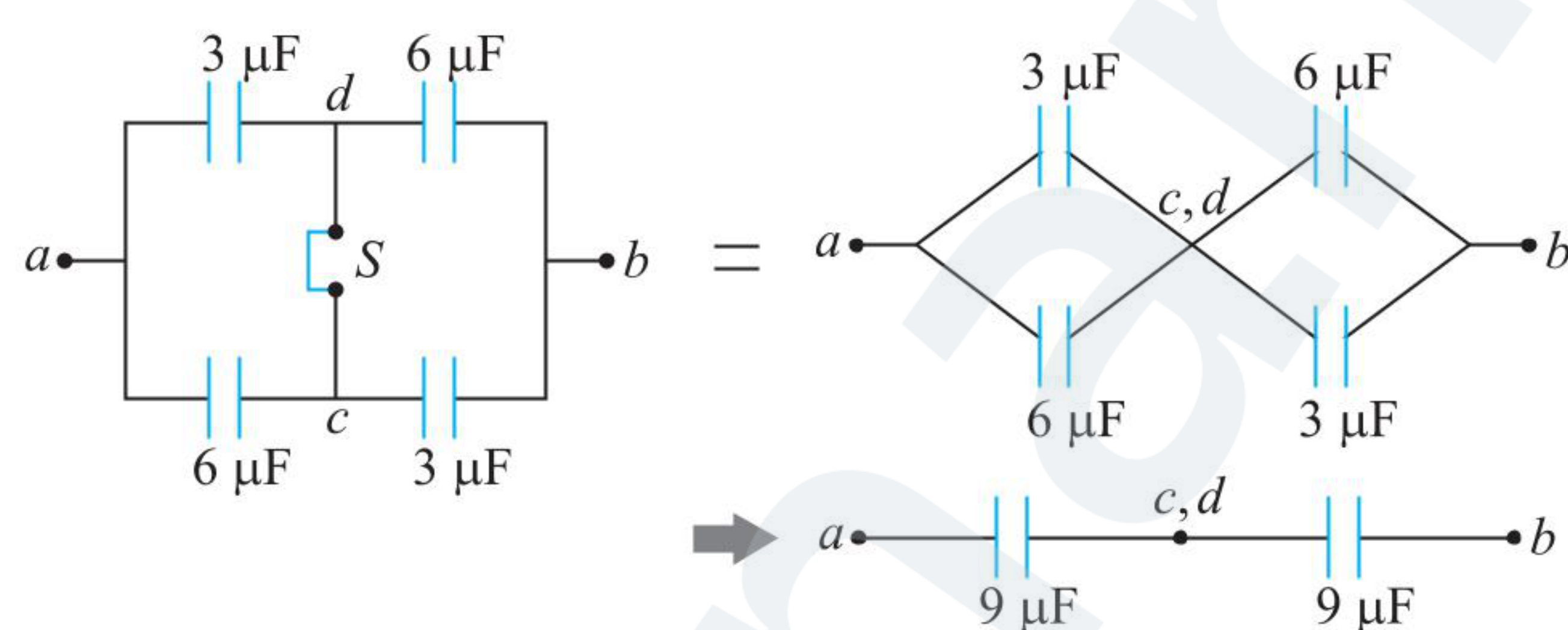
$$V_a - V_d = \frac{6}{6+3} \times 360 = 240 \text{ V} \quad \dots(i)$$

and the potential difference across a and c ,

$$V_a - V_c = \frac{3}{6+3} \times 360 = 120 \text{ V} \quad \dots(ii)$$

From (i) and (ii), $V_c - V_d = V_{cd} = 120 \text{ V}$

- (b) When switch S is closed, the circuit will be reduced as shown in figure.



$$\begin{aligned} \text{From figure, } V_a - V_c &= V_a - V_d = V_d - V_b = V_c - V_b \\ &= \frac{360}{2} = 180 \text{ V} \end{aligned}$$

This is, the potential across each capacitor is 180 V

- (c) Before closing the switch, the charge in each capacitor,

$$q = \frac{3 \times 6}{6+3} \times 360 = 720 \mu\text{C}$$

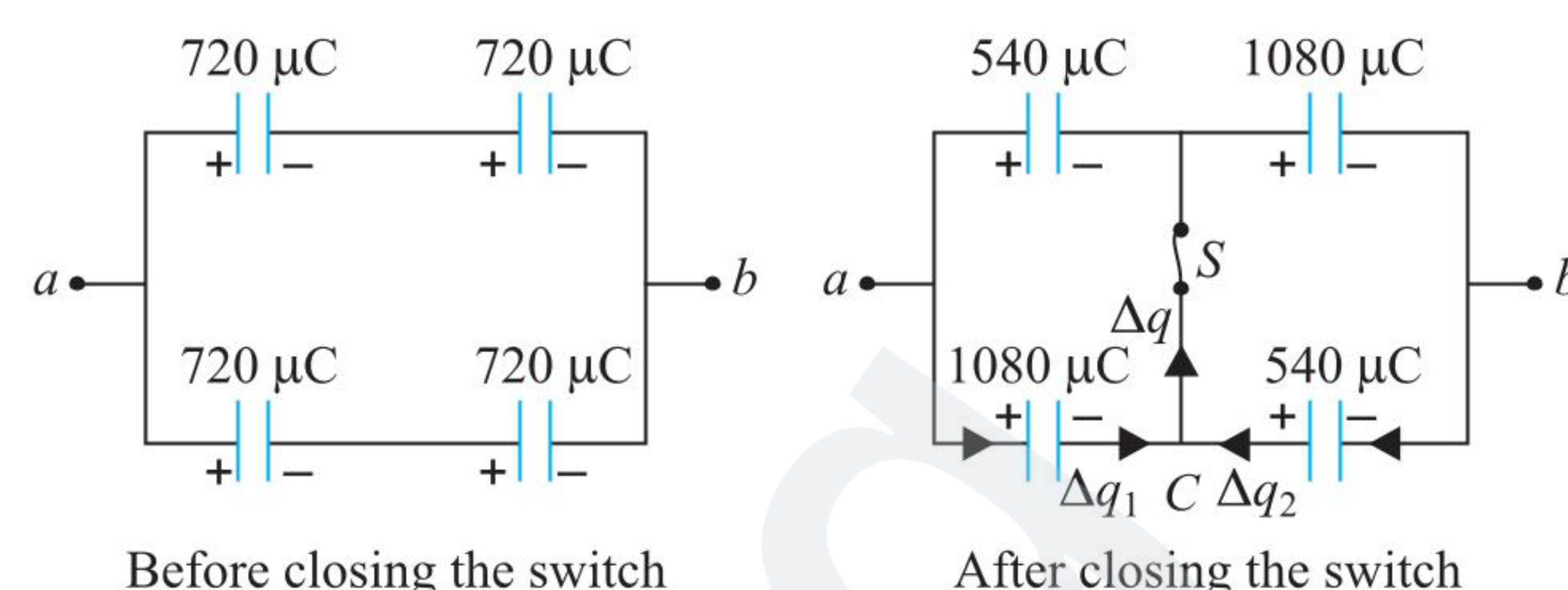
After closing the switch, the charge in the capacitors having capacitance $3 \mu\text{F}$:

$$q_{3\mu\text{F}} = 3 \times 180 = 540 \mu\text{C}$$

The charge in the capacitors having capacitance $6 \mu\text{F}$:

$$q_{6\mu\text{F}} = 6 \times 180 = 1080 \mu\text{C}$$

The charge appearing on each capacitor before and after closing the switch are shown in figure.



From figure we can observe, $\Delta q_1 = 1080 - 720 = 360 \mu\text{C}$ and $\Delta q_2 = 720 - 540 = 180 \mu\text{C}$

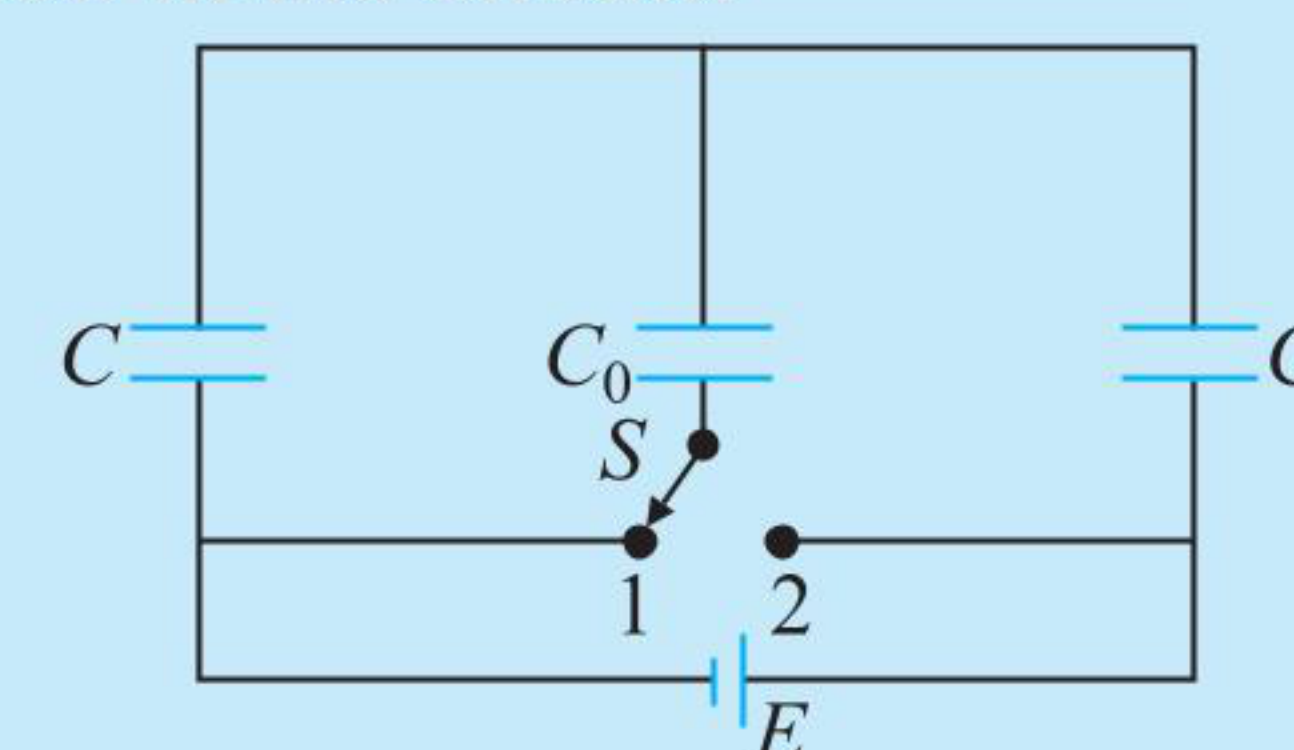
Now, at junction c incoming charge should be equal to outgoing charge.

$$\text{Hence, } \Delta q = \Delta q_1 + \Delta q_2 = 360 + 180 = 540 \mu\text{C}$$

It means, charge of $540 \mu\text{C}$ flows from c to d after closing the switch.

ILLUSTRATION 4.28

In the given figure, when switch is swapped from 1 to 2, find the heat produced in the circuit.



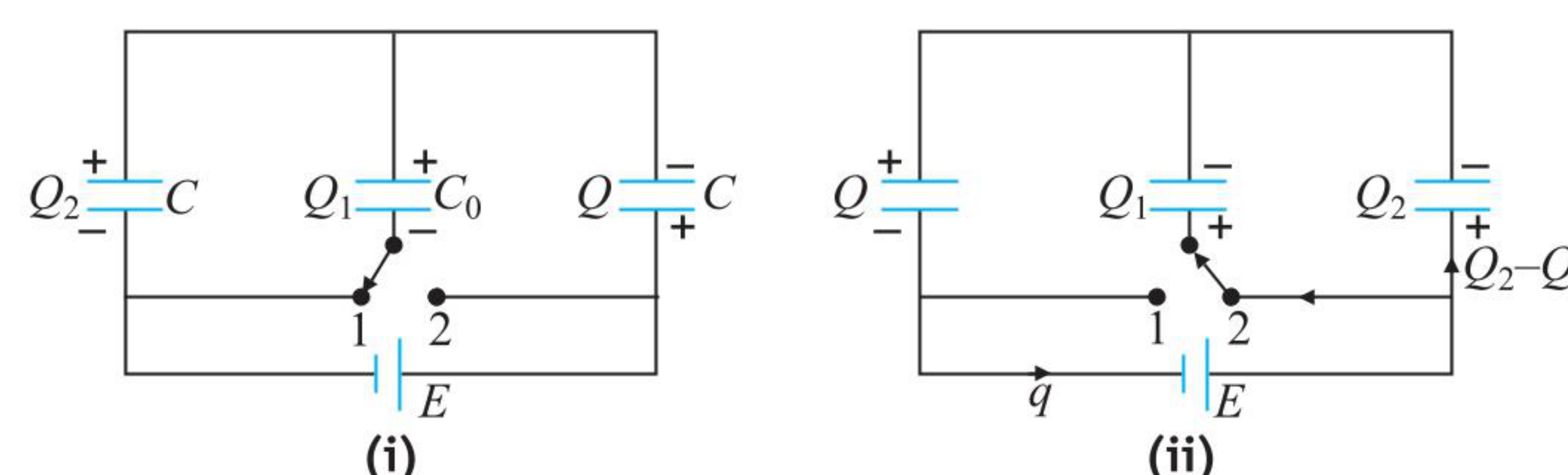
Sol. When the switch is at position 1, the circuit diagram is as shown in Fig. (i). The equivalent capacitance of the circuit,

$$C_{\text{eq}} = \frac{(C + C_0)C}{2C + C_0} \text{ and } Q = C_{\text{eq}} E$$

The charge supplied by battery, $Q = C_{\text{eq}} E = \frac{(C + C_0)C}{2C + C_0} E$

Hence the charge in other capacitors as connected in Fig. (i)

$$Q_1 = \frac{C_0 Q}{C + C_0} \text{ and } Q_2 = \frac{C Q}{C + C_0}$$



After shifting the switch from position 1 to position 2, the equivalent capacitance of the circuit will not change but the charge on different capacitors will be as shown in Fig. (ii). At this stage the charge flowing through the left most capacitor 'C' should be equal to the charge supplied by the battery in this process

Hence the charge flowing battery on reconnection,

$$q = Q - Q_2 = Q - \frac{C Q}{C + C_0} = \frac{C_0}{C + C_0} Q = \frac{C_0}{C + C_0} (C_{\text{eq}} E)$$

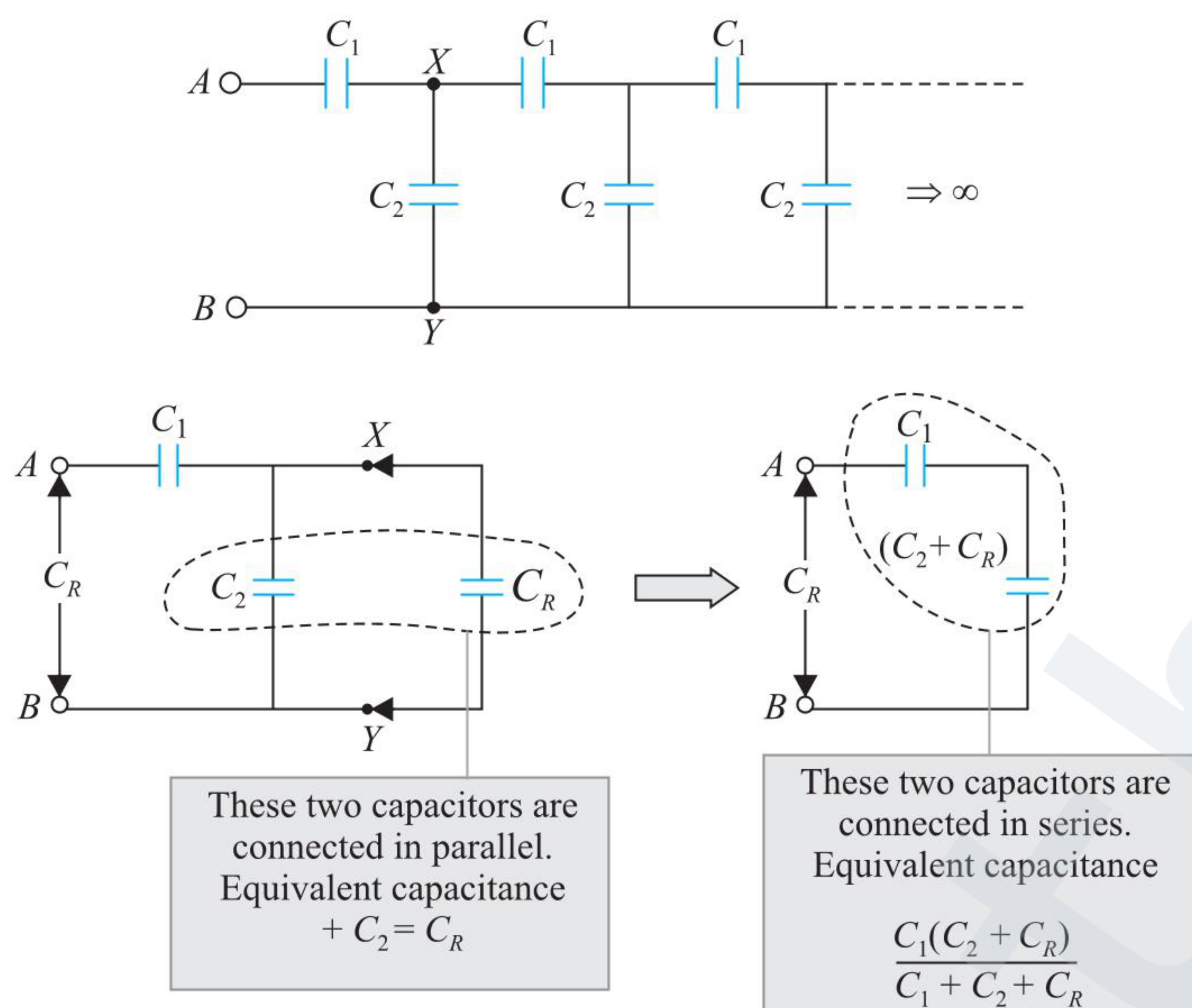
As the total energy on capacitors before and after reconnection is same. Hence whole work done by battery will go in the form of heat.

$$W_{\text{battery}} = qE = \left(\frac{C_0}{C + C_0} \times \frac{(C + C_0)C}{(2C + C_0)} E \right) E = \frac{CC_0 E^2}{2C + C_0}$$

So heat generated is $H = \frac{CC_0 E^2}{2C + C_0}$

INFINITE CHAIN OF CAPACITORS

Suppose the effective capacitance between A and B is C_R . Since the network is infinite, even if we remove one pair of capacitors from the chain, the remaining network would still have infinite pairs of capacitors, i.e., effective capacitance between X and Y would also be C_R .

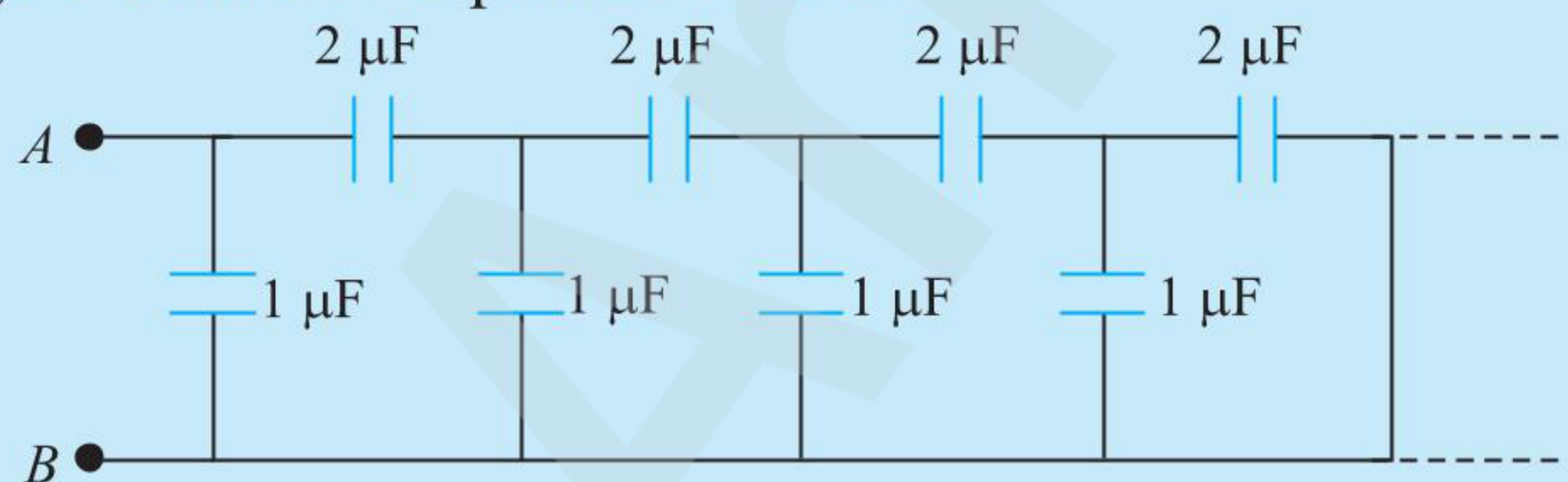


Hence, the equivalent capacitance between A and B is

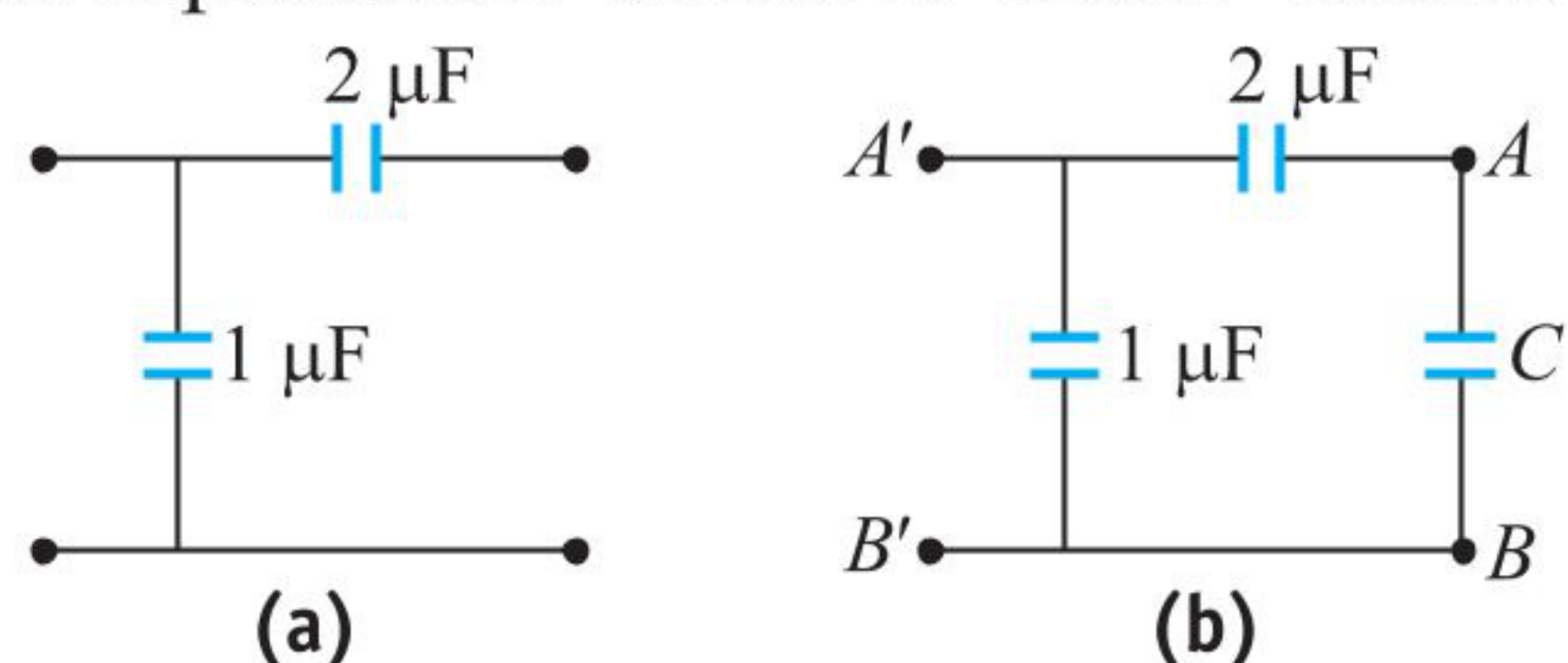
$$C_{AB} = \frac{C_1(C_2 + C_R)}{C_1 + C_2 + C_R} = C_R \text{ or } C_{AB} = \frac{C_2}{2} \left[\sqrt{1 + 4 \frac{C_1}{C_2}} - 1 \right]$$

ILLUSTRATION 4.29

Find the equivalent capacitance of the infinite ladder shown in figure between the points A and B .



Sol. First of all we need to identify repeating unit. The repeating unit is as shown in Fig. (a). Let equivalent capacitance across A and B be C . If we add one more unit as shown in figure the equivalent capacitance across A' and B' should again be C .



$$C_{A'B'} = C = 1 + \frac{2C}{2+C} \Rightarrow C = \frac{2+C+2C}{2+C}$$

$$\Rightarrow (2+C)C = (3C+2) \Rightarrow C^2 - C - 2 = 0$$

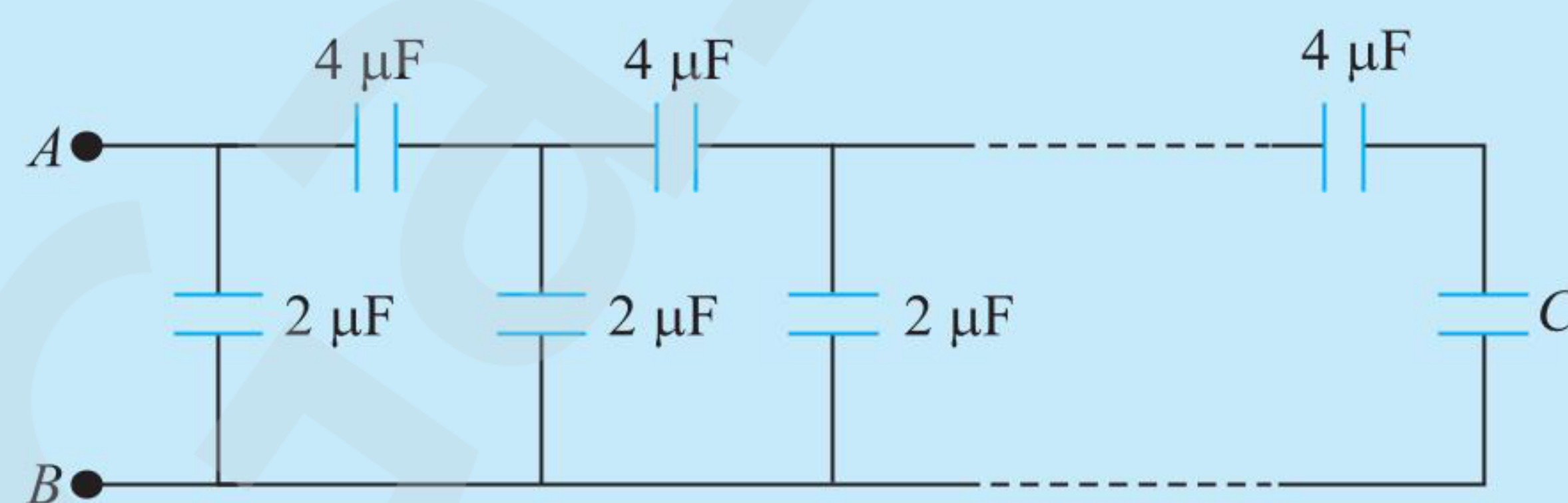
$$\Rightarrow (C-2)(C+1) = 0$$

Since $C = -1$ is not acceptable

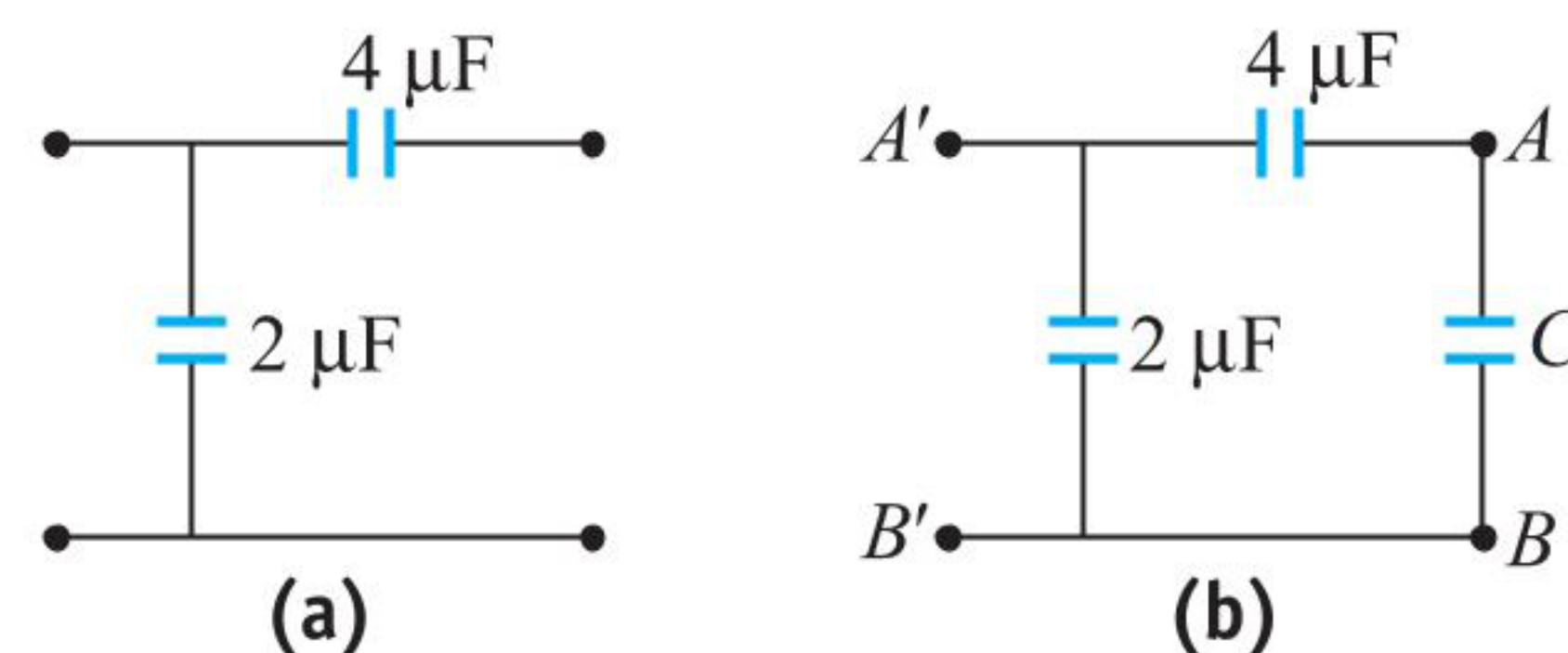
Hence equivalent capacitance, $C = 2 \mu\text{F}$.

ILLUSTRATION 4.30

An infinite ladder is constructed by connecting several sections of $2 \mu\text{F}$, $4 \mu\text{F}$ capacitor combinations as shown in figure. It is terminated by a capacitor of capacitance C . What value should be chosen for C such that the equivalent capacitance of the ladder between A and B becomes independent of the number of sections in between?



Sol. The equivalent capacitance of the given infinite ladder between A and B becomes independent of the number of units in between. It means, if we remove all the capacitors, other than terminal capacitor ' C ', the equivalent capacitance across A and B should also be ' C '. Here the repeating unit is shown in Fig. (a). If we connect one unit across A and B , as shown in Fig. (b) the equivalent capacitance across A' and B' should remain ' C '.



$$C_{A'B'} = C = 2 + \frac{4 \times C}{4+C} \Rightarrow 4C + C^2 = 8 + 2C + 4C$$

$$\Rightarrow C^2 - 2C - 8 = 0 \Rightarrow (C+2)(C-4) = 0$$

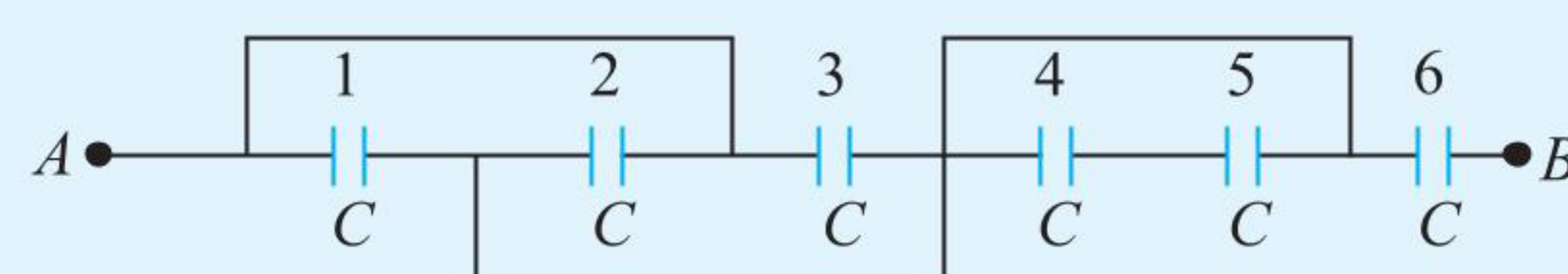
$$\Rightarrow C = -2 \text{ and } 4$$

As $C = -2$ is not acceptable.

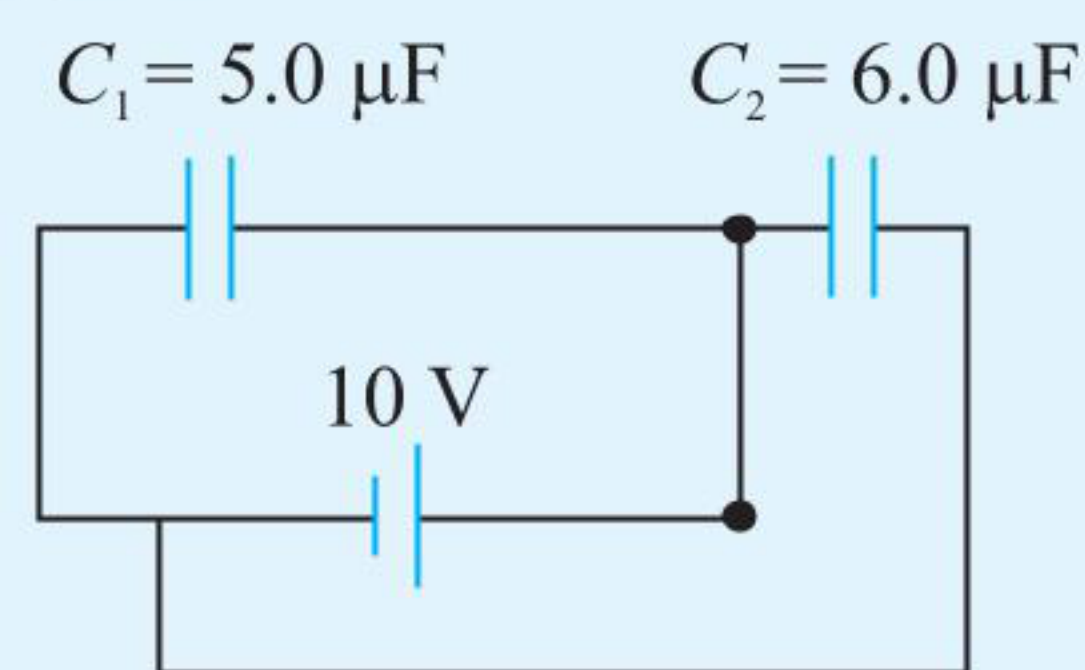
Hence equivalent capacitance, $C = 4 \mu\text{F}$.

CONCEPT APPLICATION EXERCISE 4.3

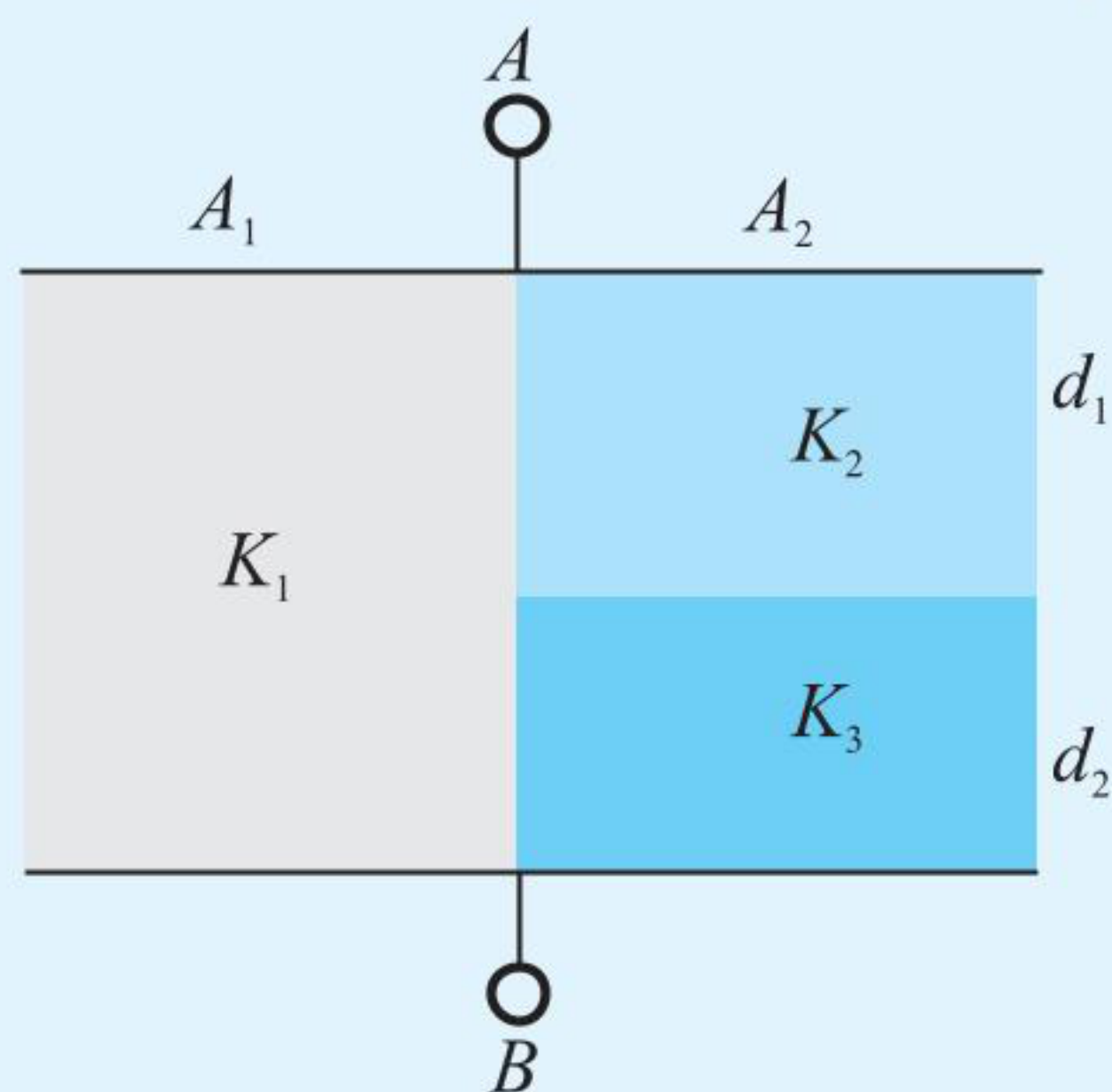
- If you have several $2.0 \mu\text{F}$ capacitors, each capable of withstanding 200 V without breakdown, how would you assemble a combination that has an equivalent capacitance of
(a) $0.4 \mu\text{F}$ (b) $1.2 \mu\text{F}$, each withstanding 1000 V ?
- N identical capacitors are connected in parallel, and then a potential difference of V is applied to them. Find the potential difference when these capacitors are reconnected in series, their charges being left undisturbed.
- Find the equivalent capacitance between points A and B as shown in figure.



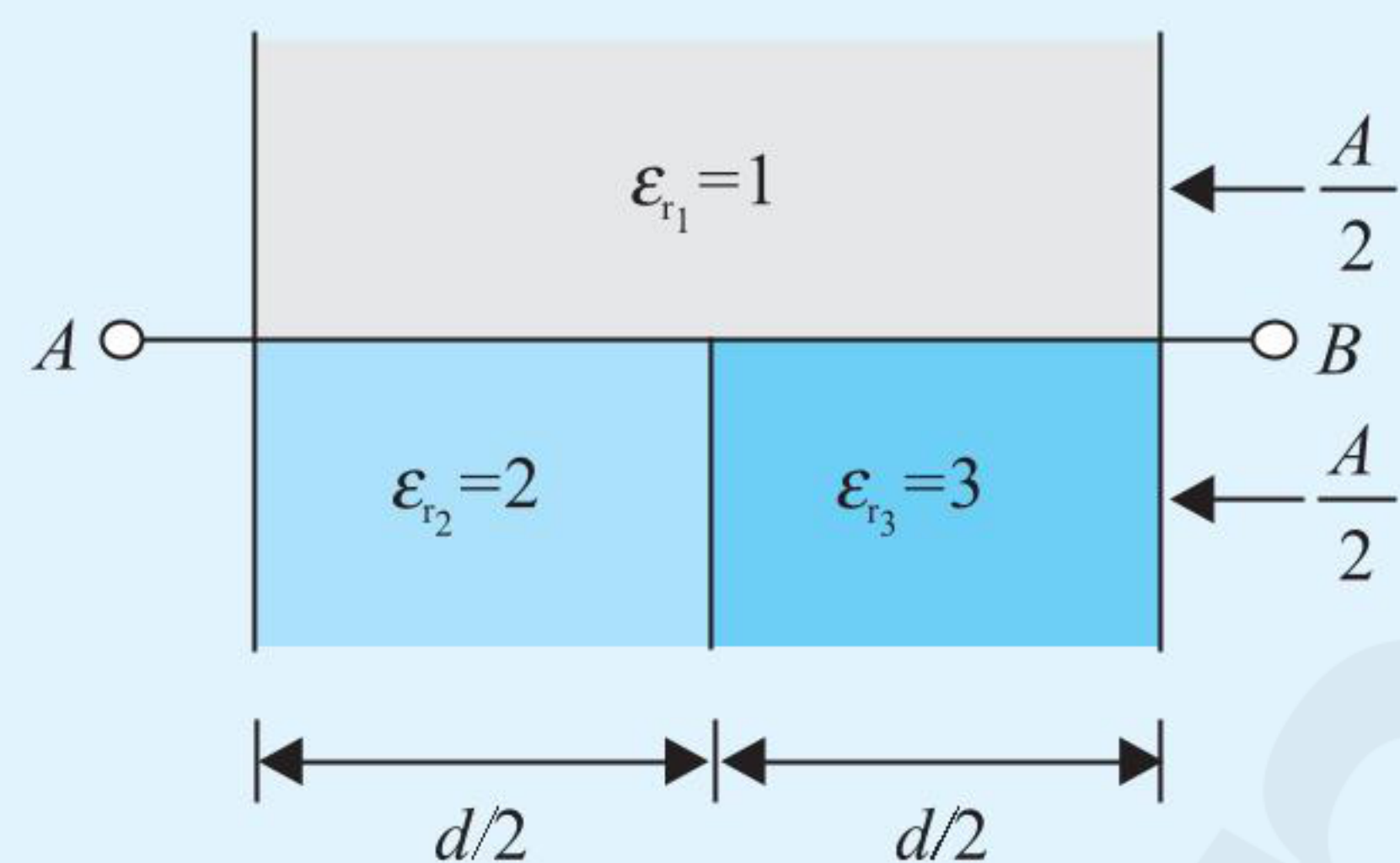
4. Find the charge supplied by the battery in the arrangement as shown in figure.



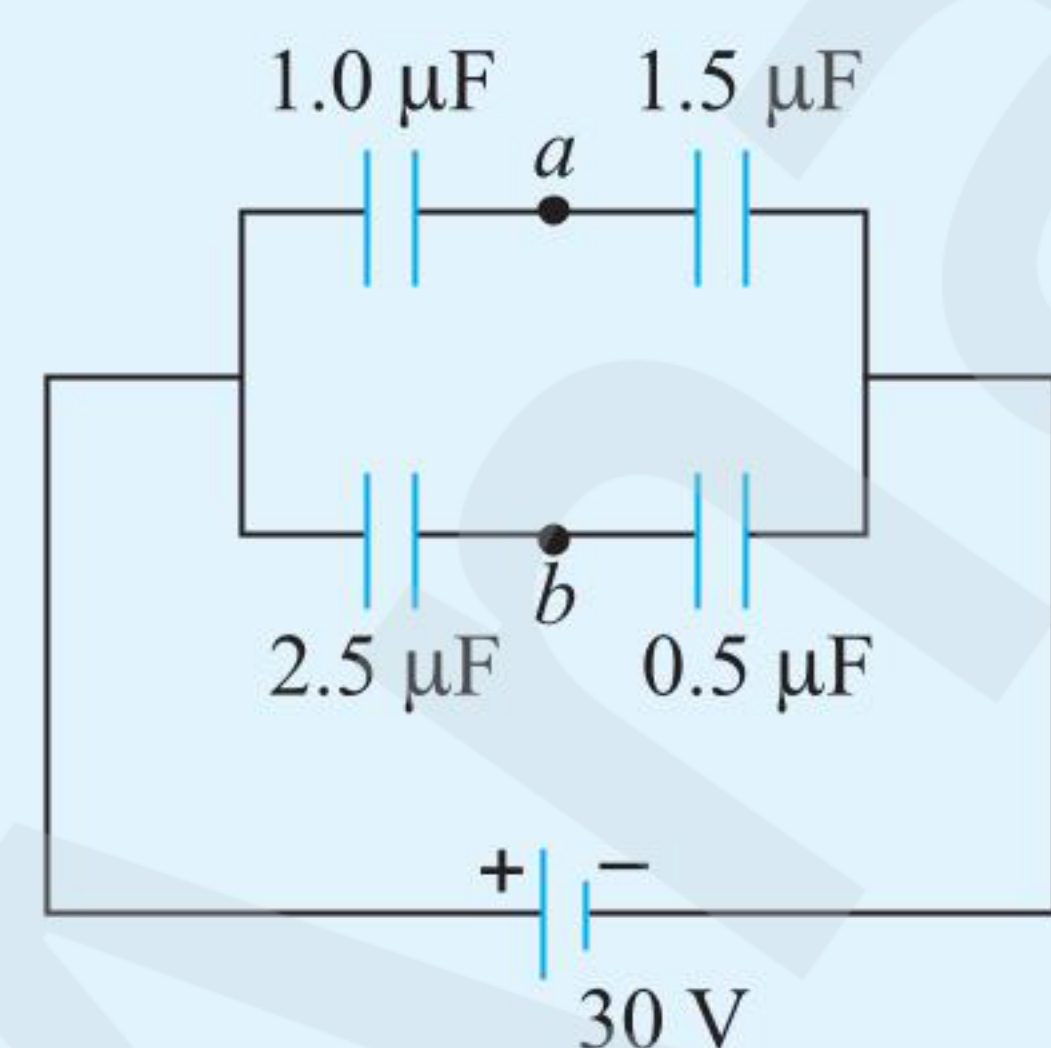
5. Find the capacitance between A and B if three dielectric slabs of dielectric constants K_1 (area A_1 and thickness d), K_2 (area A_2 and thickness d_1), and K_3 (area A_2 and thickness d_2) are inserted between the plates of a parallel plate capacitor of plate area A (figure). (Given that the distance between the two plates is $d = d_1 + d_2$.)



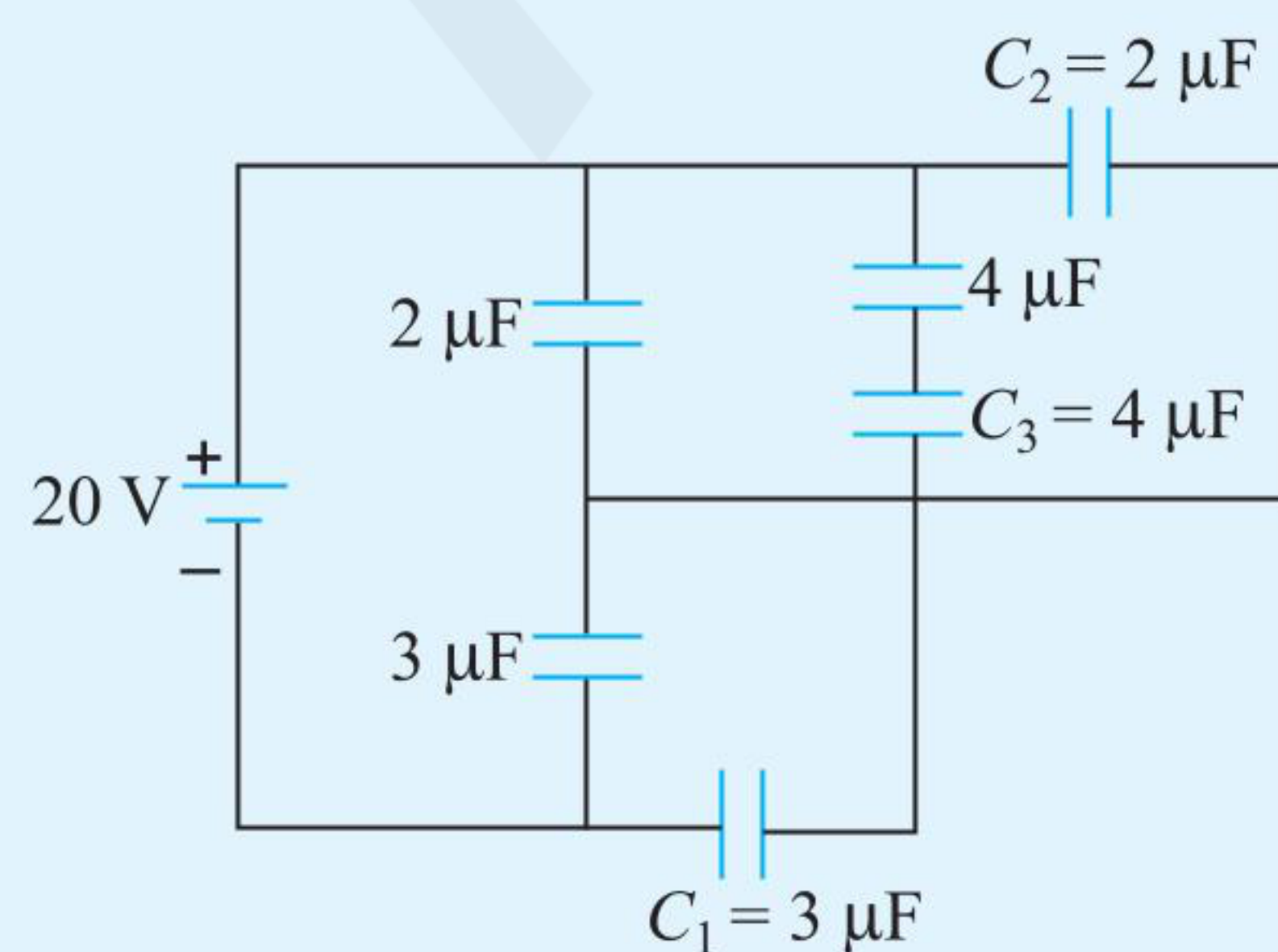
6. Three dielectrics of relative permittivities $\epsilon_{r1} = 1$, $\epsilon_{r2} = 2$, and $\epsilon_{r3} = 3$ are introduced in a parallel plate capacitor of plate area A and separation d . Find the effective capacitance between A and B .



7. Four capacitors are connected as shown in figure to a 30 V battery. Find the potential difference between points a and b .

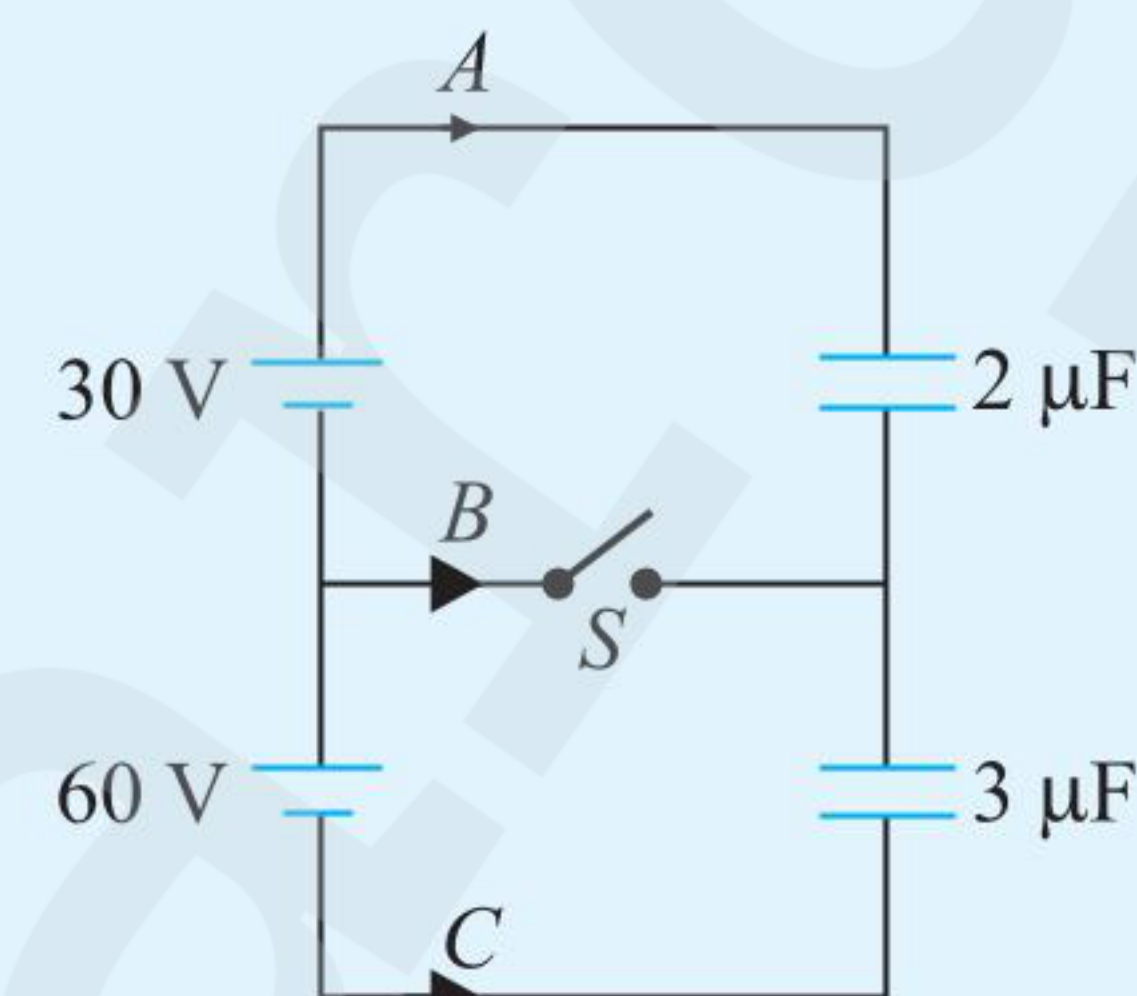


8. In figure, the battery has a potential difference of 20 V. Find

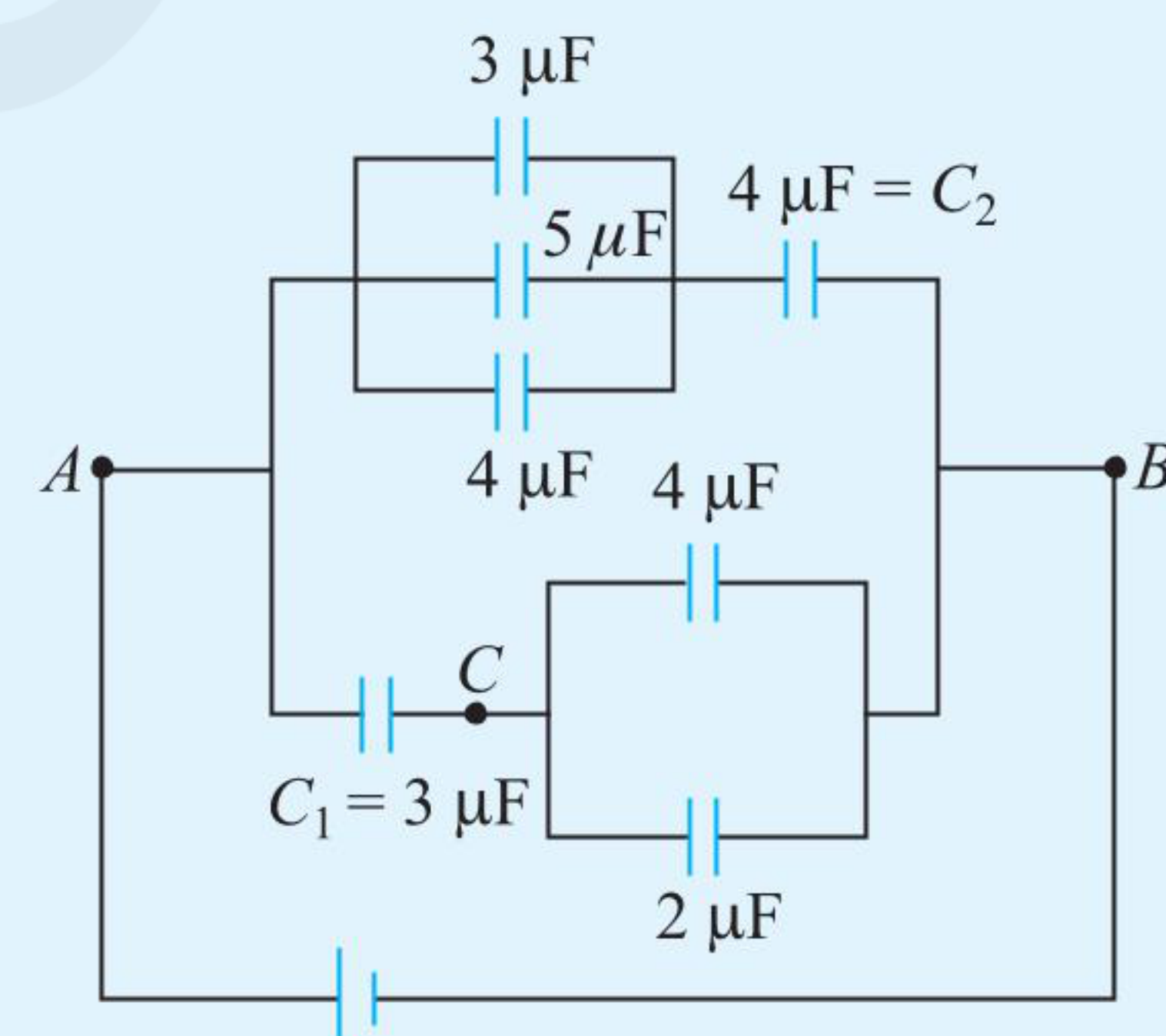


- (a) the equivalent capacitance of all the capacitors across the battery and
(b) the charge stored on that equivalent capacitance.
Find the charge on
(c) capacitor C_1 ,
(d) capacitor C_2 , and
(e) capacitor C_3 .

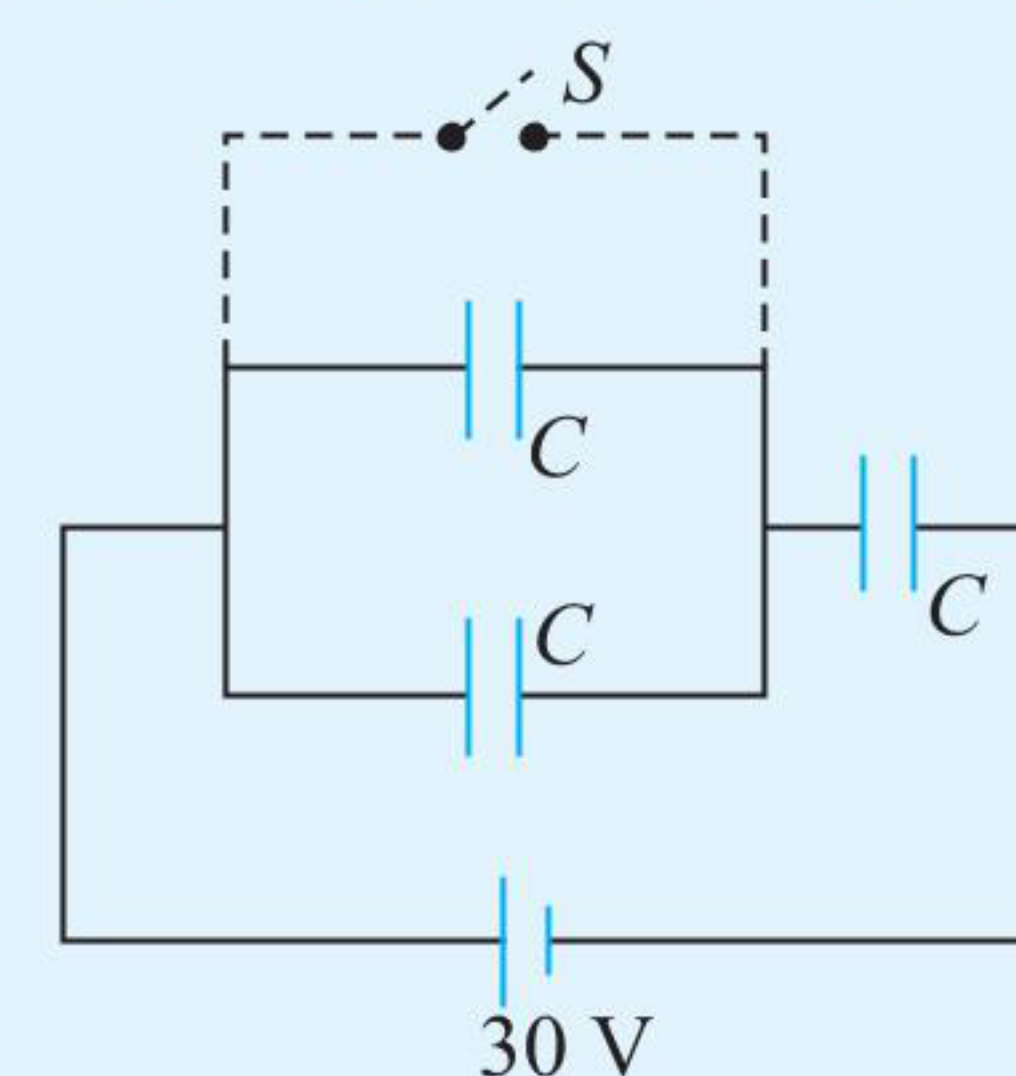
9. What charges will flow through section B of the circuit in the direction shown when switch S is closed.



10. Figure shows a network of seven capacitors. If charge on $5 \mu\text{F}$ capacitor is $10 \mu\text{C}$, find the potential difference between points A and C



11. The capacitor each having capacitance $C = 2 \mu\text{F}$ are connected with a battery of emf 30 V as shown in figure. When the switch S is closed. Find



- (a) the amount of charge flown through the battery
(b) the energy supplied by the battery
(c) the heat generated in the circuit
(d) the amount of charge flown through the switch S

ANSWERS

2. NV 3. $3C/4$ 4. $110 \mu\text{C}$

5. $\frac{A_1 k_1 \epsilon_0}{d_1 + d_2} + \frac{A_2 k_2 k_3 \epsilon_0}{k_2 d_2 + k_3 d_1}$ 6. $\frac{17 \epsilon_0 A}{10d}$ 7. 13 V

8. (a) $3 \mu\text{F}$ (b) $60 \mu\text{C}$ (c) $30 \mu\text{C}$ (d) $20 \mu\text{C}$ (e) $20 \mu\text{C}$ 9. $120 \mu\text{C}$

10. 5.33 V 11. (a) $20 \mu\text{C}$ (b) 0.6 mJ (c) 0.3 mJ (d) $60 \mu\text{C}$

KIRCHHOFF'S RULES FOR CAPACITORS

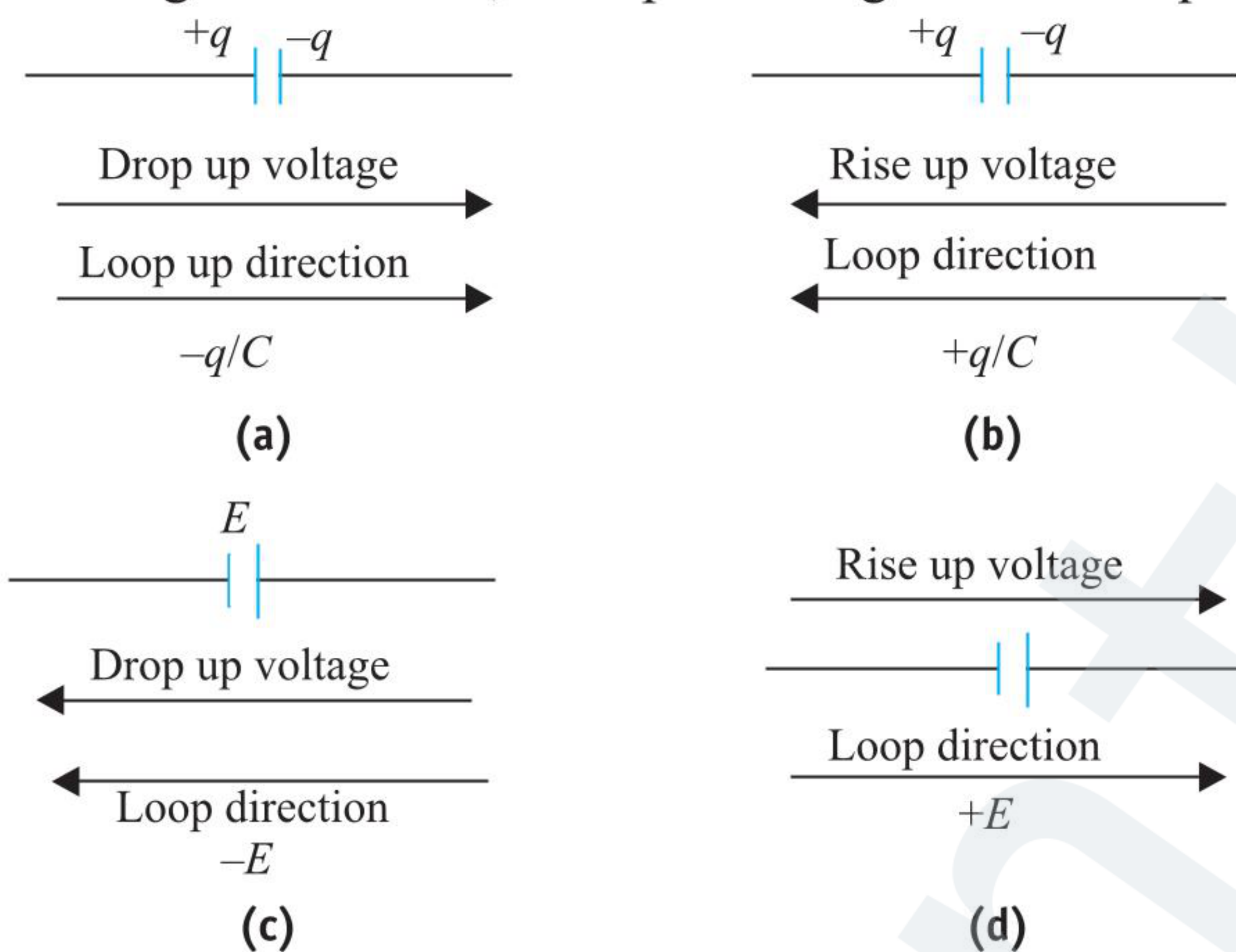
Kirchhoff's rules can be used to determine the potential difference and charge on the plates of a capacitor in any electric circuit. In a circuit with capacitors and batteries, two important rules are involved.

Junction rule: A capacitor circuit obeys the principle of conservation of charge. The incoming charge at any junction is equal to the outgoing charge from the junction, i.e., $q = q_1 + q_2$. In other words, in any isolated system, the net charge is conserved. So in the system shown in figure,

$$-q + q_1 + q_2 = 0 \Rightarrow q = q_1 + q_2$$

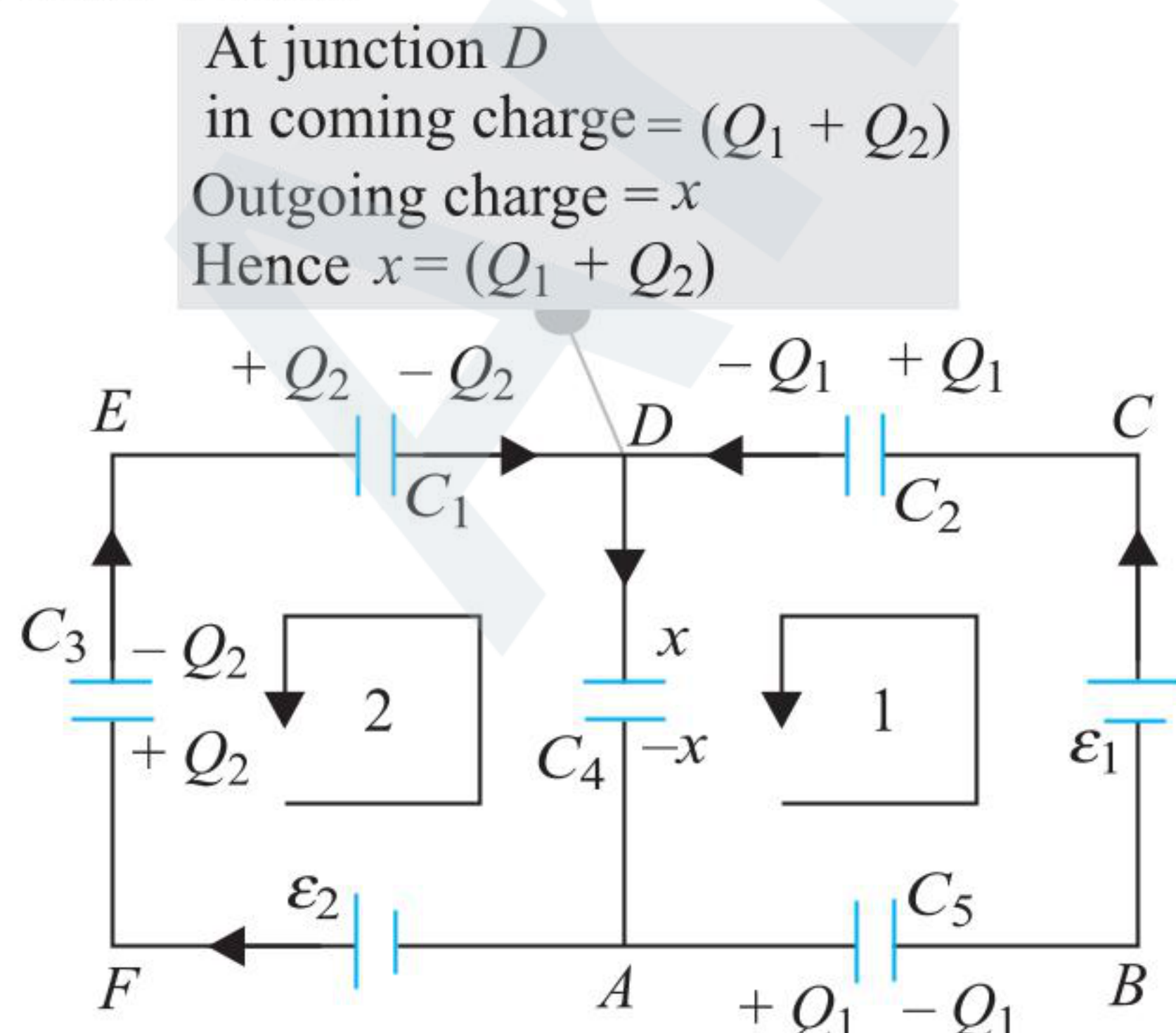
Loop rule: In a closed circuit, the algebraic sum of the rise up and drop up voltages is zero, i.e., $\Sigma V = 0$.

The direction of the loop is not specified and is chosen in a comfortable manner. When we go from a point of higher potential to a point of lower potential in the loop direction, drop up voltage occurs [Fig. (a)]. In sign convention, drop up voltage is taken as negative. If we go from a lower potential point to a higher potential point, rise up voltage occurs. In sign convention, rise up of voltage is taken as positive.



APPLICATION OF KIRCHHOFF'S LOOP RULE IN A CIRCUIT

If you travel from the positively charged plate of a capacitor or the terminal of a cell to the negatively charged plate of a capacitor or the terminal of a cell, then the sign of V or $\mathcal{E}$ is negative and vice versa.



Applying loop rule in closed loop $ABCD A$

$$-\frac{Q_1}{C_5} + \mathcal{E}_1 - \frac{Q_1}{C_2} - \frac{Q_1 + Q_2}{C_4} = 0$$

$$\text{or } Q_1 \left(\frac{1}{C_2} + \frac{1}{C_4} + \frac{1}{C_5} \right) + \frac{Q_2}{C_4} = \mathcal{E}_1 \quad \dots(i)$$

Applying loop rule in closed loop $ADEFA$

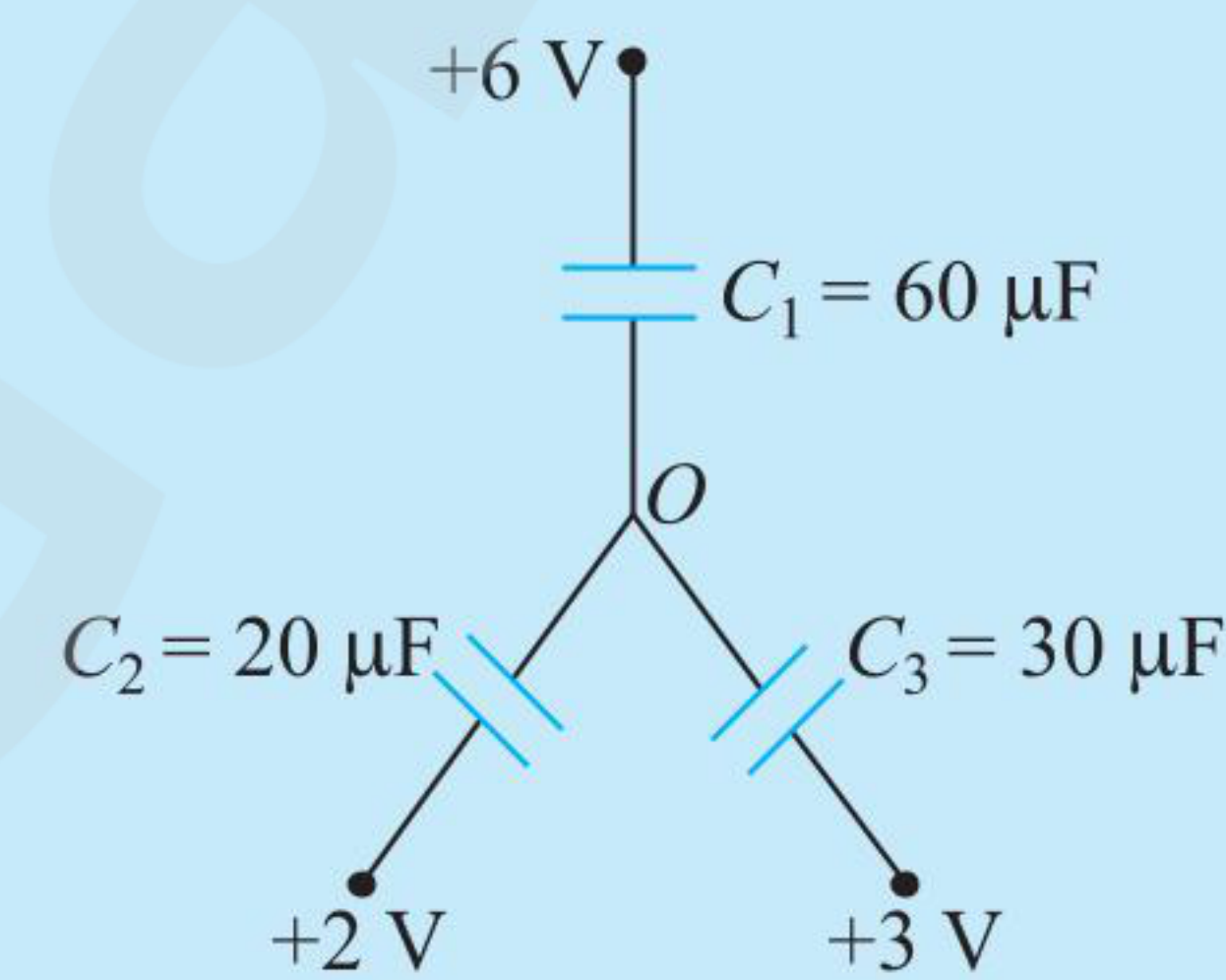
$$\frac{Q_1 + Q_2}{C_4} + \frac{Q_2}{C_1} + \frac{Q_2}{C_3} - \mathcal{E}_2 = 0$$

$$\text{or } \frac{Q_1}{C_4} + Q_2 \left[\frac{1}{C_1} + \frac{1}{C_3} + \frac{1}{C_4} \right] = \mathcal{E}_2 \quad \dots(ii)$$

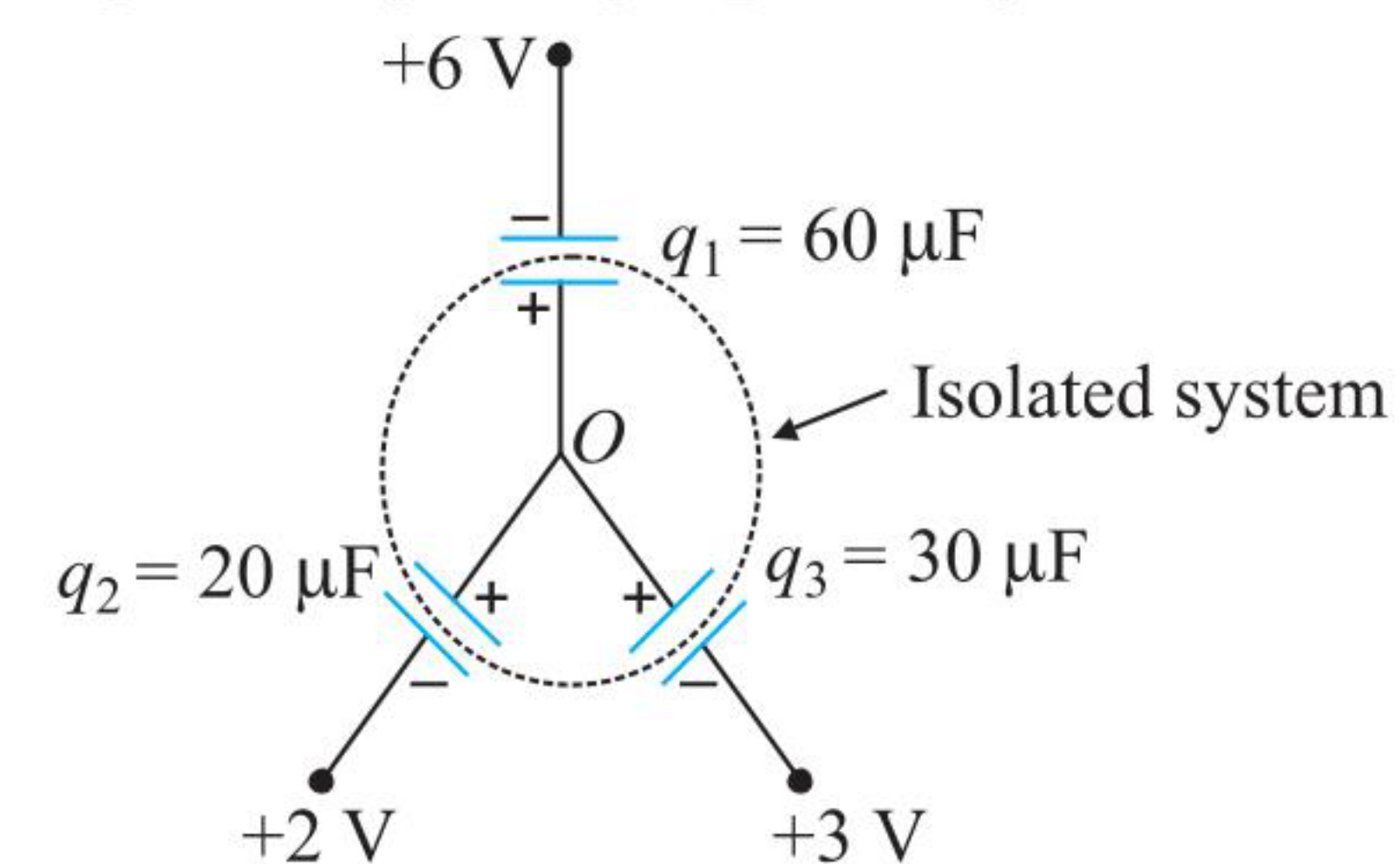
Now you have two equations and two unknowns (Q_1 and Q_2), which can be found easily by solving the equations.

ILLUSTRATION 4.31

Three uncharged capacitors of capacitance C_1 , C_2 and C_3 are connected to one another as shown in figure. Find the potential at O .



Sol. Let at junction O , the potential be V and the charges in capacitors C_1 , C_2 and C_3 be q_1 , q_2 and q_3 respectively



$$\text{At } O, q_1 + q_2 + q_3 = 0 \quad \dots(i)$$

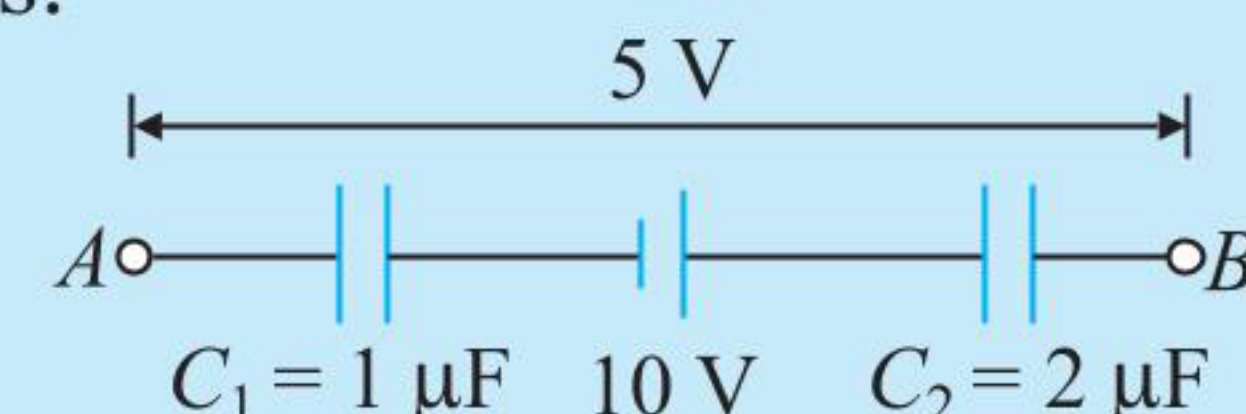
The potential difference across the capacitors C_1 , C_2 and C_3 will be $(V - 6)$, $(V - 2)$ and $(V - 3)$ respectively. Hence equation (i) can be written as

$$60(V - 6) + 20(V - 2) + 30(V - 3) = 0$$

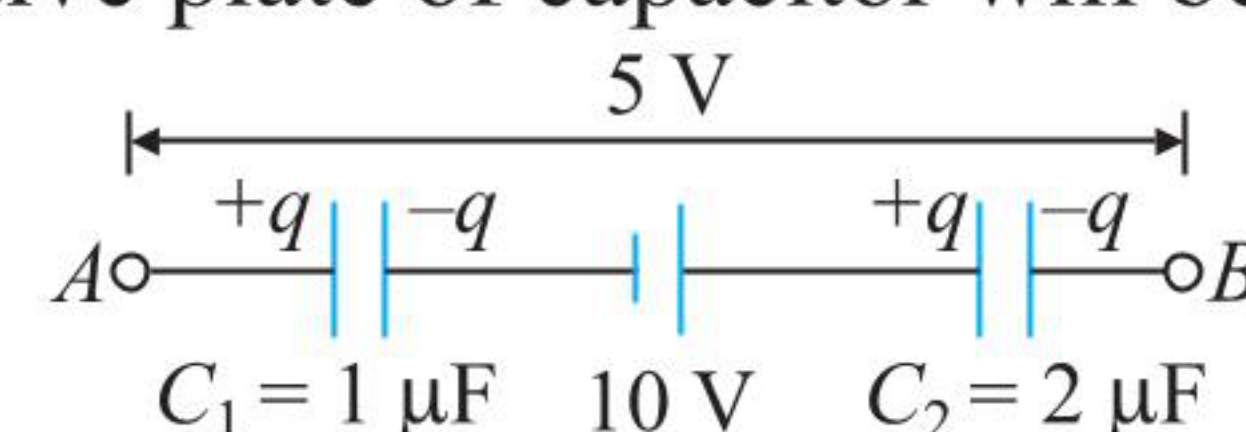
$$\text{Solving this equation, we get, } V = \frac{49}{11} \text{ V}$$

ILLUSTRATION 4.32

A circuit has a section AB shown in figure. The emf of the cell is 10 V and the capacitors have capacitances $C_1 = 1 \mu\text{F}$ and $C_2 = 2 \mu\text{F}$. The potential difference $V_{AB} = 5\text{V}$. Find the charges on the capacitors.



Sol. Let the charge on the left plate of the capacitor is q , hence the charge on right plate of capacitor C_2 will be $-q$ as charge supplied by negative plate of capacitor will be $-q$.



Moving from point A in the section of circuit AB

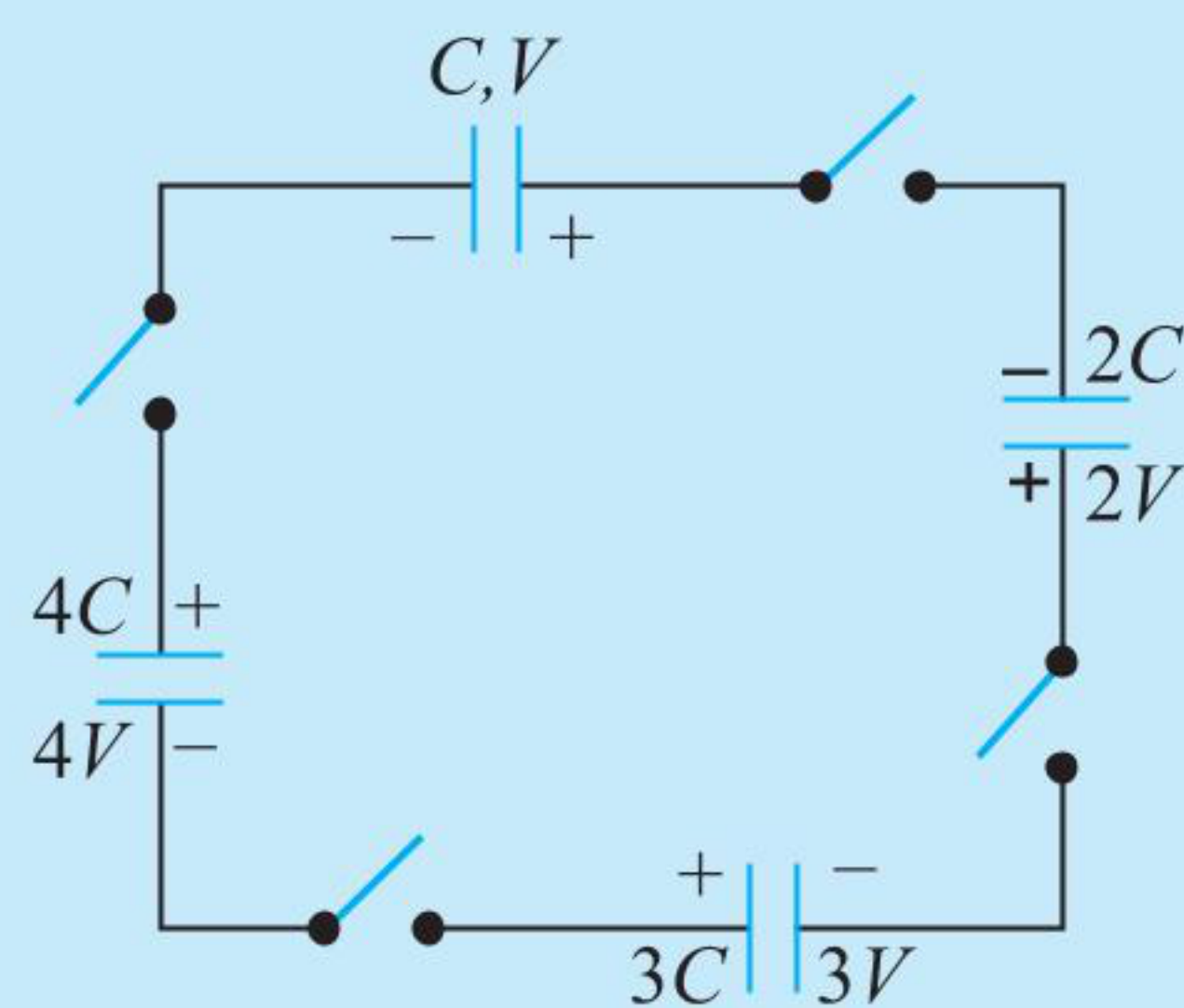
$$V_A - \frac{q}{1} + 10 - \frac{q}{2} = V_B$$

$$V_A - V_B = q + \frac{q}{2} - 10 \Rightarrow 5 = \frac{3}{2}q - 10$$

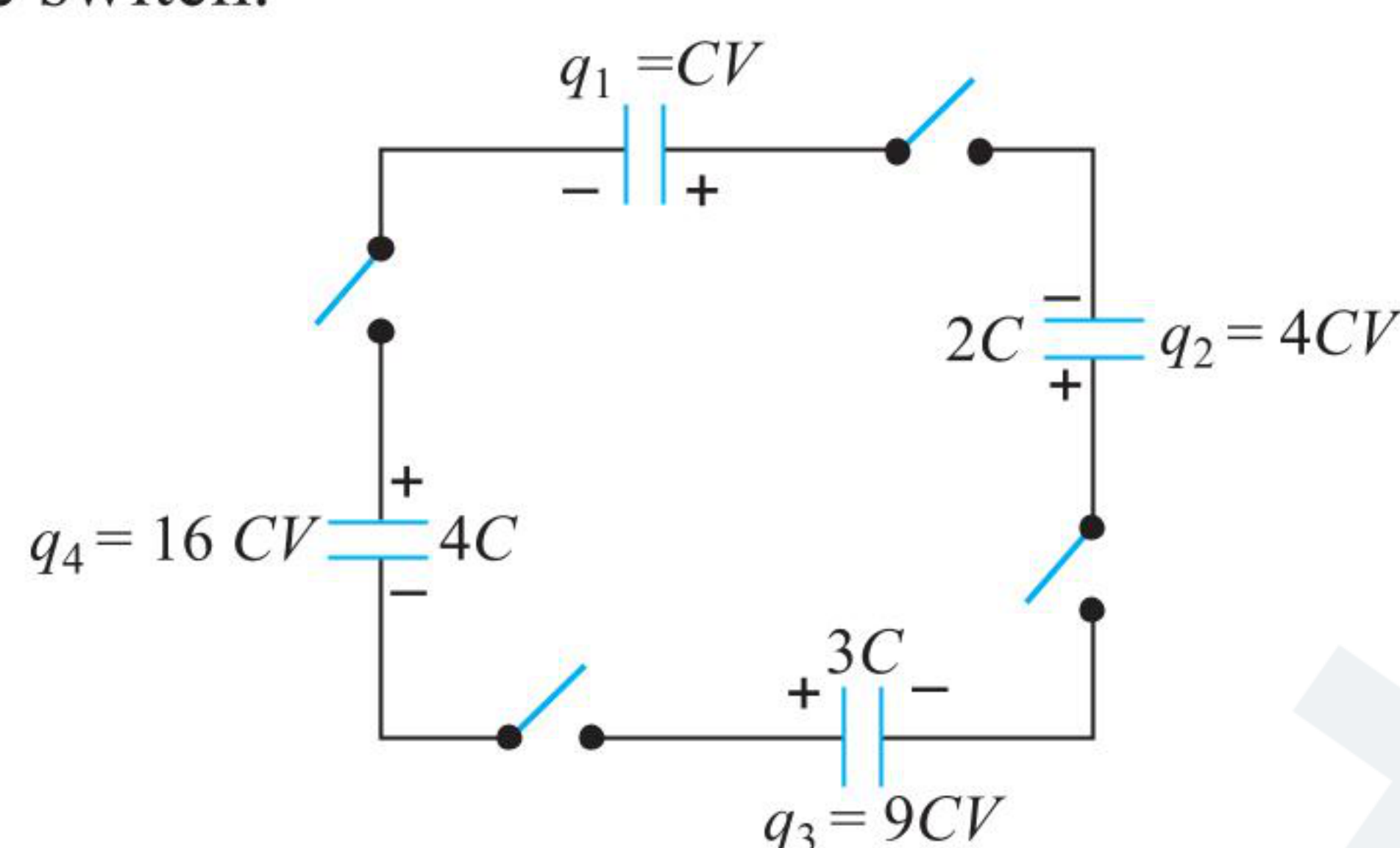
Hence the charges on the capacitors $q = 10 \mu\text{C}$

ILLUSTRATION 4.33

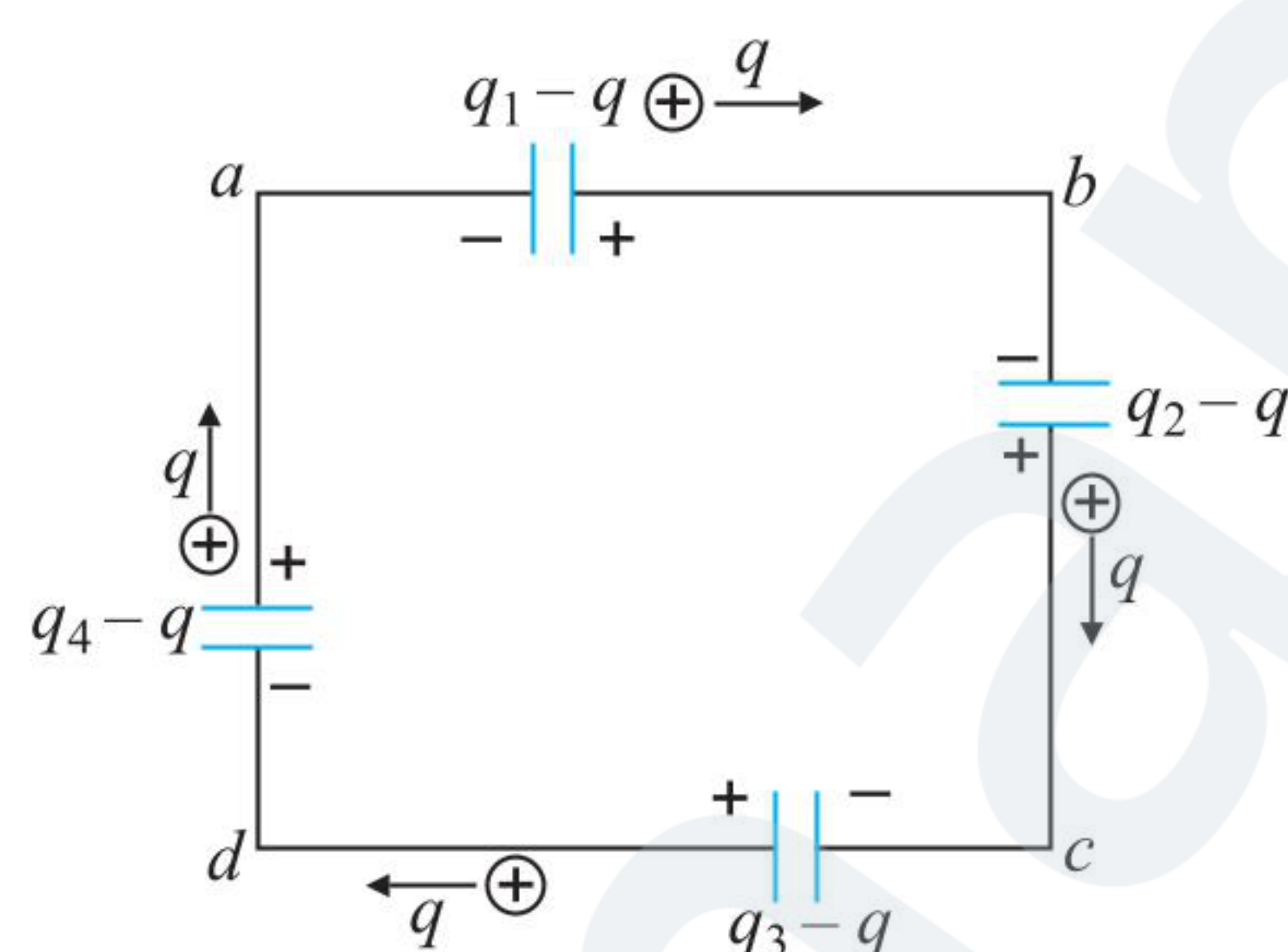
Four capacitors having capacitance C , $2C$, $3C$ and $4C$ are charged to the voltage V , $2V$, $3V$ and $4V$ correspondingly. The circuit is closed. Find the voltage on all condensers in the equilibrium.



Sol. Let a charge q flow in the clockwise direction after closing the switch.



Just before closing the switch



Just after closing the switch

Applying loop law in $abcd$, we get

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

Substituting values of q_1 , q_2 , q_3 , and q_4 , we get, $q = \frac{24}{5} CV$

The potential difference across the capacitor ' C ',

$$|\Delta V|_C = \left| \frac{q_1 - q}{C} \right| = \frac{19}{5} V$$

The potential difference across the capacitor ' $2C$ ',

$$|\Delta V|_{2C} = \left| \frac{q_2 - q}{2C} \right| = \frac{2}{5} V$$

The potential difference across the capacitor ' $3C$ ',

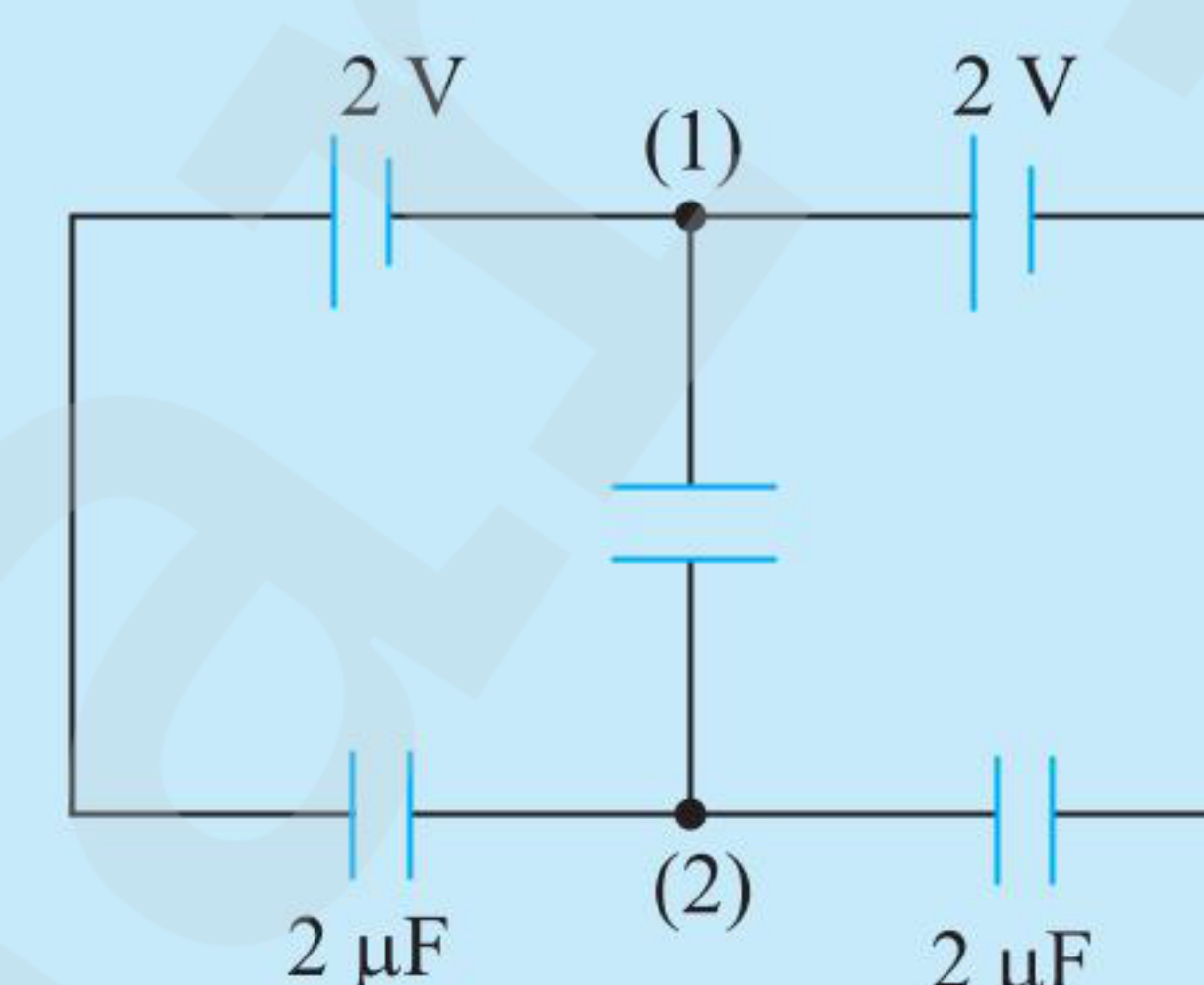
$$|\Delta V|_{3C} = \left| \frac{q_3 - q}{3C} \right| = \frac{7}{5} V$$

and the potential difference across the capacitor ' $4C$ ',

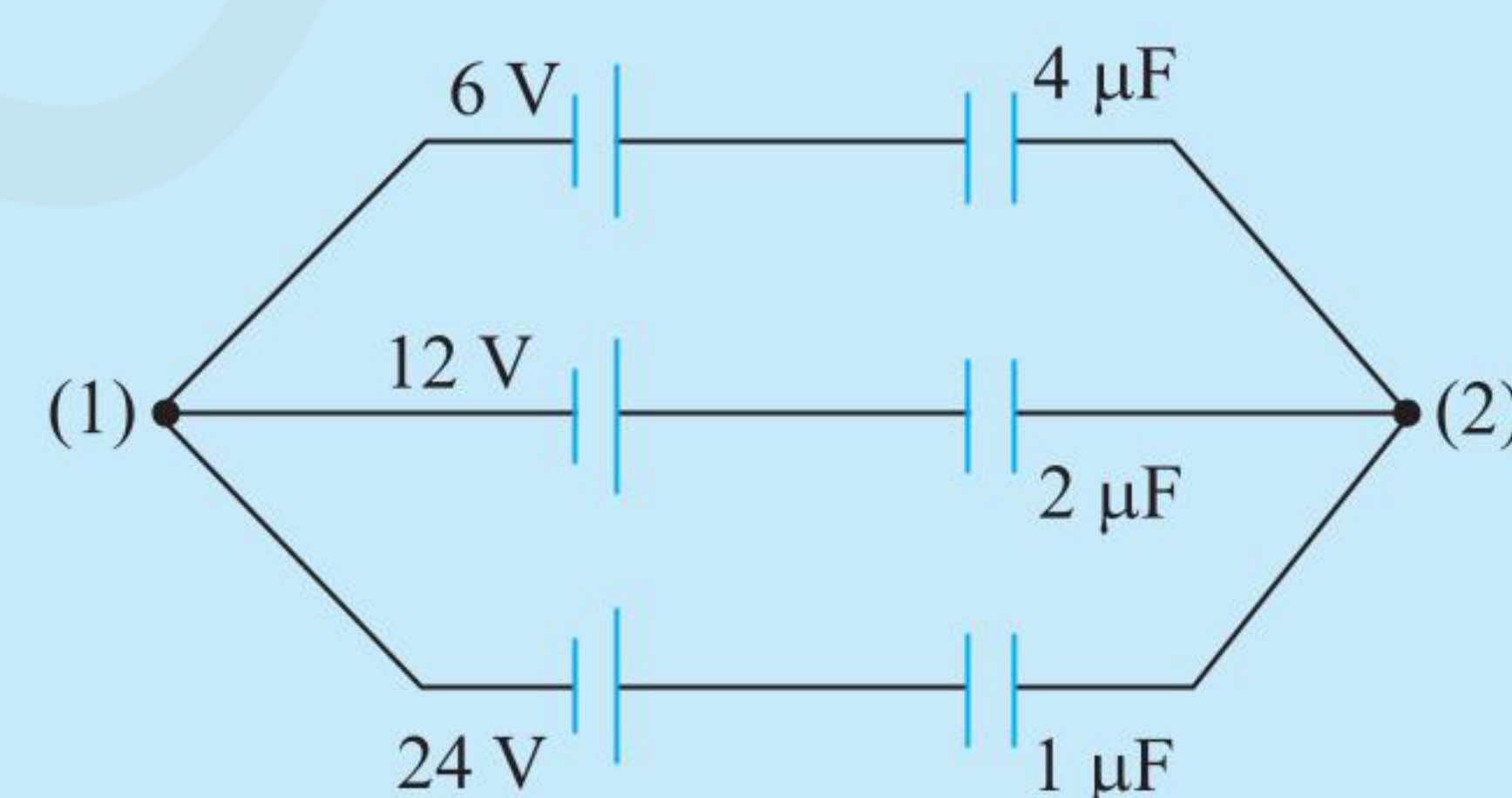
$$|\Delta V|_{4C} = \left| \frac{q_4 - q}{4C} \right| = \frac{14}{5} V$$

ILLUSTRATION 4.34

Find the potential difference $V_1 - V_2$ between the points (1) and (2) shown in each part of figure.



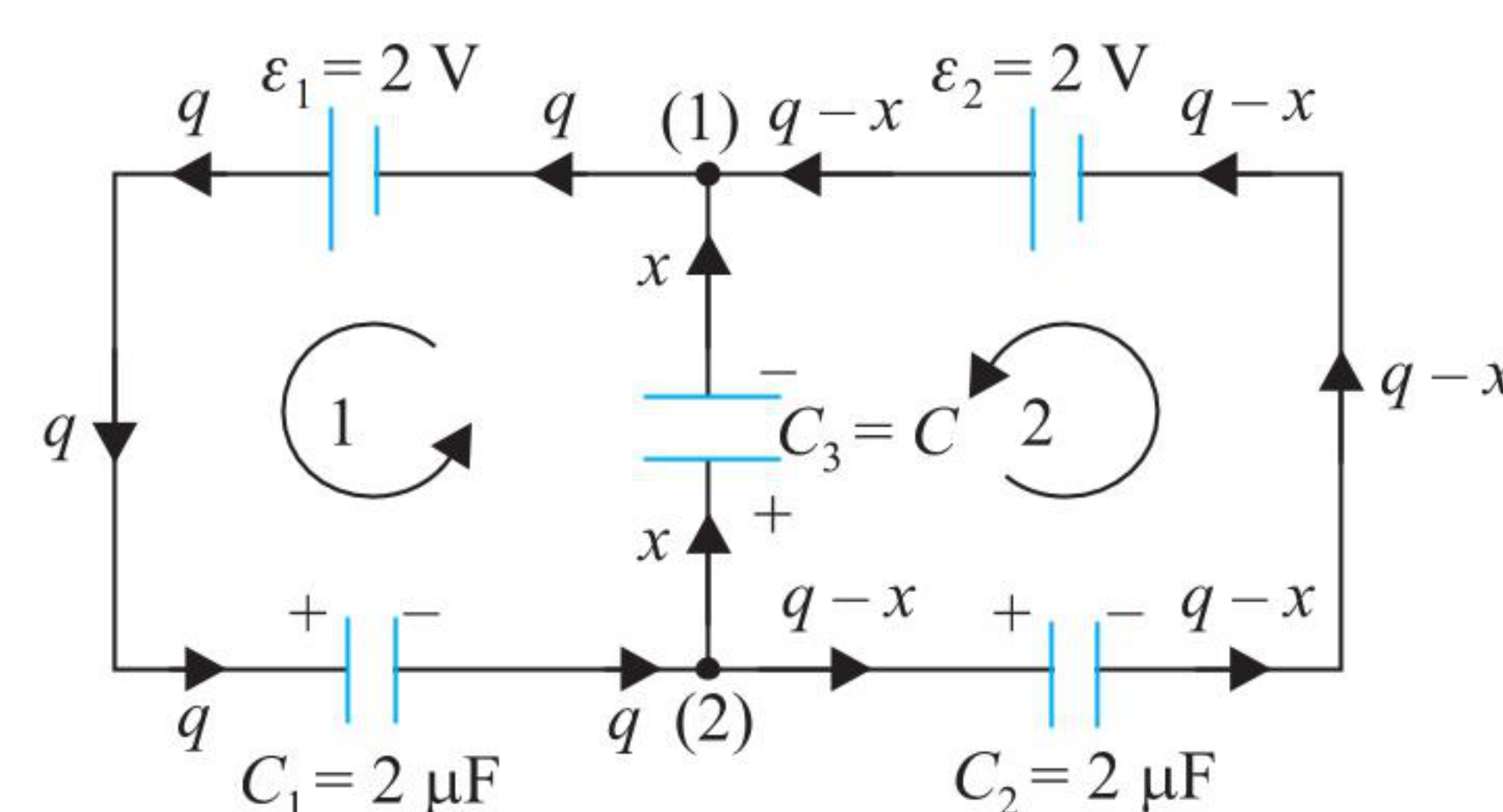
(a)



(b)

Sol.

(a) We first distribute the charges on different capacitors and branches keeping in mind Kirchhoff's junction rule. We can start from any battery. In this case, we start from battery ε_1 . Let battery ε_1 supply a charge q . The charge on the left plate of capacitor C_1 will be q as this plate is receiving the charge. Now charge q reaches junction b where it is divided into two paths. Let charge x go to capacitor C_3 , then the remaining charge $q - x$ will go to capacitor C_2 . In junction a , we can verify Kirchhoff's junction rule, i.e., the incoming charge is equal to the outgoing charge. Now we write the loop equations.



For loop 1

$$2 - \frac{q}{2} - \frac{x}{C} = 0 \quad \dots(i)$$

For loop 2

$$2 + \frac{x}{C} - \frac{(q - x)}{2} = 0$$

$$\text{or } 2 + \frac{x}{C} - \frac{q}{2} + \frac{x}{2} = 0 \quad \dots(ii)$$

From Eqs. (i) and (ii)

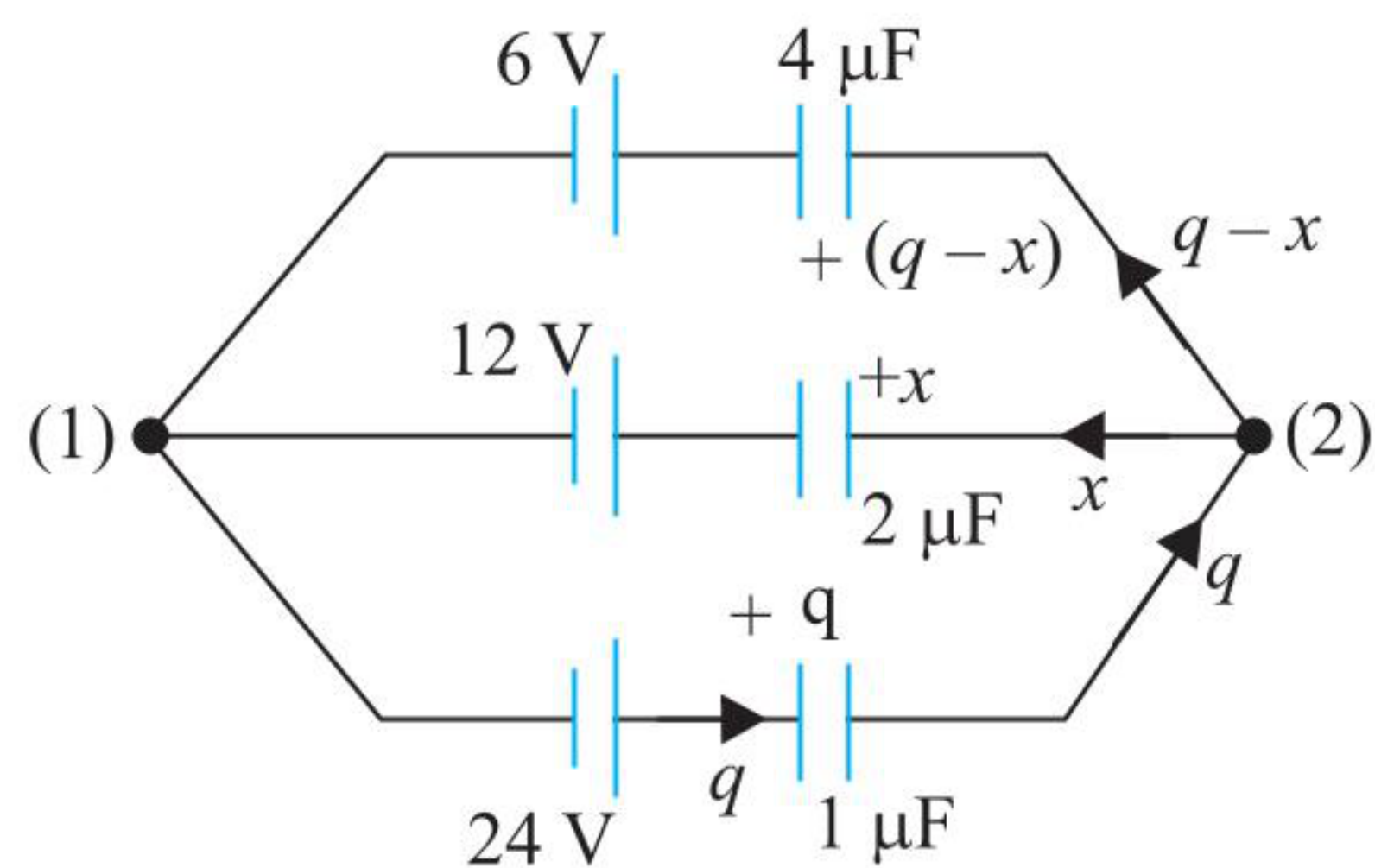
$$\frac{2x}{C} + \frac{x}{2} = 0$$

$$\text{or } x \left(\frac{2}{C} + \frac{1}{2} \right) = 0$$

$$\text{or } x = 0 \left(\text{since } \frac{2}{C} + \frac{1}{2} \neq 0 \right)$$

No charge will go to the capacitor connected across (1) and (2). As there is no charge in the capacitor connected across (1) and (2), the potential difference across (1) and (2) should be zero.

- (b) In this case also, we will first distribute the charges. Let us start from the 24 V battery. Let this battery supply a charge q , which is divided into two paths after reaching junction (2). Let x charge move toward the $2 \mu\text{F}$ capacitor, then the remaining charge $q - x$ will move toward the $4 \mu\text{F}$ capacitor, as per Kirchhoff's junction rule. Now we write the loop equations.



In the upper closed loop

$$+6 + \frac{(q-x)}{4} - \frac{x}{2} - 12 = 0 \text{ or } \frac{q}{4} - x = 6 \quad \dots(i)$$

In the lower closed loop

$$12 + \frac{x}{2} + \frac{q}{1} - 24 = 0 \text{ or } q + \frac{x}{2} = 12 \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$x = \frac{-8}{3} \mu\text{C}$$

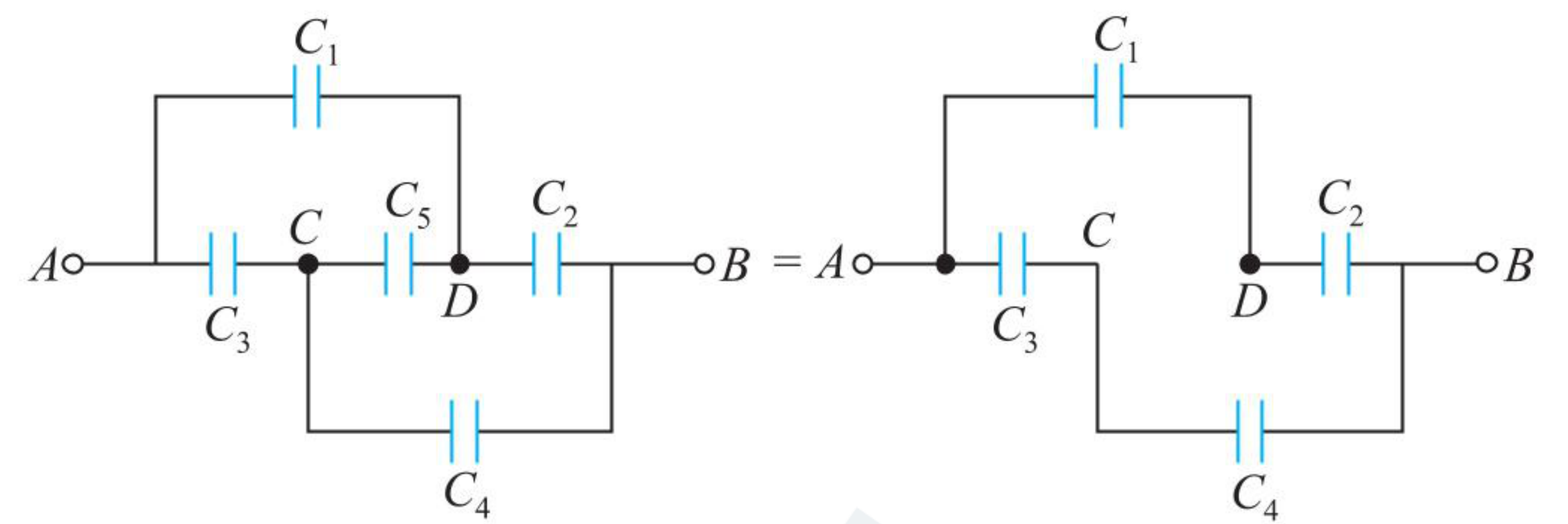
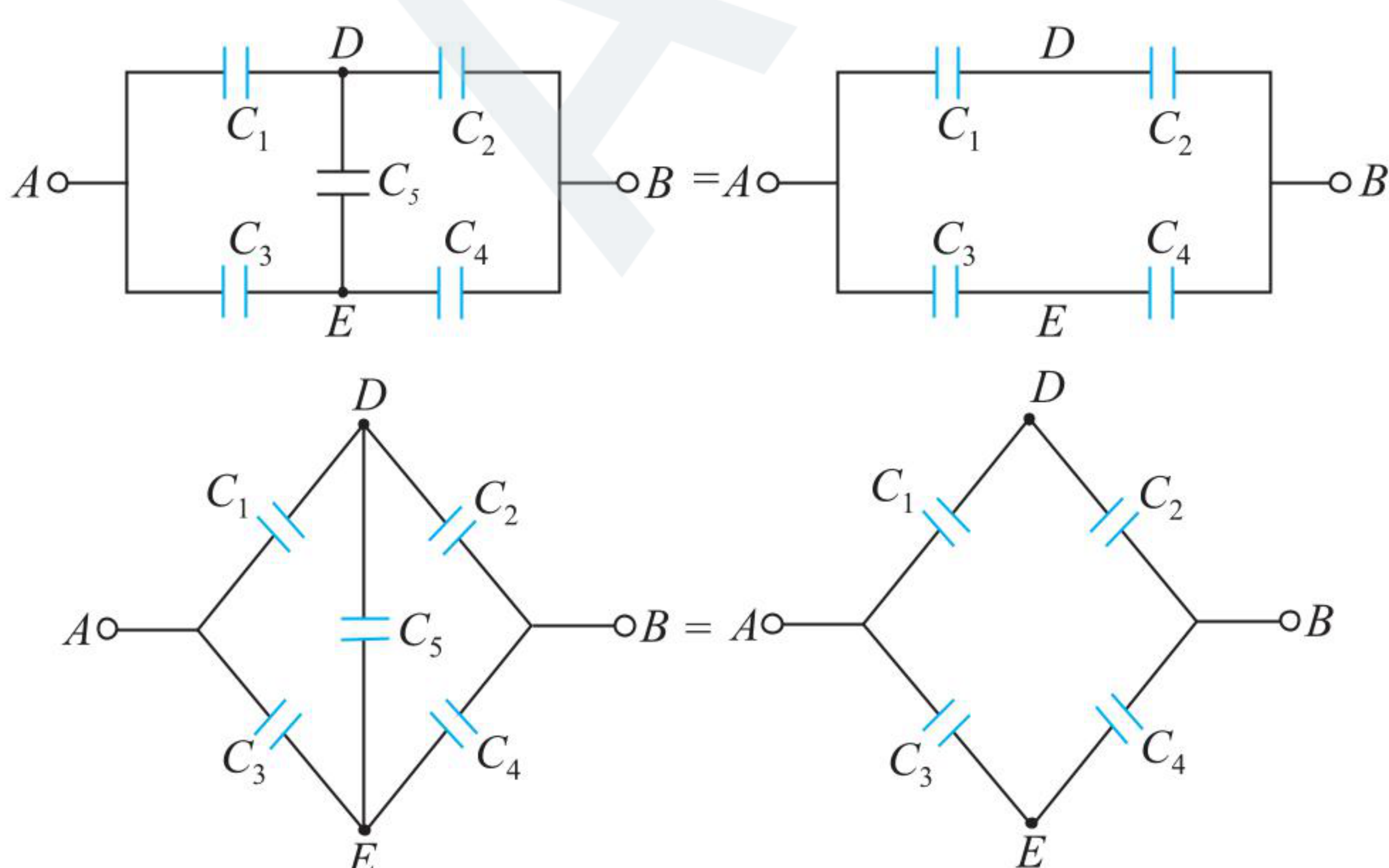
Moving from (1) to (2) through middle path

$$V_1 + 12 - \frac{8/3}{2} = V_2$$

$$\text{or } V_1 - V_2 = \frac{4}{3} - 12 = \frac{-32}{3} \text{ V}$$

CIRCUITS BASED ON WHEATSTONE BRIDGE

In following circuits if $(C_1/C_2) = (C_3/C_4)$, the potential difference across D and E will be zero.



In all above cases, the equivalent capacity between A and B is

$$C_{AB} = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

EXTENDED WHEATSTONE BRIDGE

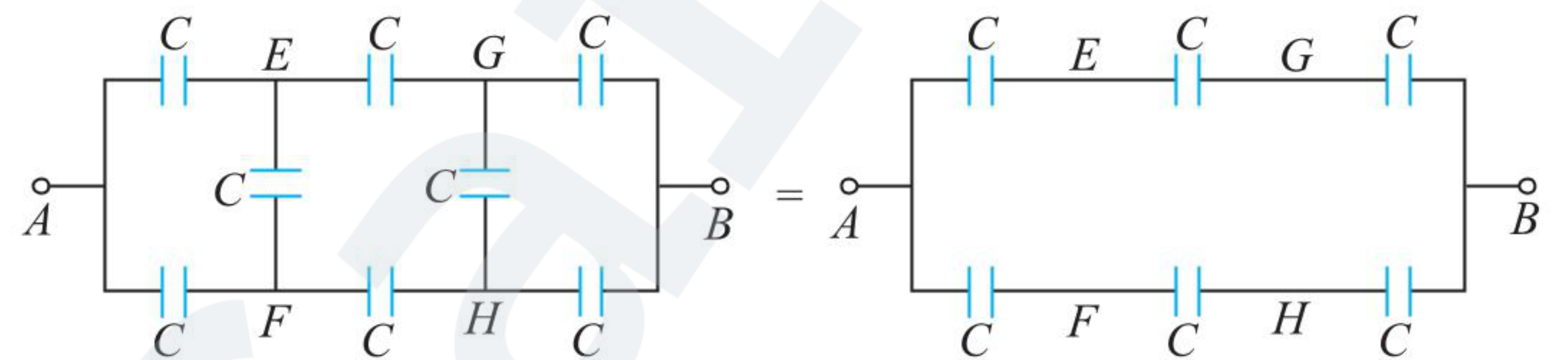


ILLUSTRATION 4.35

What is $V_A - V_B$ in the arrangement shown in figure. What is the condition such that $V_A - V_B = 0$.

Sol. Let the charges be as shown in figure. We know that capacitors in series have the same charge. Considering the loop containing C_1 , C_2 , and E , we get

$$\frac{q}{C_1} + \frac{q}{C_2} - E = 0$$

$$\text{or } q = E \left[\frac{C_1 C_2}{C_1 + C_2} \right]$$

From the loop containing C_3 , C_4 , and E , we get

$$\frac{q'}{C_3} + \frac{q'}{C_4} - E = 0 \text{ or } q' = E \left[\frac{C_3 C_4}{C_3 + C_4} \right]$$

Now,

$$V_A - V_B = \frac{q}{C_2} - \frac{q'}{C_4} = E \left[\frac{C_1}{C_1 + C_2} - \frac{C_3}{C_3 + C_4} \right]$$

$$= E \left[\frac{C_1 C_4 - C_3 C_2}{(C_1 + C_2)(C_3 + C_4)} \right]$$

For $V_A - V_B = 0$, $C_1 C_4 = C_2 C_3 = 0$. If $(C_1/C_2) = (C_3/C_4)$, the potential difference across A and B will be zero. This statement is very important in capacitor circuit solving. This type of the situation is called balanced Wheatstone bridge.

ILLUSTRATION 4.36

Six capacitors $C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C$ are arranged as shown in figure. Determine the equivalent capacitance between A and B .

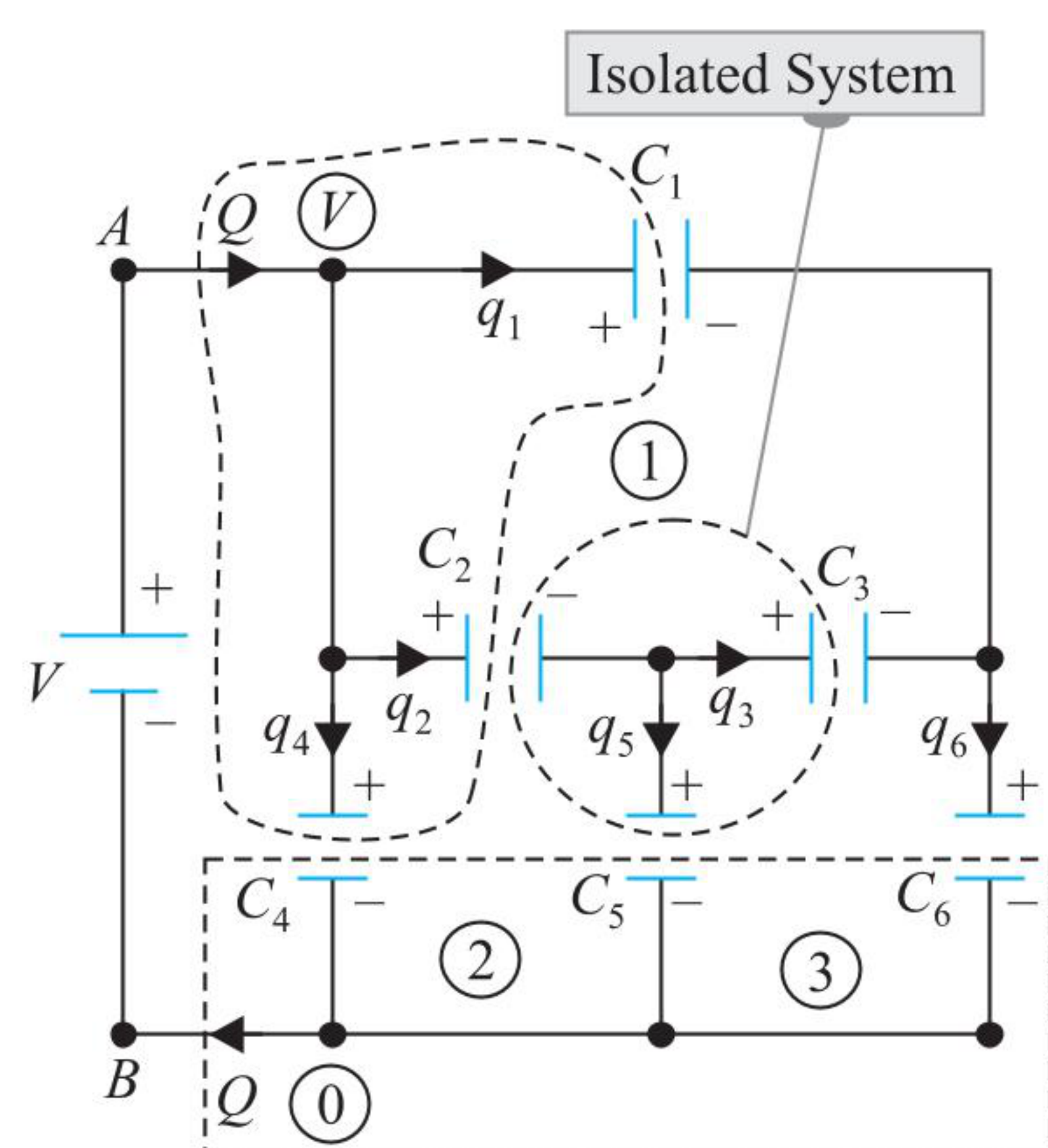
Sol. Let the potential difference across the battery terminals be V and the charge supplied be Q . We have to find the capacitance

of the system and the capacitance of a capacitor that would have the same charge Q on its plates as the battery at voltage V . Hence,

$$C_{\text{equivalent}} = \frac{Q}{V_A - V_B} = \frac{Q}{V}$$

The incoming charge Q is equal to the charge received by the plates of the capacitors C_1 , C_2 , and C_4 . Thus,

$$Q = q_1 + q_2 + q_4$$

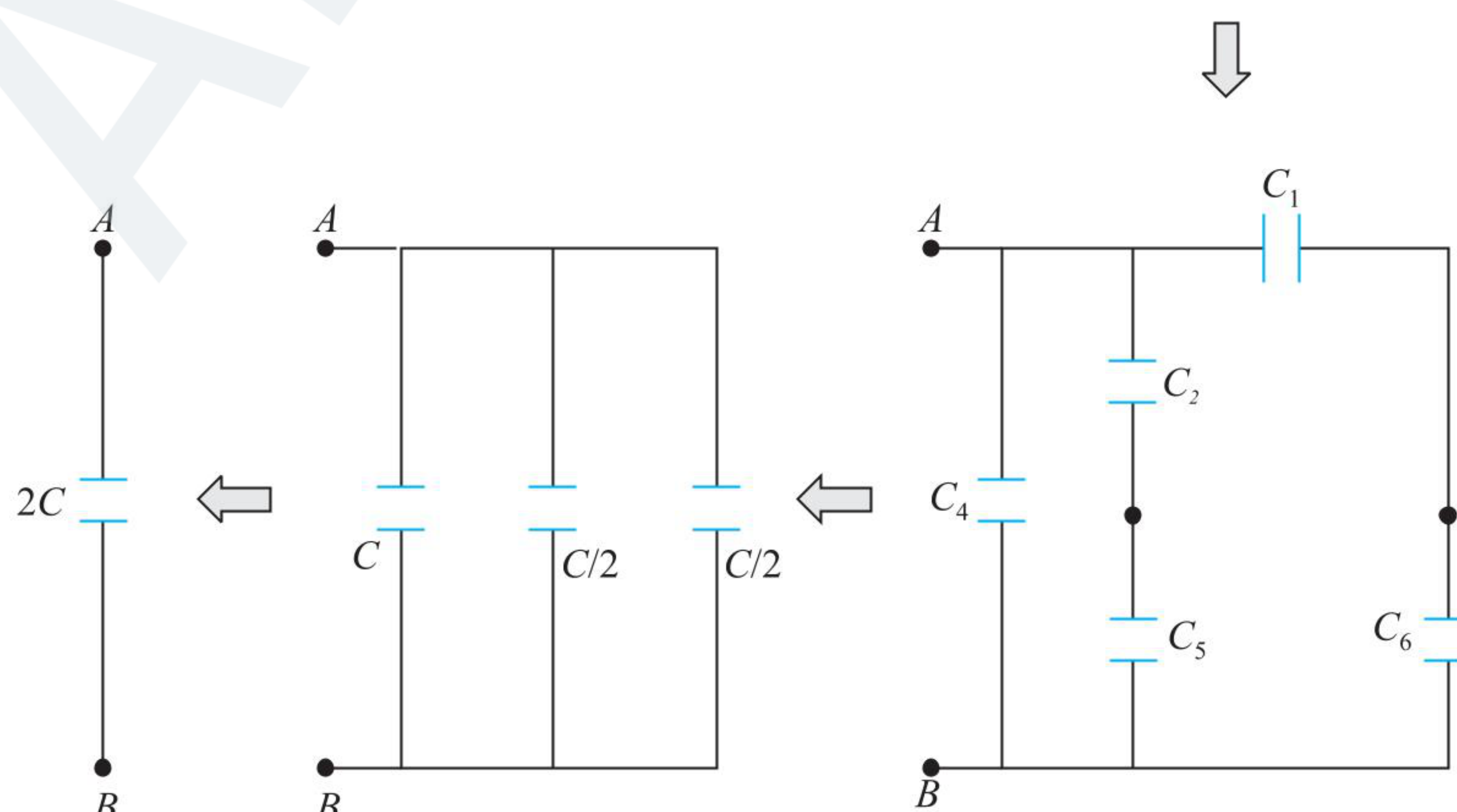
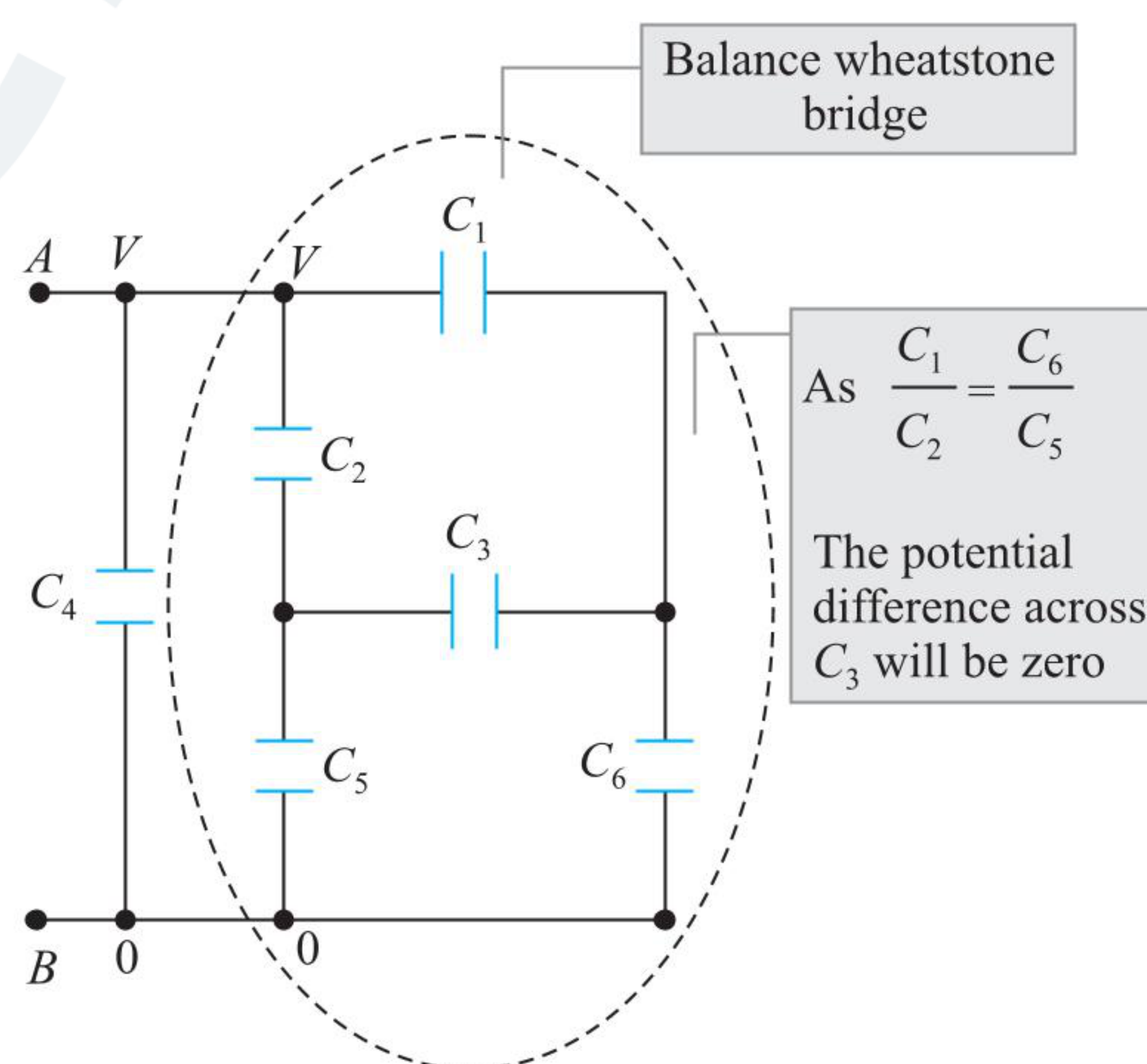
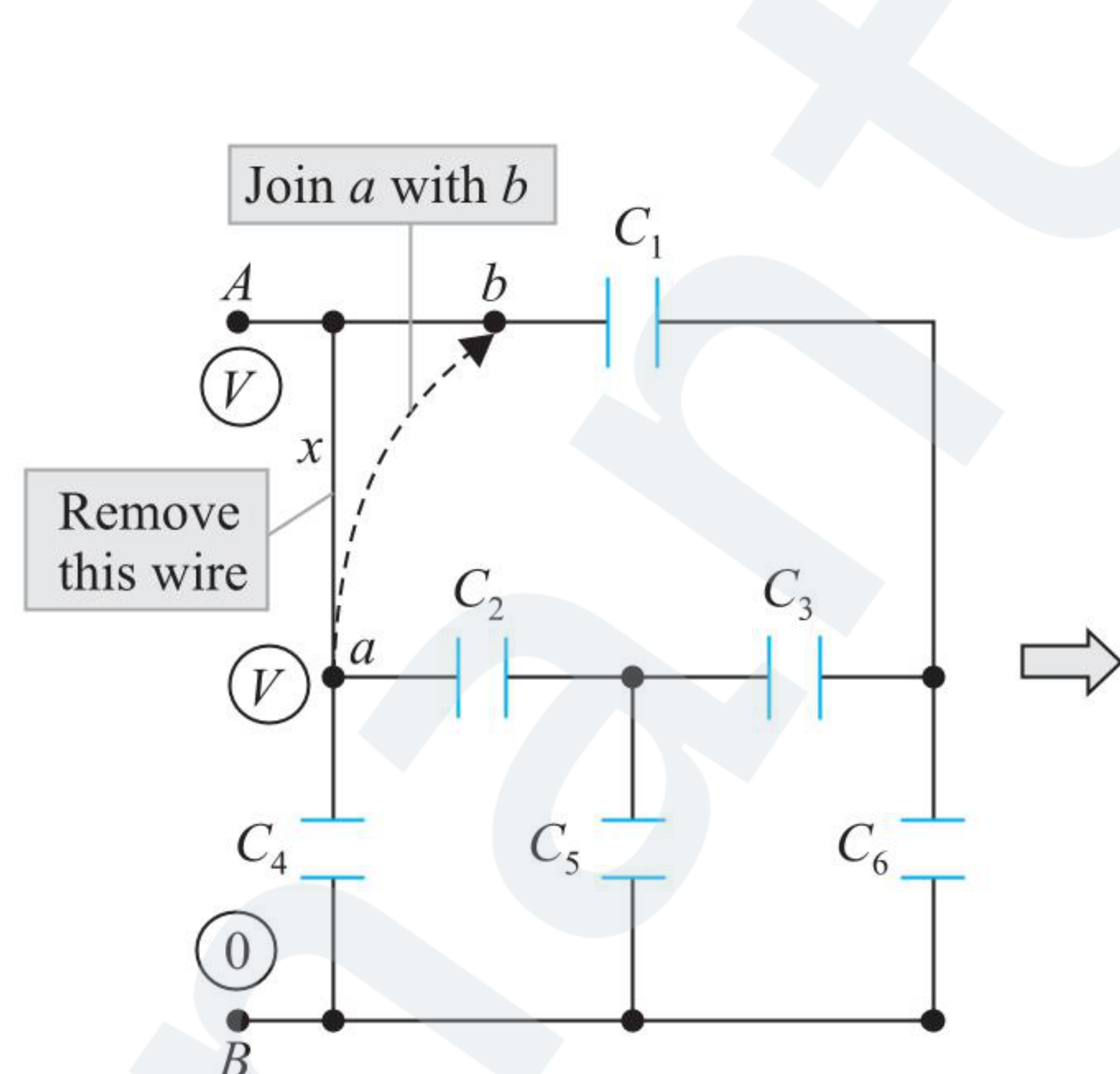


Similarly, outgoing charge Q is equal to charge coming out from the plates of the capacitors C_4 , C_5 , and C_6 .

$$Q = q_4 + q_5 + q_6$$

The above equations give

$$q_1 + q_2 = q_5 + q_6 \quad \dots(i)$$



In a closed loop, the net potential drop must be zero, according to Kirchhoff's voltage law. Therefore, for loop 1, loop 2, and loop 3, we have

$$\left[\begin{aligned} \text{Loop 1: } \frac{q_1}{C} - \frac{q_2}{C} - \frac{q_3}{C} &= 0 \Rightarrow q_1 = q_2 + q_3 \\ q_1 + q_2 &= 2q_2 + q_3 \end{aligned} \right] \quad \dots(ii)$$

$$\text{Loop 2: } \frac{q_2}{C} - \frac{q_4}{C} + \frac{q_5}{C} = 0 \Rightarrow q_4 = q_2 + q_5 \quad \dots(iii)$$

$$\left[\begin{aligned} \text{Loop 3: } \frac{q_3}{C} - \frac{q_5}{C} + \frac{q_6}{C} &= 0 \Rightarrow q_5 = q_3 + q_6 \\ q_5 + q_6 &= 2q_6 + q_3 \end{aligned} \right] \quad \dots(iv)$$

From the isolated system shown in figure, we can conclude

$$q_3 + q_5 - q_2 = 0 \quad \dots(v)$$

From Eqs. (i), (ii), and (iv), we conclude $q_2 = q_6$ and $q_1 = q_5$.

If $q_1 = q_5$, then Eq. (v) becomes

$$q_2 - q_1 = q_3 \quad \dots(vi)$$

Solving Eqs. (ii) and (vi), we get

$$q_3 = 0, q_1 = q_2 = q_5 = q_6 = \frac{q_4}{2}, \text{ and } Q = 2q_4$$

We can write

$$V = \frac{q_4}{C} = \frac{Q}{2C} \text{ or } \frac{Q}{V} = 2C$$

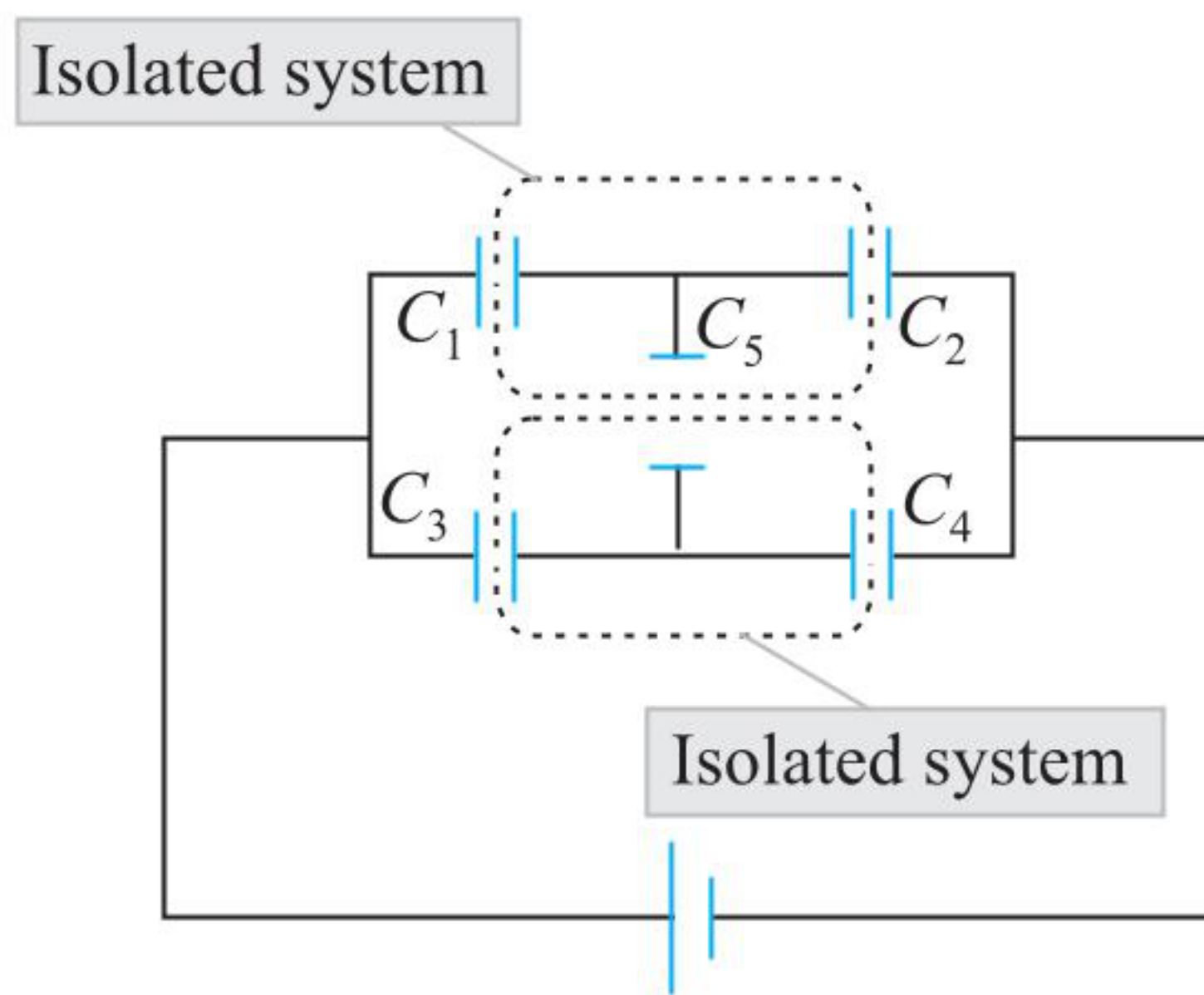
Hence,

$$C_{\text{equivalent}} = \frac{Q}{V_A - V_B} = \frac{Q}{V} = 2C$$

Alternative method: Connect point a to point b and note the indicated Wheatstone bridge (figure). Now simplify the circuit to obtain the desired result.

NODAL ANALYSIS OF CAPACITIVE CIRCUITS

Nodal analysis is based on conservation of charge. At any isolated part of a circuit, charge is conserved. $\Sigma q_i = 0$ if the capacitors are initially uncharged, and Σq_i is equal to the sum of the initial charges of the plates of the capacitors connected to the node if the capacitors have initial charge.



The following steps should be followed in problem solving:

- Mark different nodes in the circuit and assign their potentials.
- Identify the isolated systems in the circuit and write the equation of conservation of charges.
- Solve the equations to get the values of potentials. Let us learn this matter through some illustrations.

Step 1: Assign a node as a reference and assume its potential to be zero.

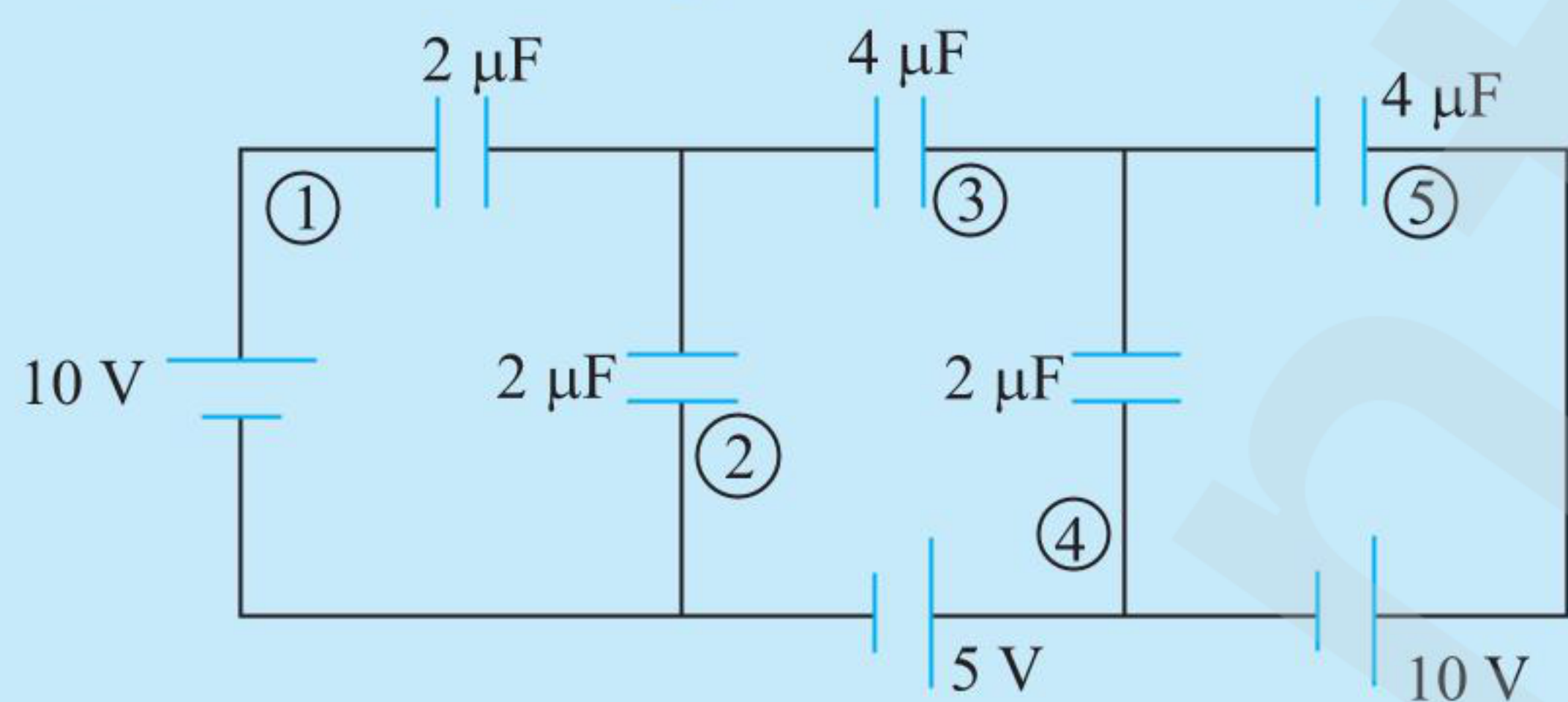
Step 2: Assign the potential of every junction of the circuit as an unknown S .

Step 3: Apply Kirchhoff's current law at the junction corresponding to the unknown potential.

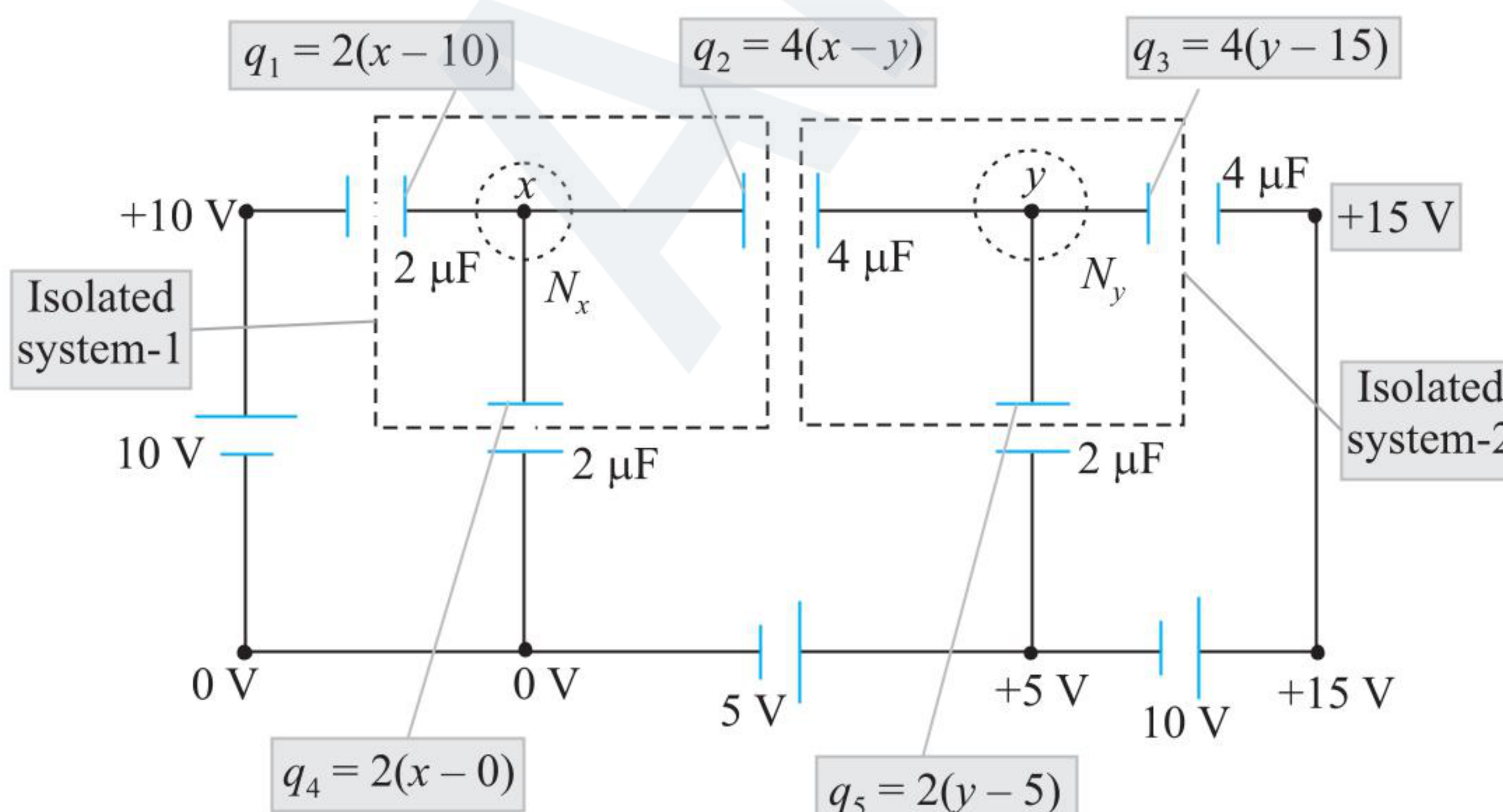
Step 4: Solve the equations to get the desired unknown potentials. Let us learn this matter through some illustrations.

ILLUSTRATION 4.37

In figure, what are the charges on all the four capacitors?



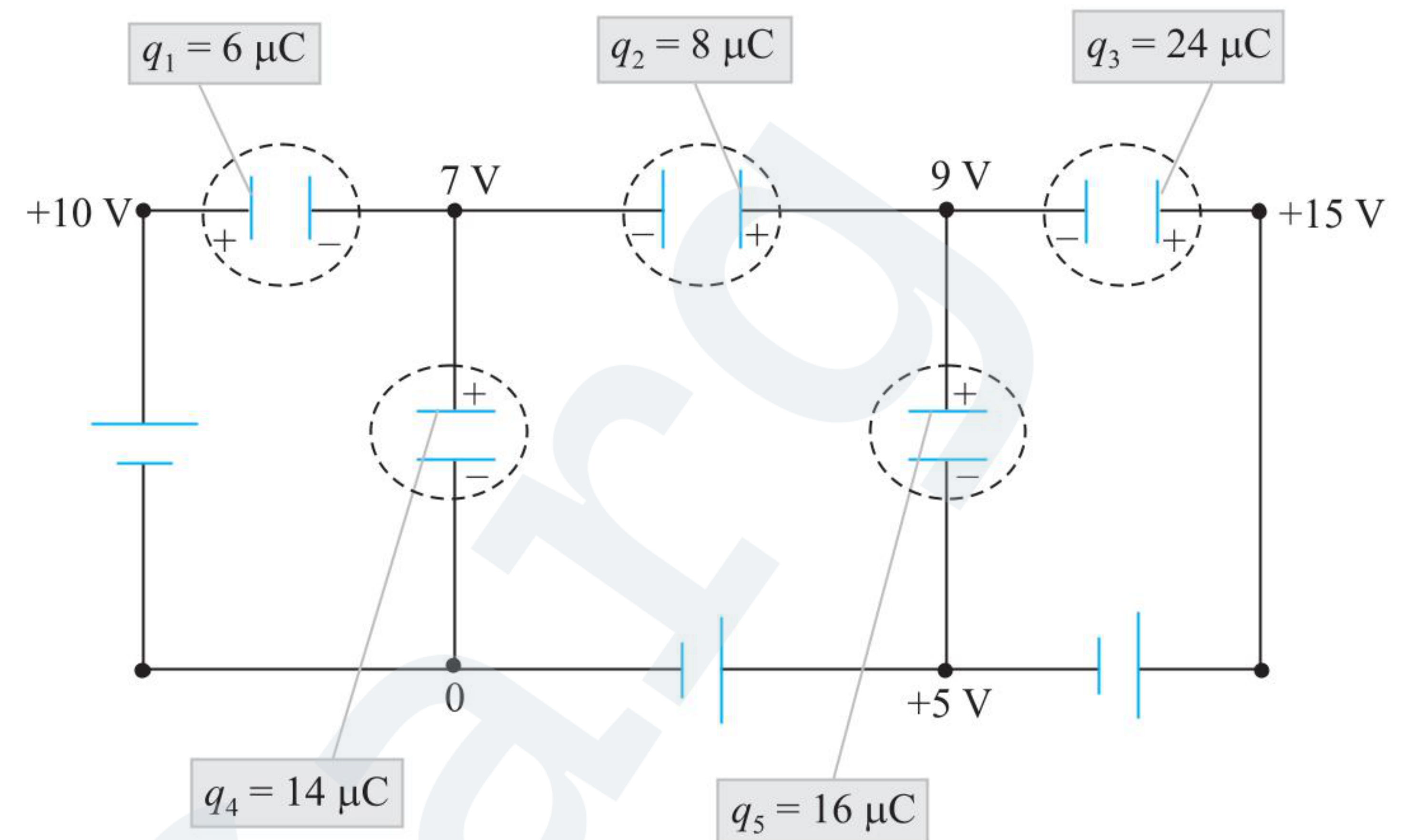
Sol. Let us assign the potentials at nodes N_1 and N_2 as x and y , respectively. We have identified two isolated systems in the circuits as marked in figure. The potentials at different junctions are marked according to the emf of the batteries as shown in the figure. The charge appearing in each capacitors is marked as per the formula $q = C\Delta V$



In the isolated systems (1) and (2), the net charge should be conserved. For the isolated system (1),

$$2(x - 10) + 4(x - y) + 2x = 0$$

$$\text{or } 8x - 4y - 20 = 0 \text{ or } 2x - y = 5 \quad \dots(i)$$



For the isolated system (2),

$$-4(x - y) + 4(y - 15) + 4(y - 5) = 0$$

$$\text{or } -4x + 12y - 80 = 0 \text{ or } -2x + 6y = 40 \quad \dots(ii)$$

Adding Eqs. (i) and (ii), we get

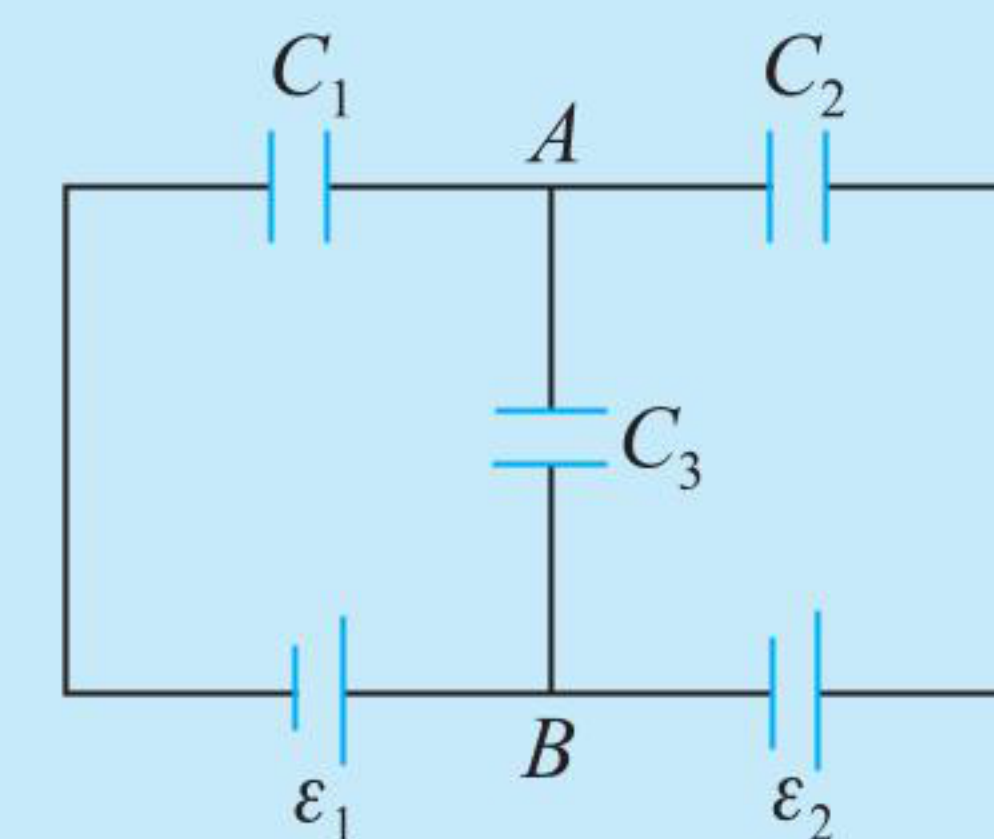
$$5y = 45$$

$$\text{or } y = 9 \text{ V and } x = 7 \text{ V}$$

Hence, the charges on different capacitors are shown in figure.

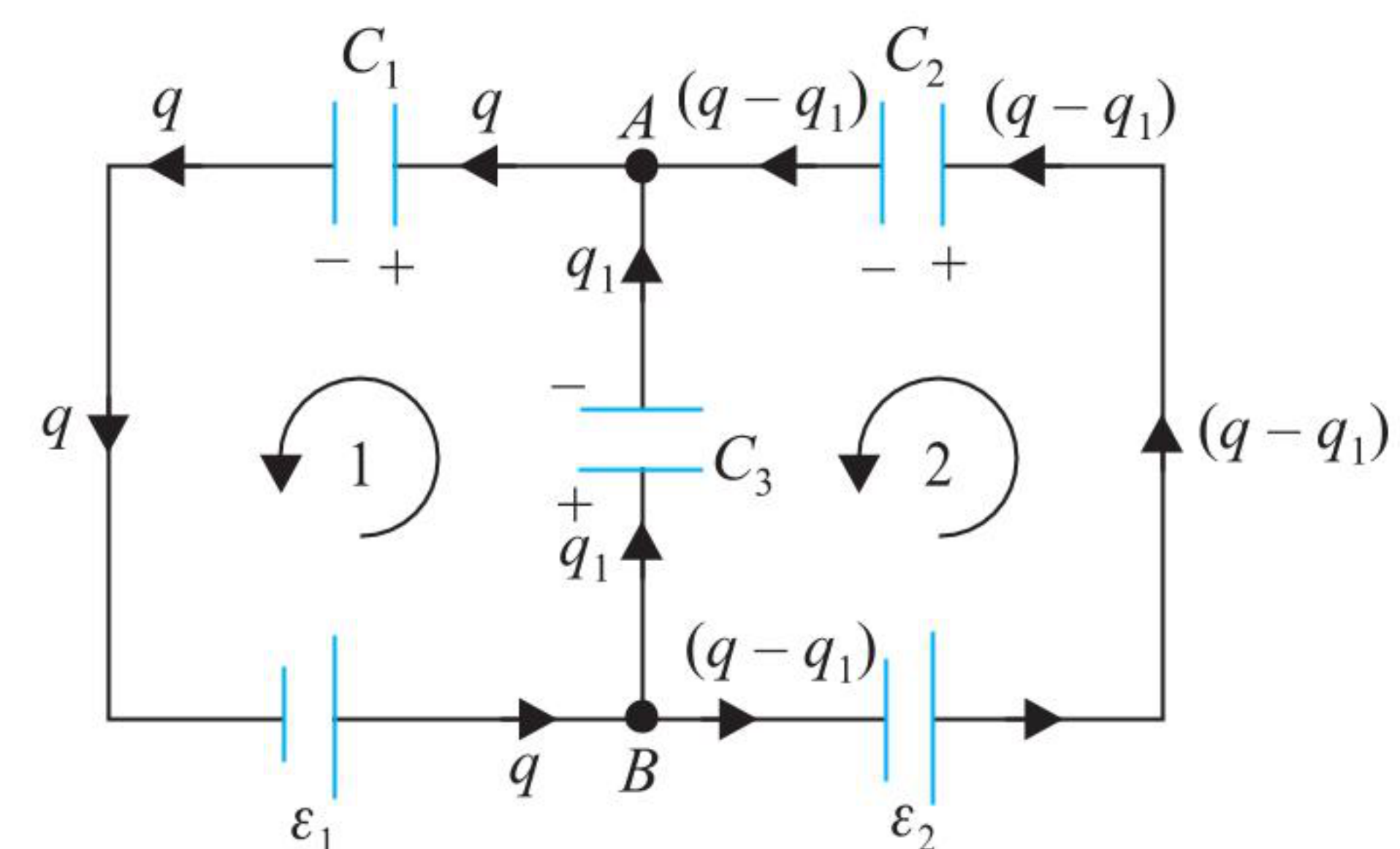
ILLUSTRATION 4.38

Find the potential difference $V_A - V_B$ between points A and B of the circuit shown in figure.



Sol.

Method 1: The distribution of electric charge is shown in figure.



In loop (1),

$$\epsilon_1 - \frac{q_1}{C_3} - \frac{q}{C_1} = 0 \quad \dots(i)$$

In loop (2),

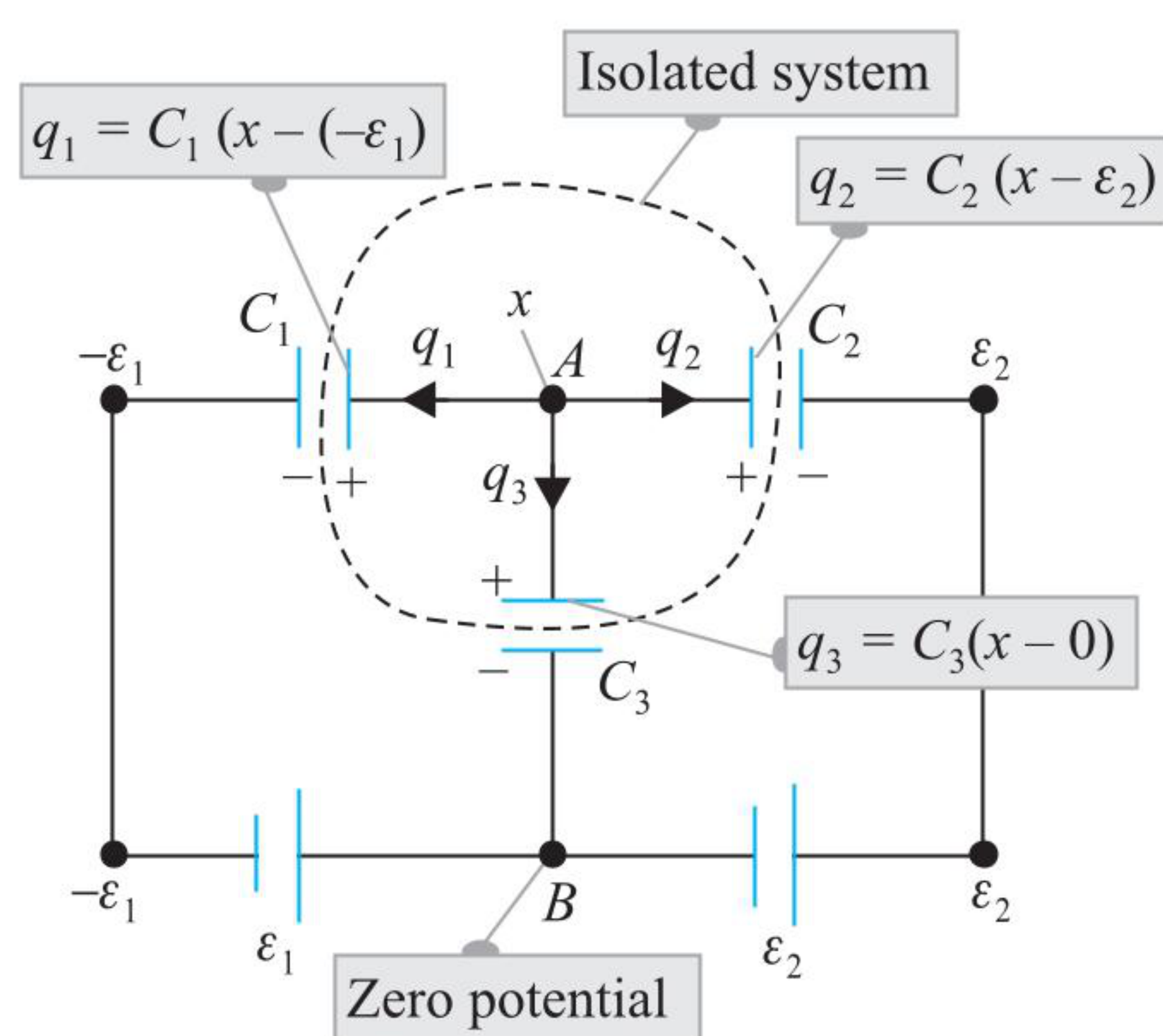
$$-\left(\frac{q - q_1}{C_2}\right) + \frac{q_1}{C_3} + \epsilon_2 = 0 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$q_1 = \frac{-C_3(C_2\varepsilon_2 - C_1\varepsilon_1)}{(C_1 + C_2 + C_3)}$$

$$\therefore V_A - V_B = -\left\{\frac{q_1}{C_3}\right\} = \left(\frac{C_2\varepsilon_2 - C_1\varepsilon_1}{C_1 + C_2 + C_3}\right)$$

Method 2: The algebraic sum of the charge at junction A should be zero. Suppose the charges q_1 , q_2 , and q_3 are moving away from the junction. Let us assume that the potential of junction B is zero. Now we will write the potentials of various junctions as shown in figure. After knowing the potential difference across the capacitors, we can write the relations of q_1 , q_2 , and q_3 in terms of their capacitance and potential difference.



At junction A,

$$\Sigma q = 0$$

or $q_1 + q_2 + q_3 = 0$

or $C_1(x + \varepsilon_1) + C_2(x - \varepsilon_2) + C_3(x - 0) = 0$

or $x(C_1 + C_2 + C_3) = \varepsilon_2 C_2 - \varepsilon_1 C_1$

$$\therefore x = \frac{\varepsilon_2 C_2 - \varepsilon_1 C_1}{C_1 + C_2 + C_3}$$

$$\therefore V_A - V_B = x - 0 = x = \frac{\varepsilon_2 C_2 - \varepsilon_1 C_1}{C_1 + C_2 + C_3}$$

Method 3: The electric charge enclosed by the isolated system shown in figure should be zero. Therefore,

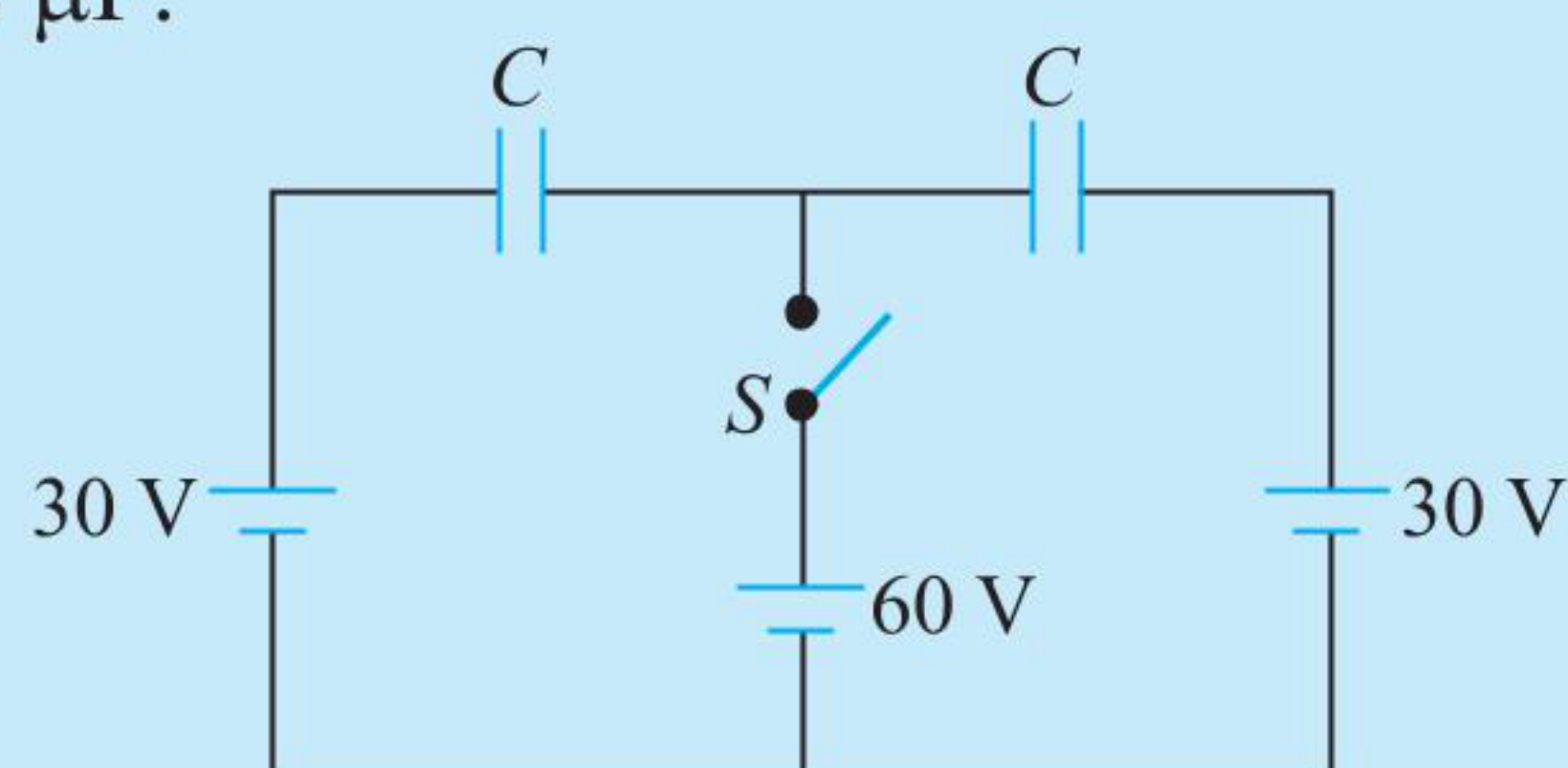
$$q_1 + q_2 + q_3 = 0$$

or $\frac{x + \varepsilon_1}{C_1} + \frac{x - \varepsilon_2}{C_2} + \frac{x - 0}{C_3} = 0$ or $x = \frac{C_2\varepsilon_2 - C_1\varepsilon_1}{C_1 + C_2 + C_3}$

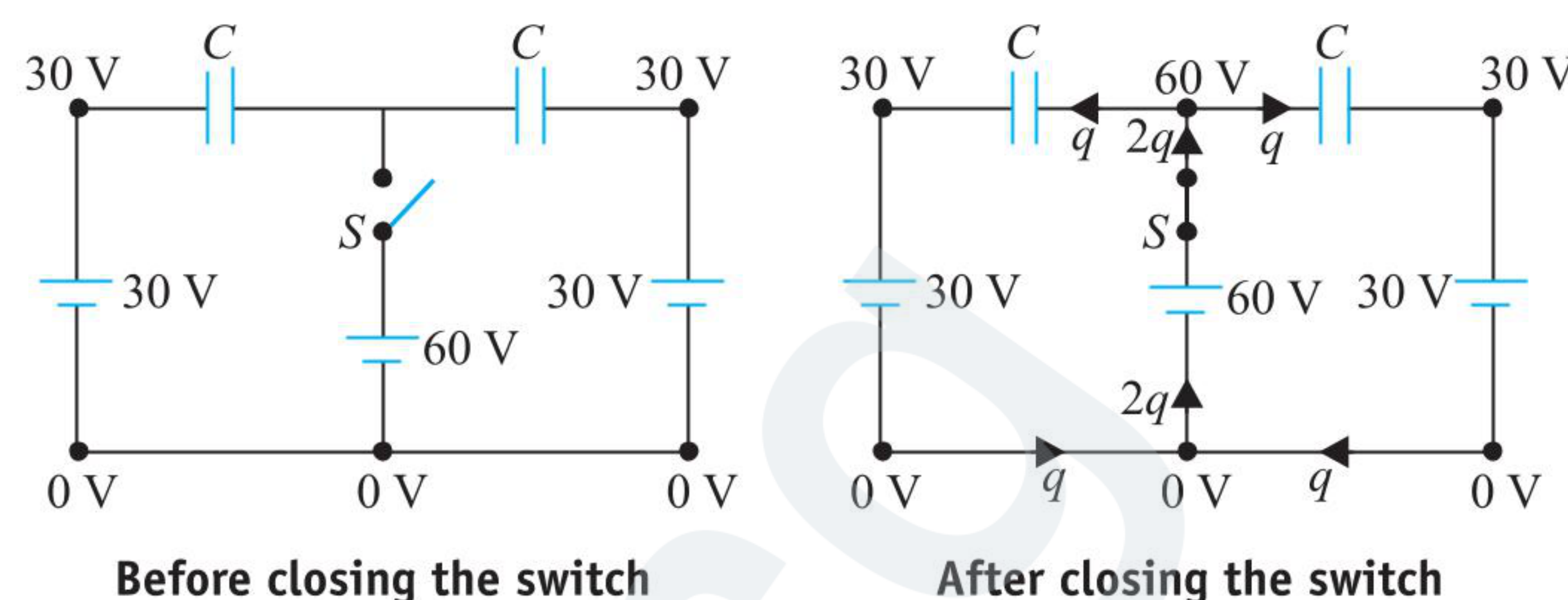
$$\therefore V_A - V_B = x - 0 = \frac{C_2\varepsilon_2 - C_1\varepsilon_1}{C_1 + C_2 + C_3}$$

ILLUSTRATION 4.39

In the given circuit diagram, initially the switch S is opened. Find heat dissipated in the circuit after switch S is closed. It is given $C = 2 \mu\text{F}$.



Sol. When switch S is open the potential difference across the group of capacitors is zero. Hence the initial charge in both the capacitors will be zero.



When the switch S is closed, potential difference across both of them is 30 V. Charge on each of them is

$$q = (2 \mu\text{F})(30 \text{ V}) = 60 \mu\text{C}$$

The heat developed in the circuit

$$H = \sum \frac{(\Delta Q)^2}{2C} = \frac{(60 - 0)^2}{2 \times 2} = 180 \mu\text{J} = 1.8 \text{ mJ}$$

Approach 2: The charge flow in various path has also been shown in figure. The charge supplied by 60 V cell $= 2q = 2 \times 60 = 120 \mu\text{C}$. Energy supplied by 60 V cell, $W_{60\text{V}} = 60 \times 120 = 7200 \mu\text{J}$. Energy absorbed by each of 30 V cell,

$$W_{30\text{V}} = -30 \times 60 = 1800 \mu\text{J}$$

Energy stored in each capacitor

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times 30^2 = 900 \mu\text{J}$$

Applying conservation of energy principle,

$$W_{\text{battery}} = \Delta U + \text{Heat} \Rightarrow \text{Heat} = W_{\text{battery}} - \Delta U$$

$$\therefore \text{Heat dissipated, } H = (7200 - 2 \times 1800) - 2 \times 900 = 180 \mu\text{J} = 1.8 \text{ mJ}$$

CONCEPT APPLICATION EXERCISE 4.4

1. For the network of capacitors as shown in figure.

(a) Find the potential of junction B,

(b) Find the potential of junction D,

(c) Find the charge on $2 \mu\text{F}$ capacitor.

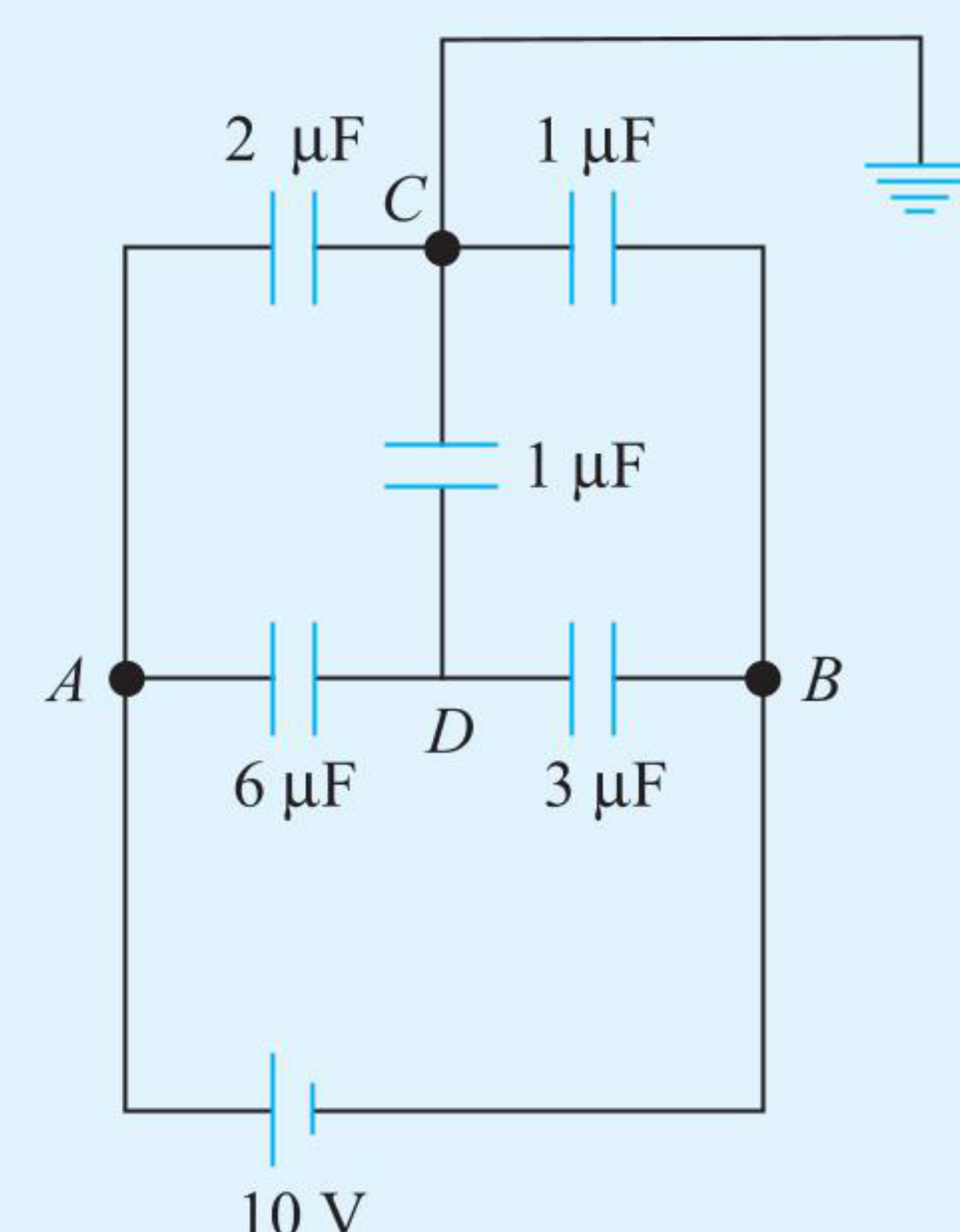
2. In figure, the system is in steady state. Then

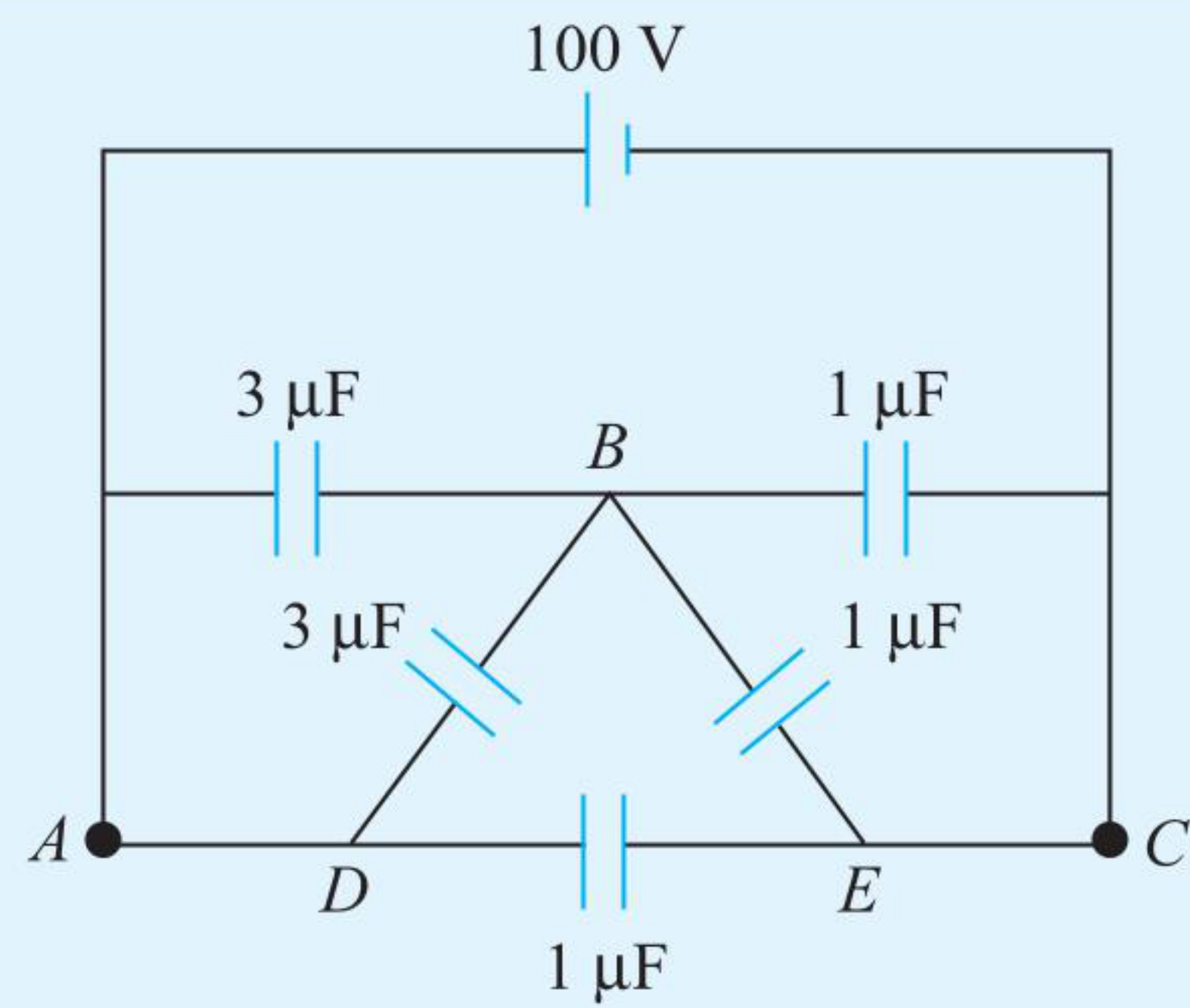
(a) $V_A - V_B =$ _____

(b) $V_B - V_C =$ _____

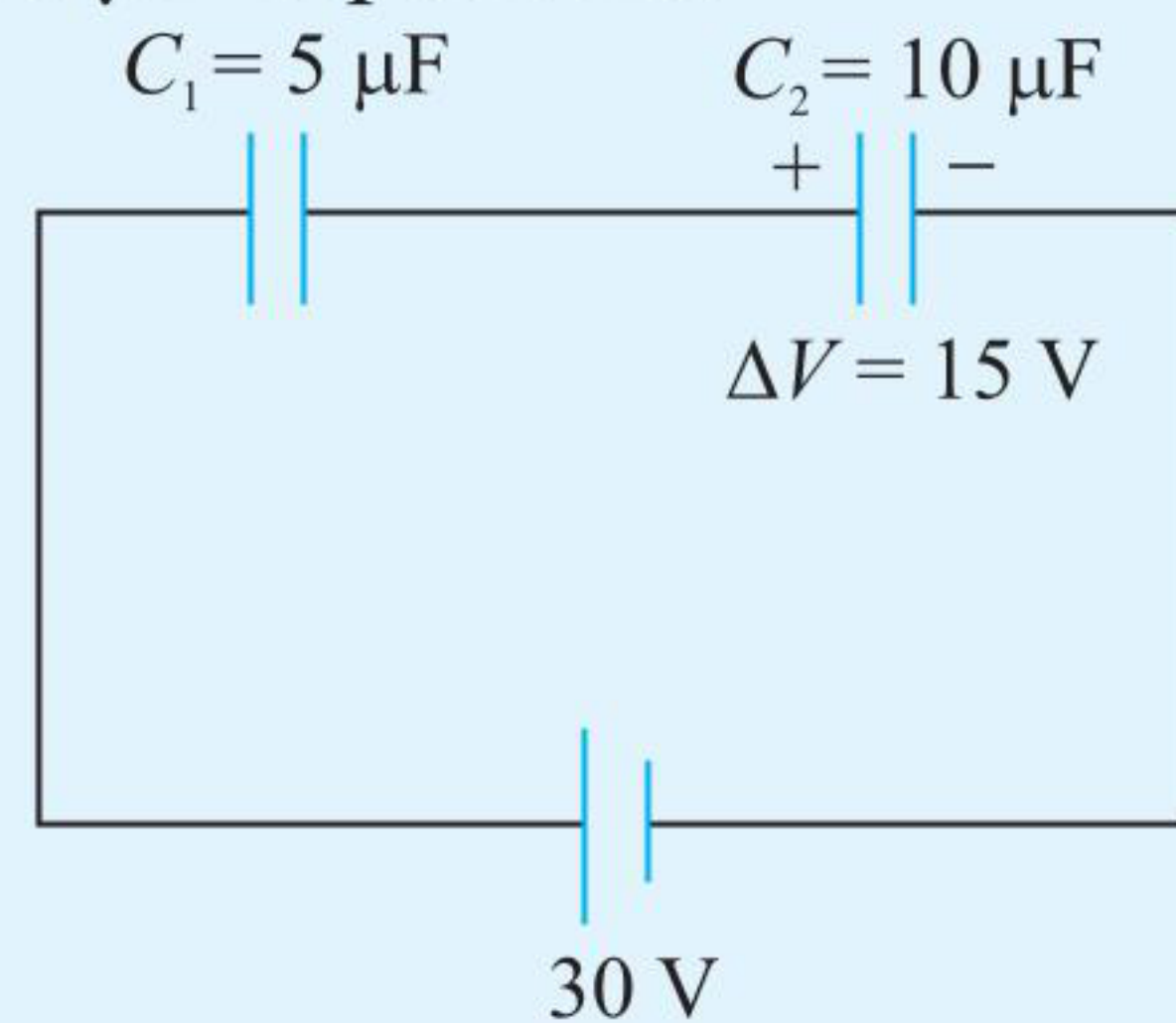
(c) $V_D - V_E =$ _____

(d) The energy stored in the circuit is _____

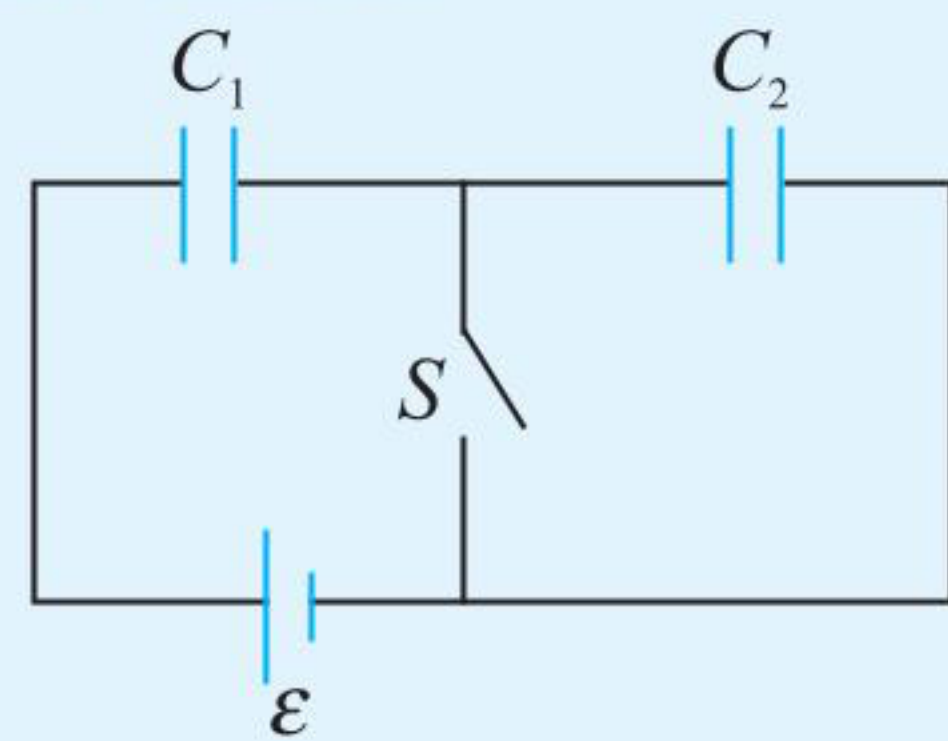




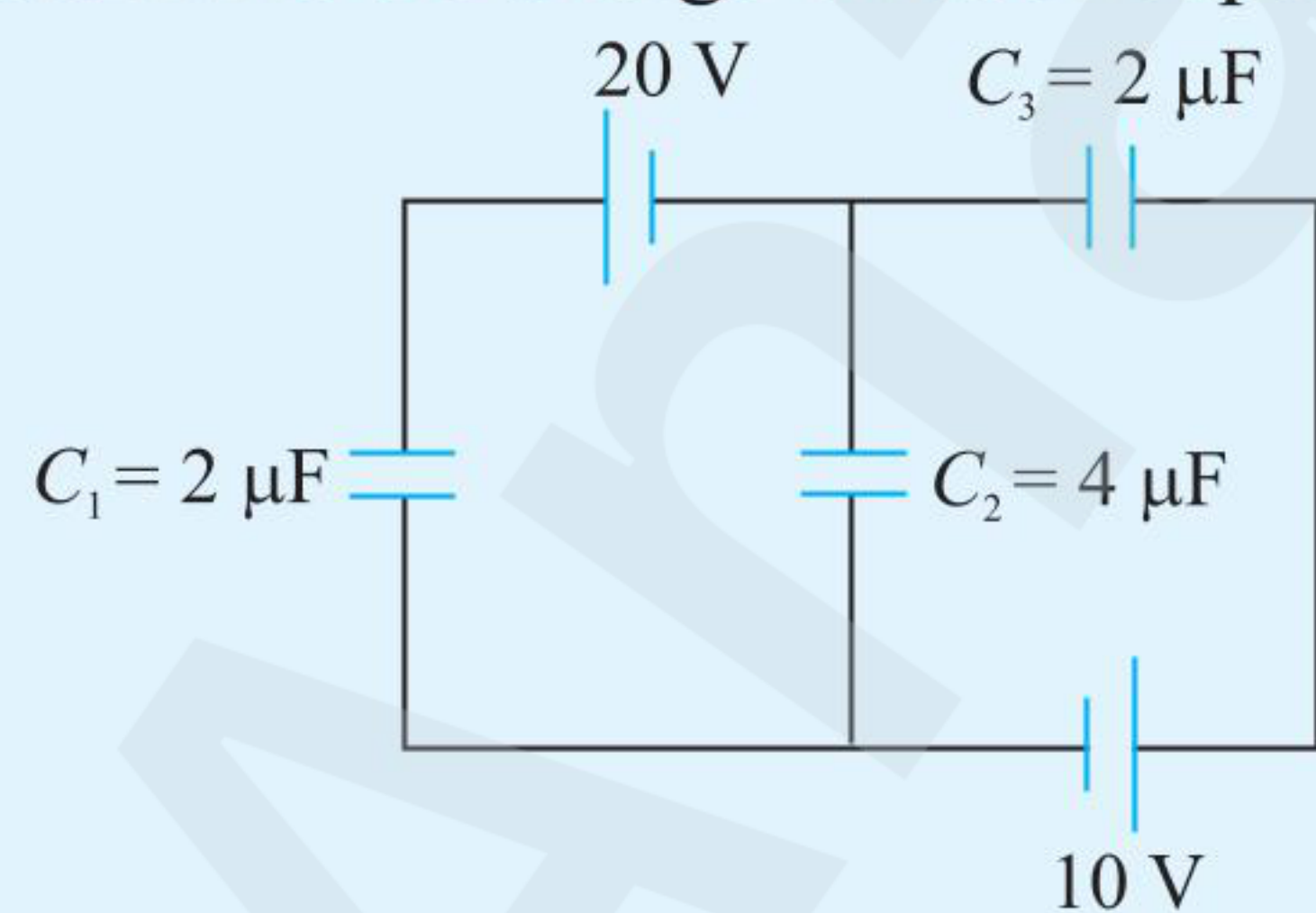
3. A $10\ \mu\text{F}$ capacitor is charged to $15\ \text{V}$. It is next connected in series with an uncharged $5\ \mu\text{F}$ capacitor. The series combination is finally connected across a $30\ \text{V}$ battery, as shown in figure. Find the new potential difference across the $5\ \mu\text{F}$ and $10\ \mu\text{F}$ capacitors.



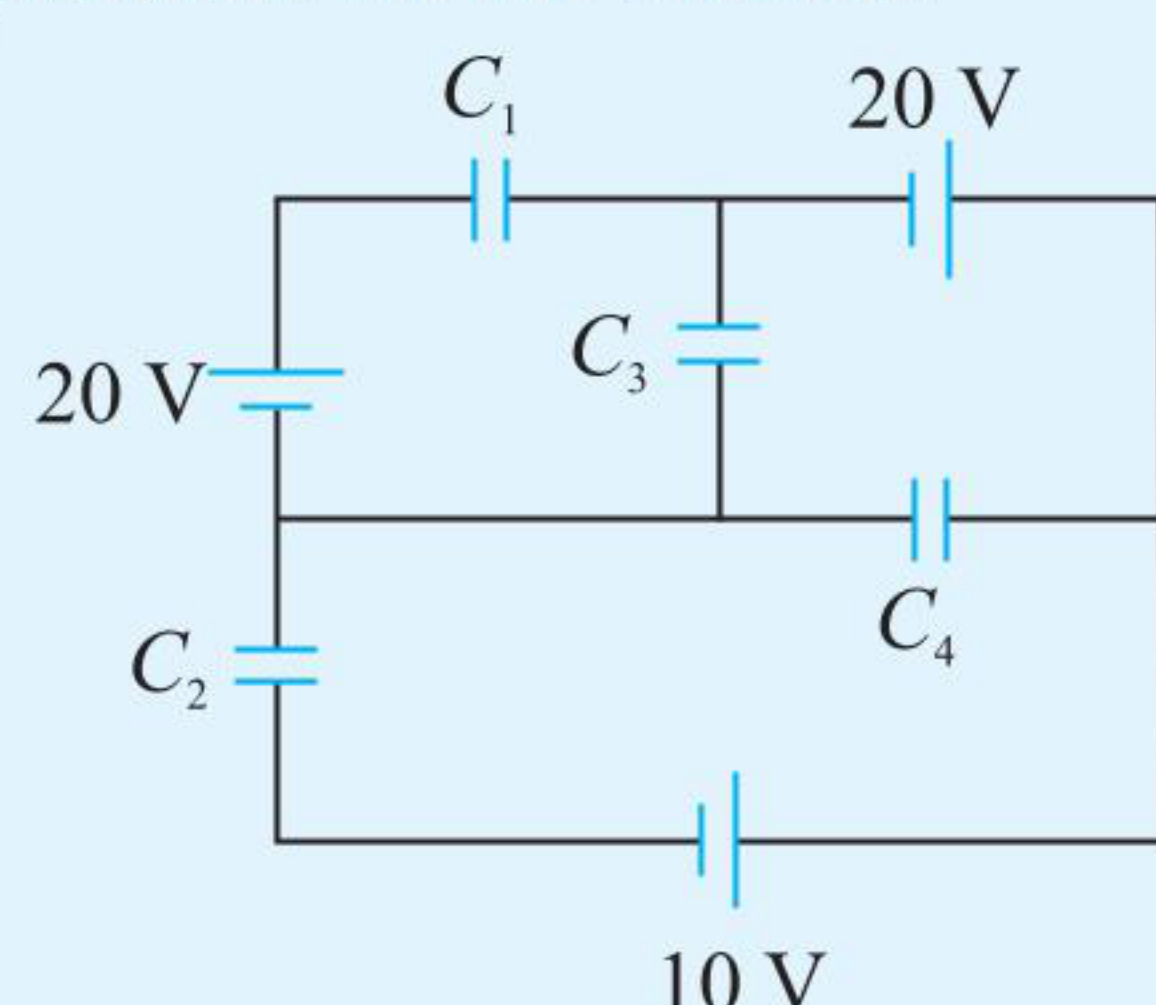
4. Two capacitors $C_1 = C_2 = C$ are connected with a battery of emf ε as shown in figure. The switch S is open for a long time and then closed.



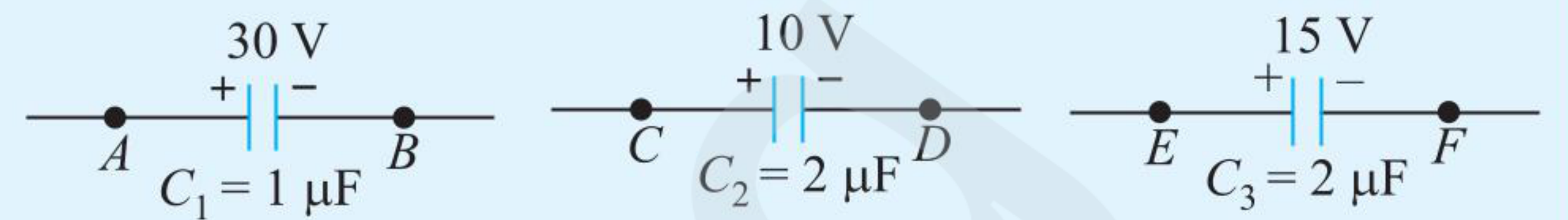
- (a) Find the charge flowing through the battery when the switch S is closed.
 (b) Find the work done by the battery.
 (c) Find the change in energy stored in the capacitance.
 (d) Find the heat developed in the system.
5. Three capacitors C_1 , C_2 , and C_3 are arranged as shown in figure. Determine the charge on each capacitor.



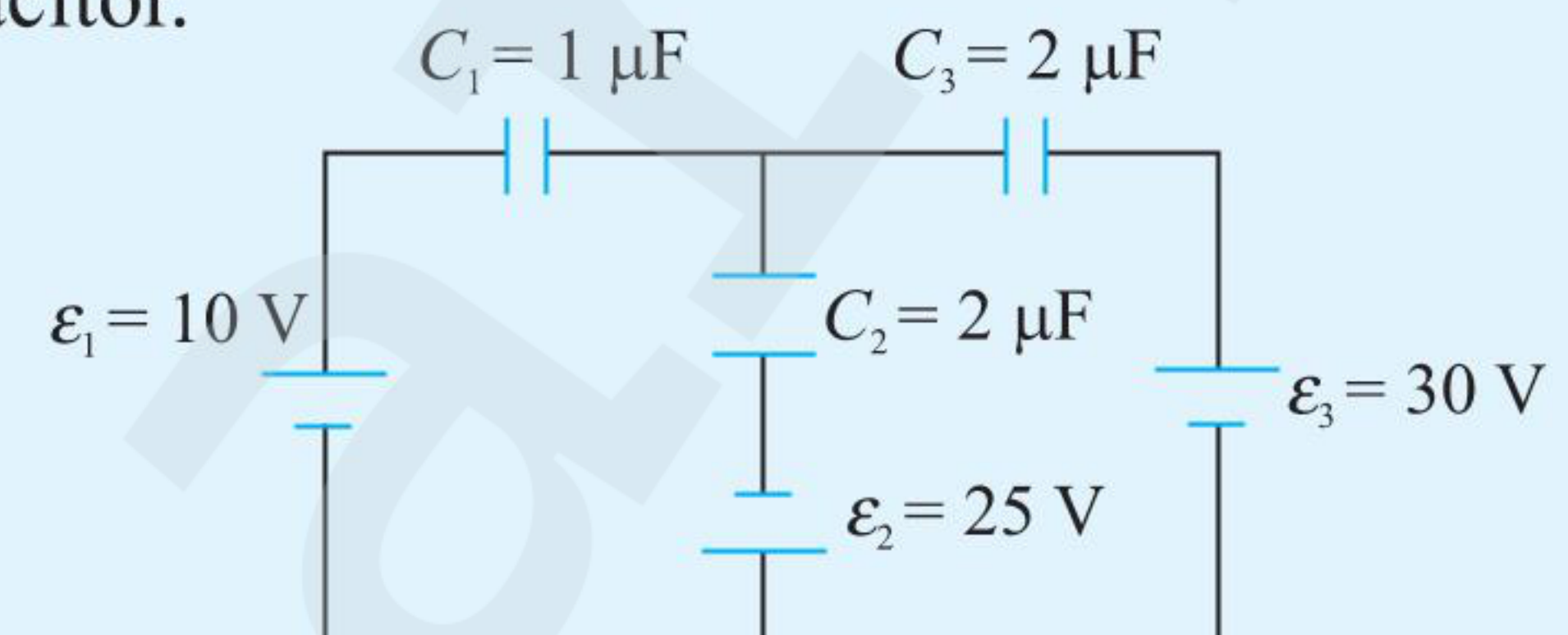
6. Four capacitors $C_1 = 8\ \mu\text{F}$, $C_2 = 2\ \mu\text{F}$, $C_3 = 6\ \mu\text{F}$, and $C_4 = 6\ \mu\text{F}$ are arranged as shown in figure. Find the charge on all the capacitors in the circuit.



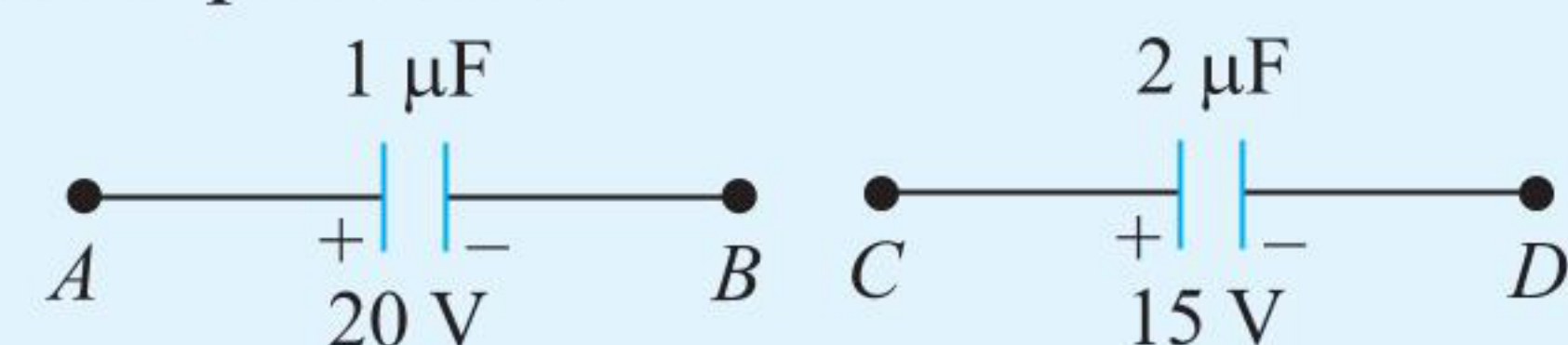
7. Three capacitors of capacitances $1\ \mu\text{F}$, $2\ \mu\text{F}$, and $2\ \mu\text{F}$ are charged up to the potential difference $30\ \text{V}$, $10\ \text{V}$, and $15\ \text{V}$, respectively. If terminal A is connected with D , C is connected with E , and F is connected with B , then find the charge flow in the circuit and the final charges on the capacitors.



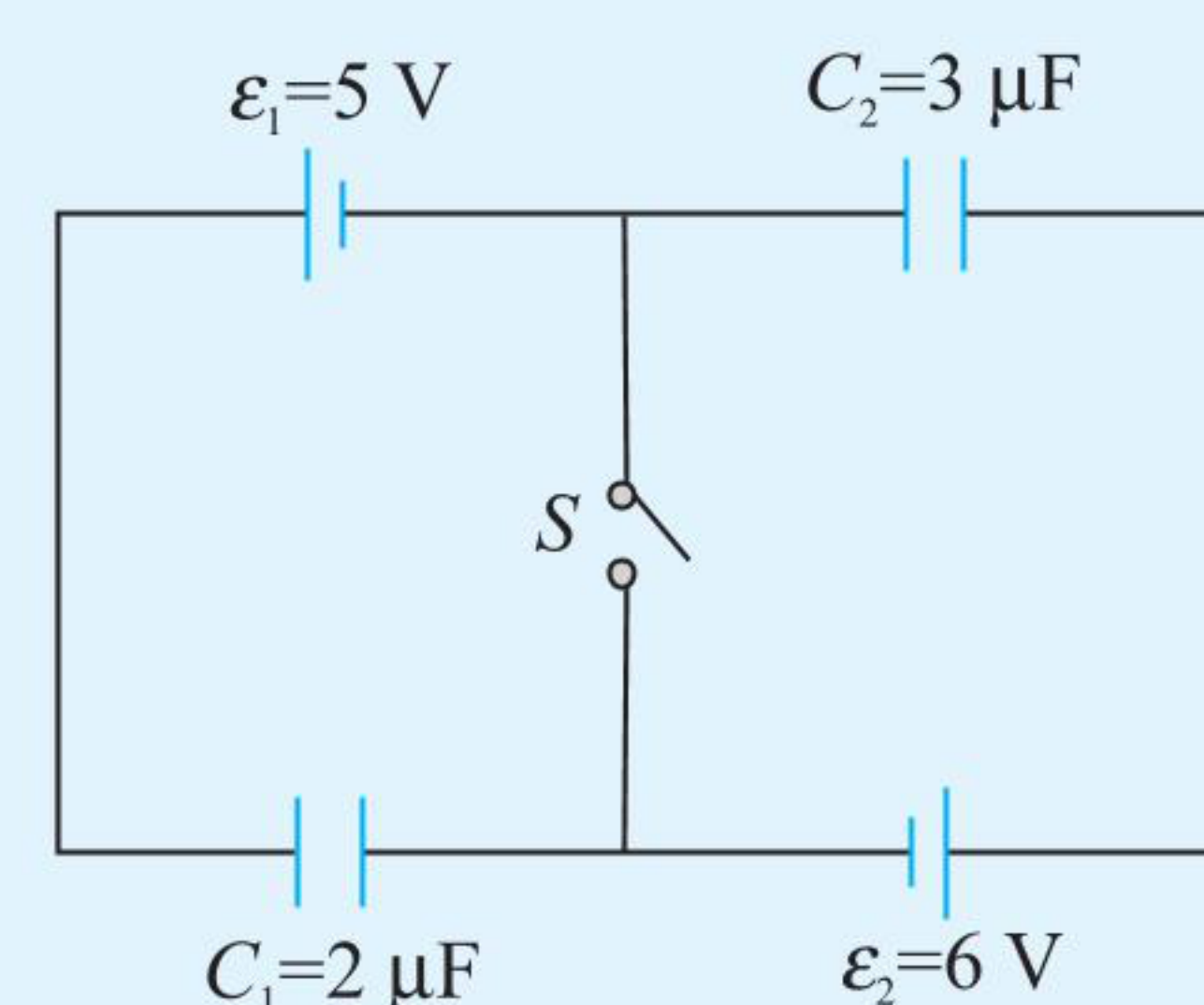
8. Three initially uncharged capacitors are arranged with batteries as shown in figure. Find the charge on each capacitor.



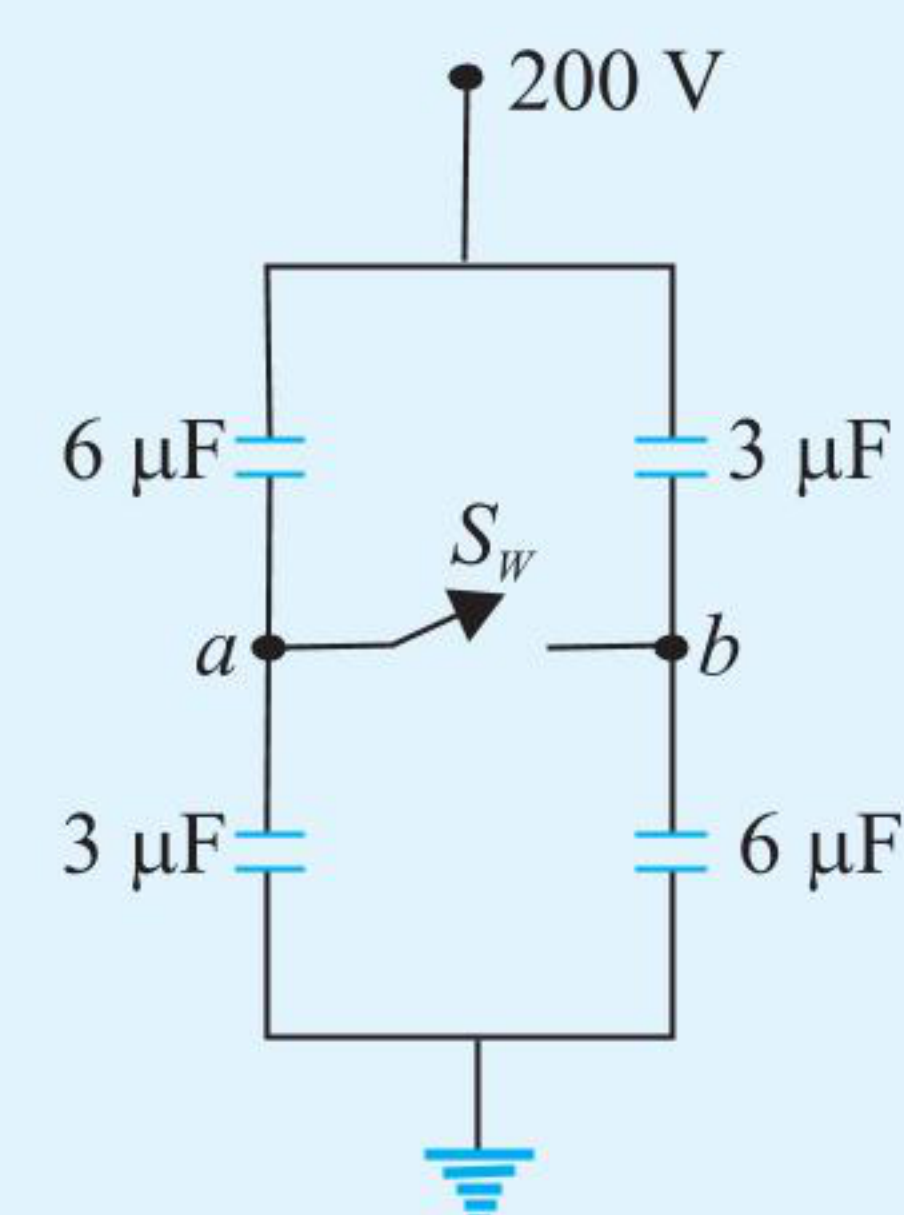
9. Two capacitors of capacitances $1\ \mu\text{F}$ and $2\ \mu\text{F}$ are charged to potential differences $20\ \text{V}$ and $15\ \text{V}$ as shown in figure. If terminals B and C are connected together and terminals A and D are connected with the positive and negative ends of the battery, respectively, then find the final charges on both the capacitors.



10. Two capacitors C_1 and C_2 are connected with two batteries of emf ε_1 and ε_2 . The circuit components are connected with a switch S as shown in figure. Initially the switch is open and the capacitors are charged. Find the heat produced when switch S is closed.



11. Figure shows a capacitive circuit, with a switch S_w .



- (a) What is the potential difference between a and b when the switch is open?
 (b) What charge flows through the switch when it is closed?

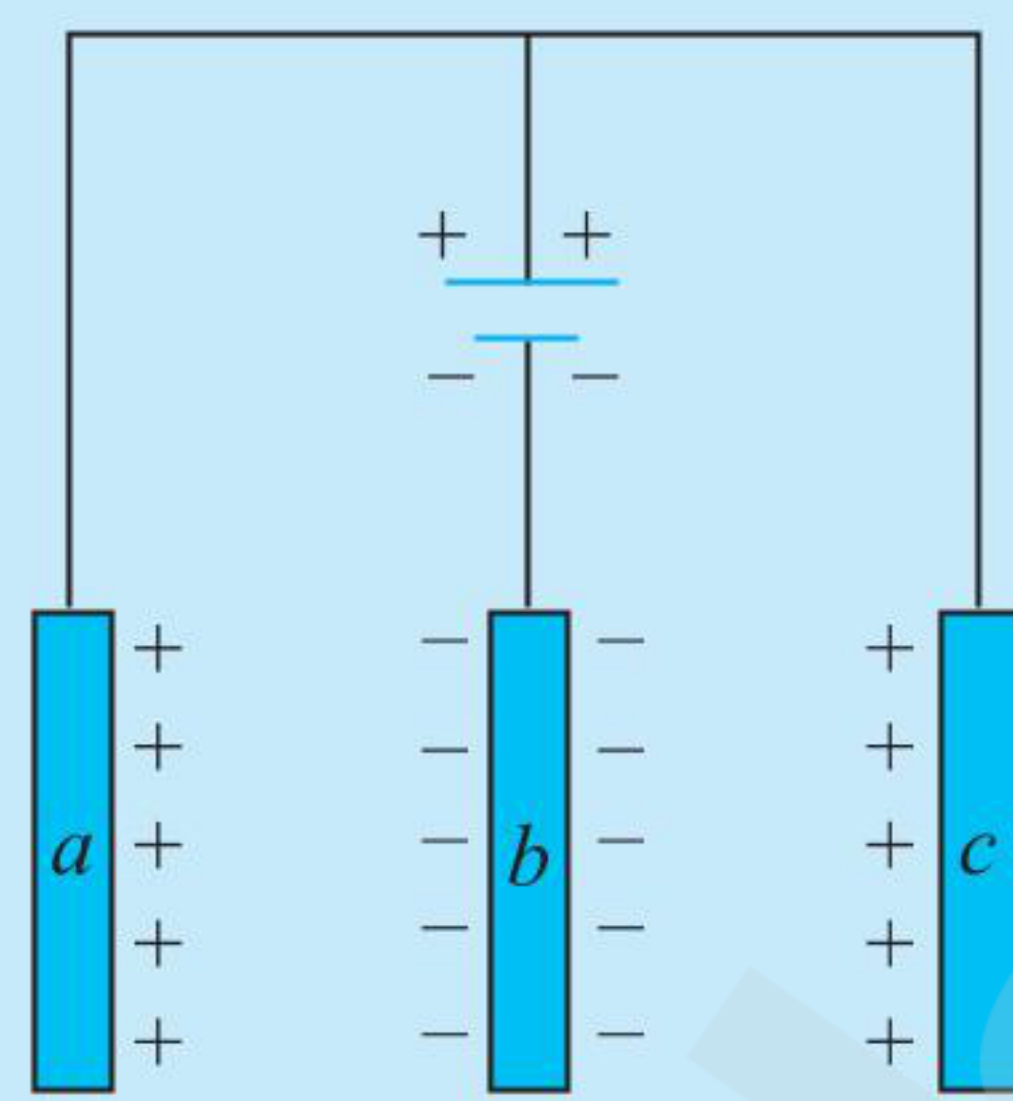
ANSWERS

1. (a) $-\frac{20}{3}$ V (b) 0 (c) $\frac{20}{3}$ μ C
 2. (a) 25 V (b) 75 V (c) 100 V (d) 125×10^{-4} J
 3. 10 V, 20 V 4. (a) $\frac{C\varepsilon}{2}$ (b) $\frac{C\varepsilon^2}{2}$ (c) $\frac{1}{4}C\varepsilon^2$ (d) $\frac{1}{4}C\varepsilon^2$
 5. 35 μ C, 10 μ C and 25 μ C
 6. 100 μ C, 10 μ C, 30 μ C and 60 μ C
 7. 12.5 μ , 17.5 μ C, 42.5 μ C, 7.5 μ C 8. 52 μ C, 58 μ C, 6 μ C
 9. $\frac{50}{3}$ μ C, $\frac{80}{3}$ μ C 10. 40 μ J 11. (a) $\frac{200}{3}$ V (b) 300 μ C

PROBLEMS INVOLVING PLATES

ILLUSTRATION 4.40

Each of the three plates shown in figure has 2.0×10^{-2} m² area on one side, and the gap between the adjacent plates is 0.2 mm. The emf of the battery is $\varepsilon = 20$ V. Find the distribution of charge on various surfaces of the plates. What is the equivalent capacitance of the system between the terminal points?



Sol. As the potentials of a and c are equal, the capacitors C_{ab} and C_{bc} are in parallel. Therefore,

$$C_{ab} = C_{bc} = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 2 \times 10^{-2}}{0.2 \times 10^{-3}} = 100 \varepsilon_0 (F)$$

Equivalent capacitance is

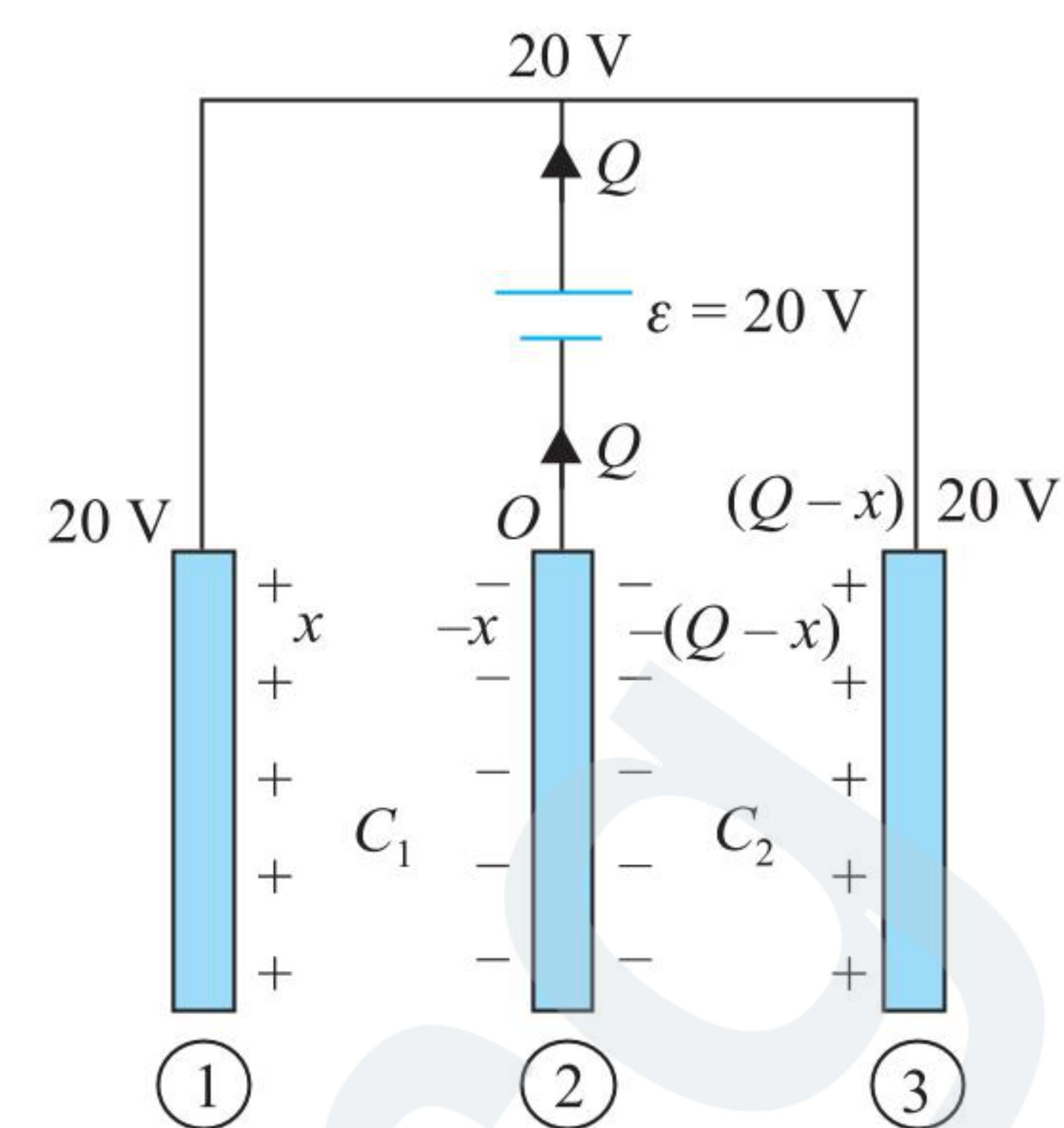
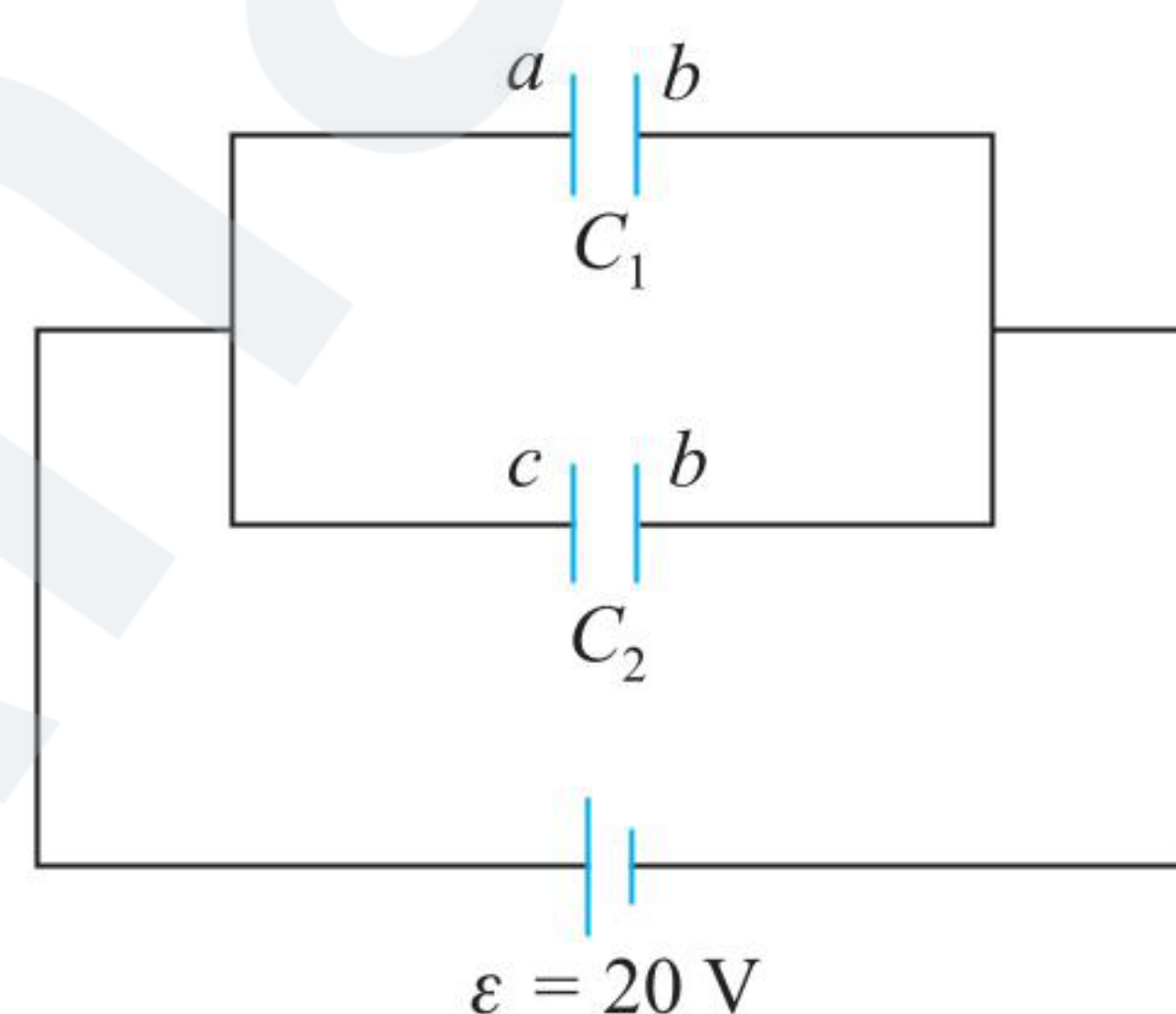
$$C = C_{ab} + C_{bc} = \frac{2\varepsilon_0 A}{d} = \frac{2 \times \varepsilon_0 \times 2 \times 10^{-2}}{0.2 \times 10^{-3}} = 200 \varepsilon_0 (F)$$

The charge on plate b is negative on both faces. Thus, the charge on faces a and c is

$$q_a = q_c = \frac{\varepsilon_0 A}{d} \varepsilon = 100 \varepsilon_0 \times 20 = 2000 \varepsilon_0 (C)$$

$$\therefore Q = 2 \times 2000 \varepsilon_0 (C) = 4000 \varepsilon_0 (C)$$

Alternative method: Let us assume that the potential of the middle plate be zero. Then the potential of the left and right plates will be 20 V each. Let the positive terminal of the battery supply a charge Q . This charge is divided into two plates as shown in figure. The charge on the outermost surfaces will be zero.



Equal and opposite charges will appear on both surfaces of the mid plate (plate 2). The sum of the charges on both the surfaces should be $-Q$, which will flow toward the negative terminal of the battery. We can observe two capacitors C_1 and C_2 . For capacitor C_1

$$x = C(20 - 0) \quad \dots(i)$$

For capacitor C_2

$$(Q - x) = C(20 - 0) \quad \dots(ii)$$

From (i)

$$x = \frac{\varepsilon_0 A}{d} \times 20 = \frac{\varepsilon_0 (2 \times 10^{-2})}{0.2 \times 10^{-3}} \times 20 = 2000 \varepsilon_0 (C)$$

Also from (ii),

$$(Q - x) = 2000 \varepsilon_0 (C)$$

The net charge on the mid plate is

$$-x + (Q - x) = -2 \times 2000 \varepsilon_0 = -4000 \varepsilon_0 (C)$$

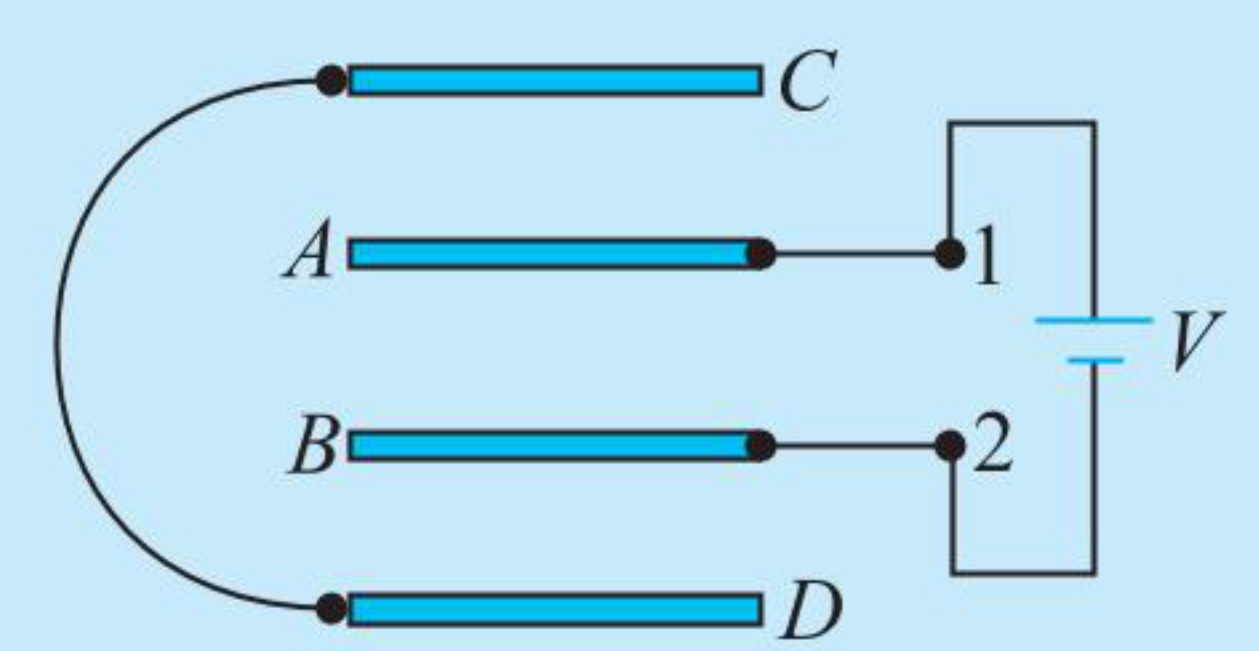
Hence, the equivalent capacity is

$$C_{eq} = \frac{Q}{\varepsilon} = \frac{4000 \varepsilon_0}{20}$$

$$\text{or } C_{eq} = 200 \varepsilon_0 (F)$$

ILLUSTRATION 4.41

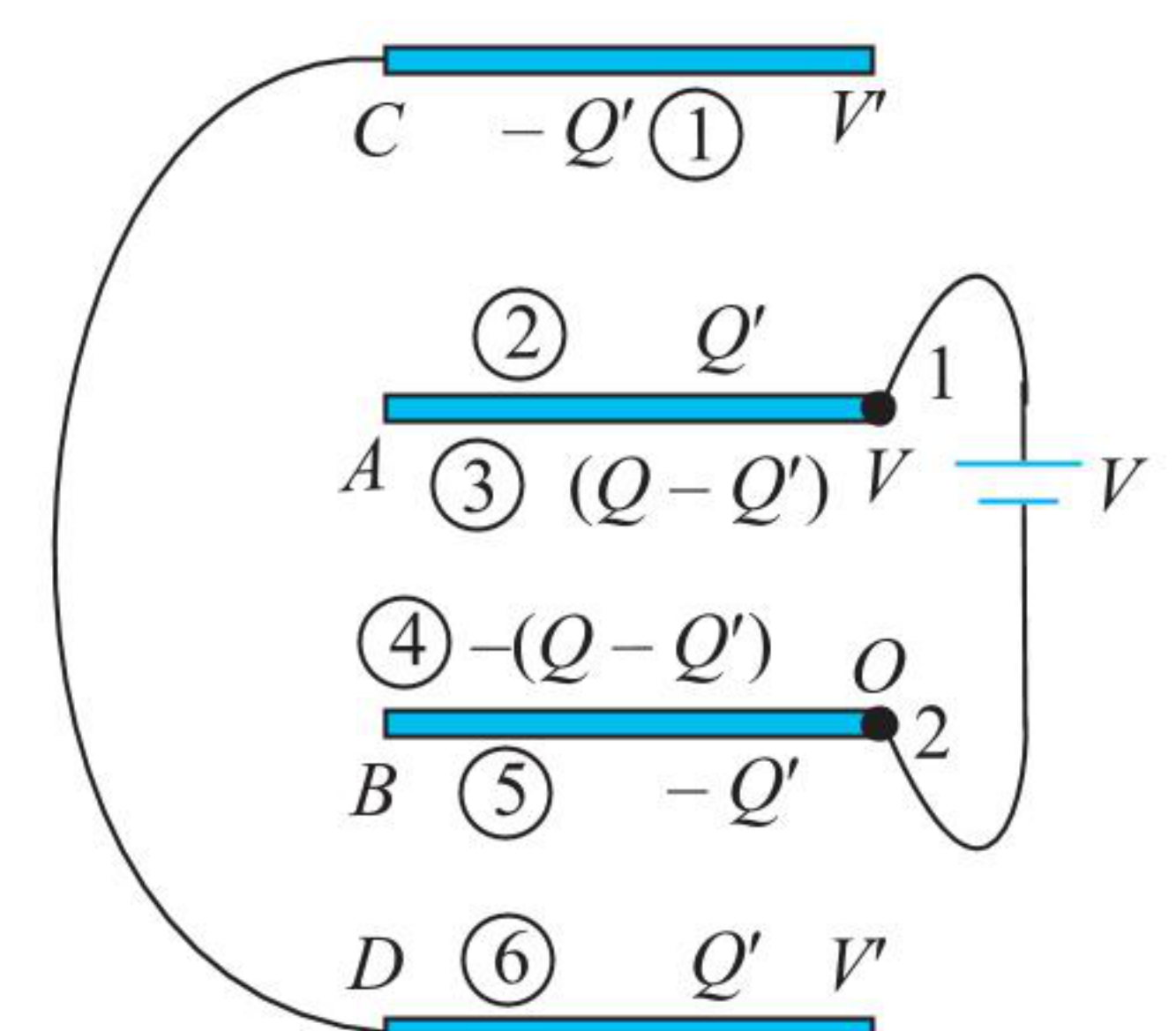
Four identical metal plates are arranged in air at equal distance d from each other. The area of each plate is A . A battery of emf V is connected across plates 1 and 2. Discuss the charge distribution and find the capacitance of the system between points 1 and 2 if the other two plates are connected by a conducting wire as shown in figure.



Sol. As the battery is connected to A and B, there is a charge $+Q$ on plate A and $-Q$ on plate B. The charges on A and B will induce a charge $-Q'$ on C and a charge $+Q'$ on D, so that the net charge on plates C and D remains zero.

Consequently, charge Q on A is divided into two parts: $+Q'$ on the upper side of plate A and $(Q - Q')$ on the lower side. Similarly, charge $-Q$ on B is also divided: $-Q'$ on the lower side and $-(Q - Q')$ on the upper side on plate B.

Let us assume that the potential of plate A be V , then the potential



of plate B is zero. As plates C and D are connected, they will be at a common potential, say V' . So the capacity of the system can be written as follows

$$C = \frac{Q}{V_+ - V_-} = \frac{Q}{V} \quad \dots(i)$$

We can write the equation for facing surfaces (1) and (2) as

$$Q' = C_0(V - V') \quad \dots(ii)$$

For facing surfaces (3) and (4)

$$(Q - Q') = C_0(V - 0) \quad \dots(iii)$$

For facing surfaces (5) and (6)

$$Q' = C_0(V' - 0) \quad \dots(iv)$$

From Eqs. (ii), (iii), and (iv)

$$\frac{Q}{V} = \frac{3}{2}C_0 = \frac{3}{2} \frac{\epsilon_0 A}{d}$$

$$\text{and } Q' = \frac{C_0 V}{2} = \frac{\epsilon_0 A V}{2d}$$

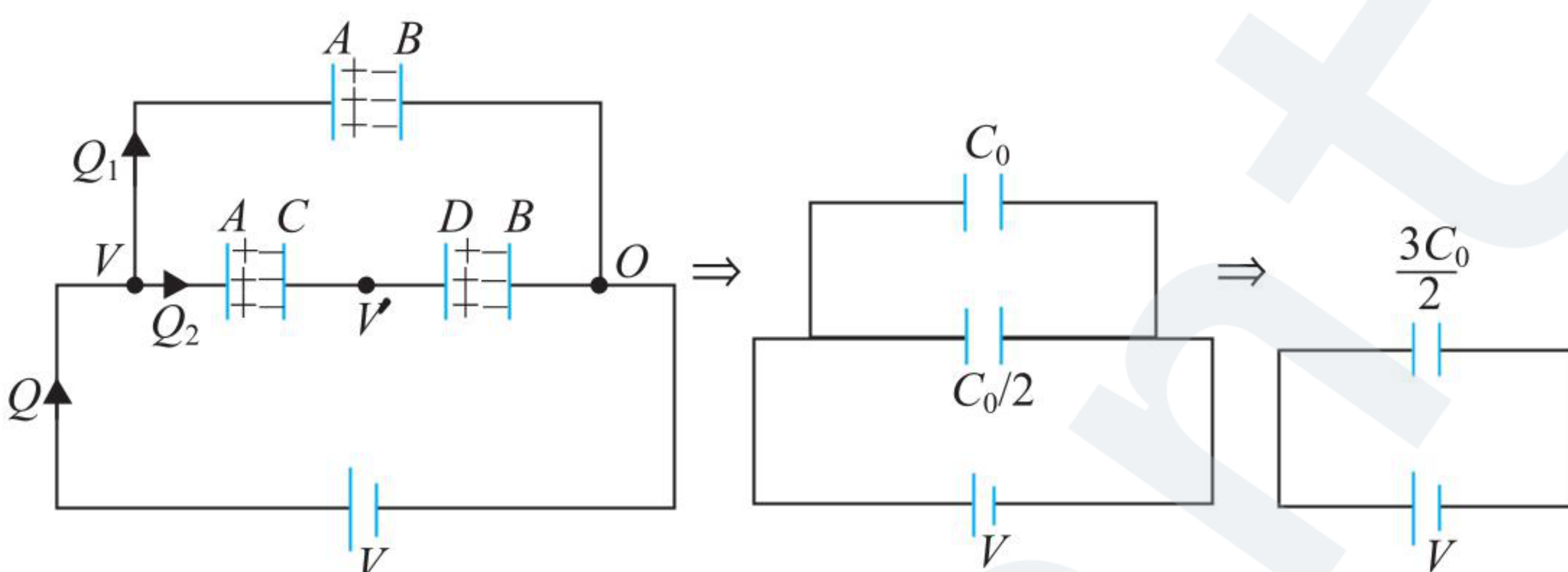
Hence, the equivalent capacitance is

$$C_{eq} = \frac{3}{2} \frac{\epsilon_0 A}{d}$$

Charges on different surfaces are given in the following table:

Surfaces	(1), (5)	(2), (6)	(3)	(4)
Charges	$-\frac{\epsilon_0 A V}{2d}$	$+\frac{\epsilon_0 A V}{2d}$	$\frac{\epsilon_0 A V}{d}$	$-\frac{\epsilon_0 A V}{d}$

Method 2: An equivalent circuit can be drawn as shown in figure.



Hence, the equivalent capacitance is

$$C_{eq} = \frac{3}{2}C_0 = \frac{3}{2} \frac{\epsilon_0 A}{d}$$

The charge supplied by the battery is

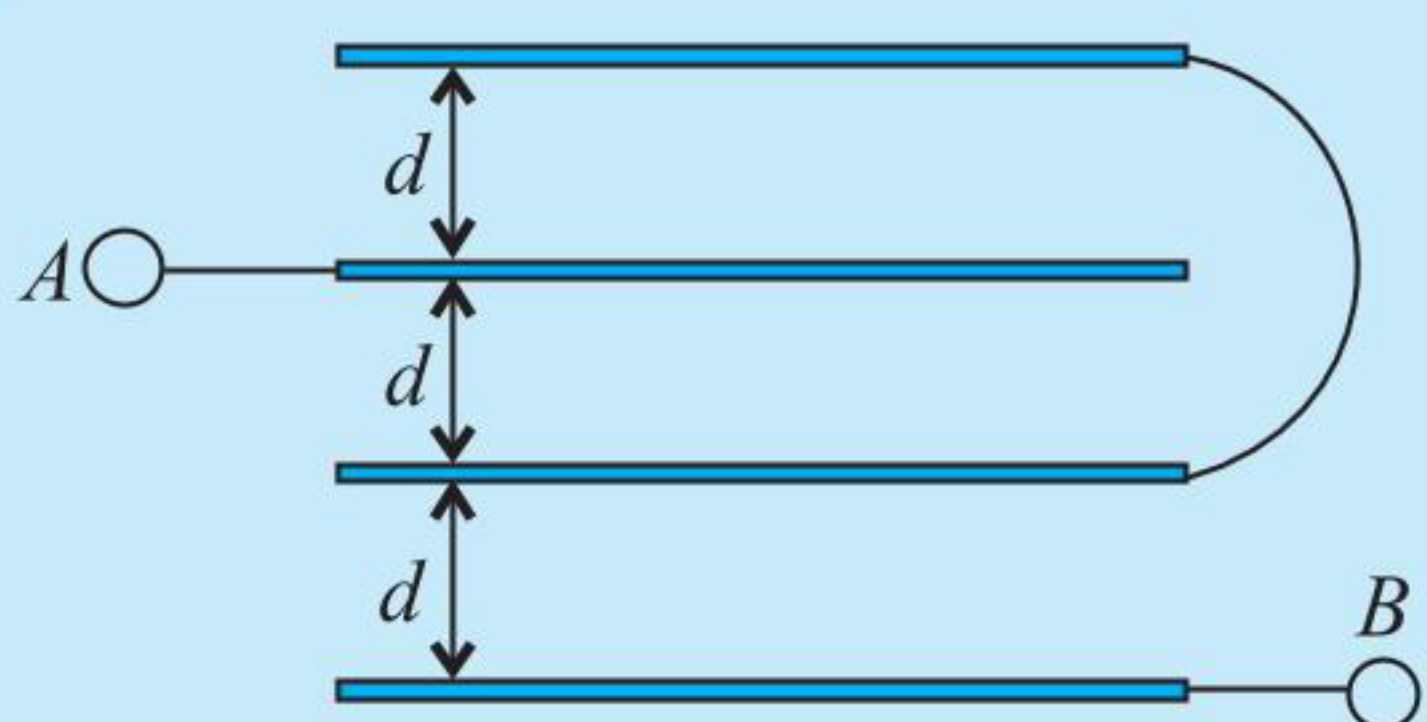
$$Q = \frac{3}{2} \frac{\epsilon_0 A}{d} \cdot V$$

$$\therefore Q_1 = Q \left(\frac{C_0}{C_0 + C_0/2} \right) = \frac{2}{3}Q \text{ and } Q_2 = \frac{Q}{3}$$

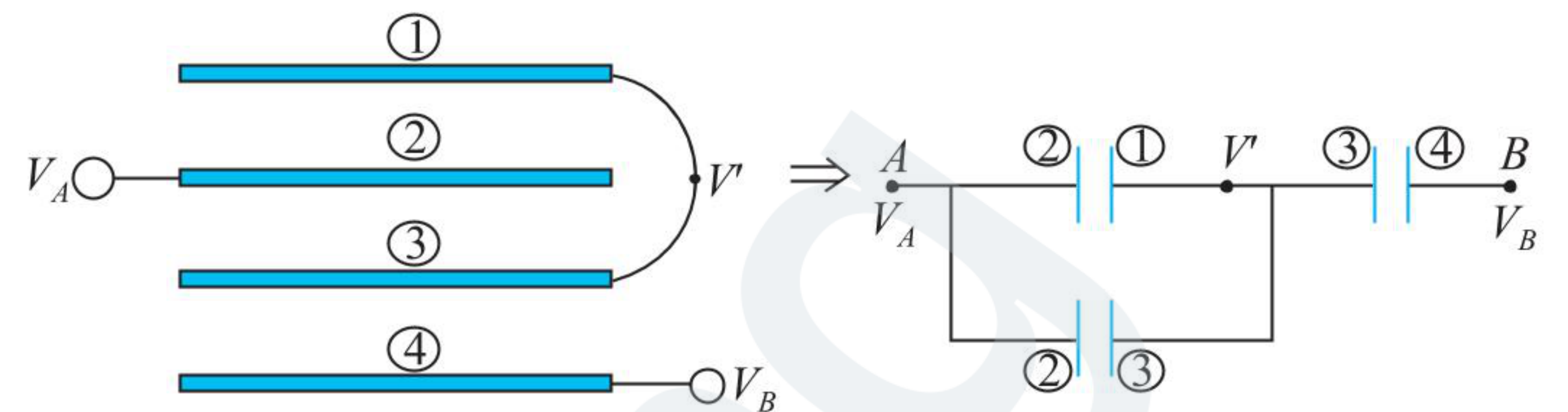
From these results, the charge on different surfaces can be calculated.

ILLUSTRATION 4.42

Four identical plates, each having area A , are arranged as shown in figure. Find the equivalent capacity of the structure between A and B .



Sol. Method 1: Let the potentials of plates A and B be V_A and V_B , respectively. As the plates (1) and (3) (figure) are connected together, they will have a common potential, say V' . Now we will make an equivalent circuit diagram by connecting the different plates across the assumed potential difference.



The equivalent capacitance between A and B can easily be calculated as follows:

$$C_{eq} = \frac{2}{3}C = \frac{2}{3} \frac{\epsilon_0 A}{d}$$

Method 2: Let us assume that a battery is connected between A and B . Let the potentials of A and B be V and 0 . Plates (1) and (2) are connected together and both have the same potential V' . Let the charge supplied by the positive terminal of the battery to plate (1) be Q ; same charge will come out from plate (4). The outermost surface of the plates will carry no charge. Hence, $-Q$ charge will appear on the inner surface of plate (4). The charge distribution on different surfaces of the plates can be done by using conservation of charge. The charge appearing on the facing surfaces should be equal and opposite. The charges appearing on different surfaces are shown in figure. The capacitance of the system of plates can be given by

$$C = \frac{Q}{(V - 0)} \quad \dots(i)$$

Between plates (1) and (2),

$$(Q - x) = C(V - V') \quad \dots(ii)$$

Between plates (2) and (3),

$$x = C(V - V') \quad \dots(iii)$$

Between plates (3) and (4),

$$Q = C(V' - 0) \quad \dots(iv)$$

From (ii) and (iii),

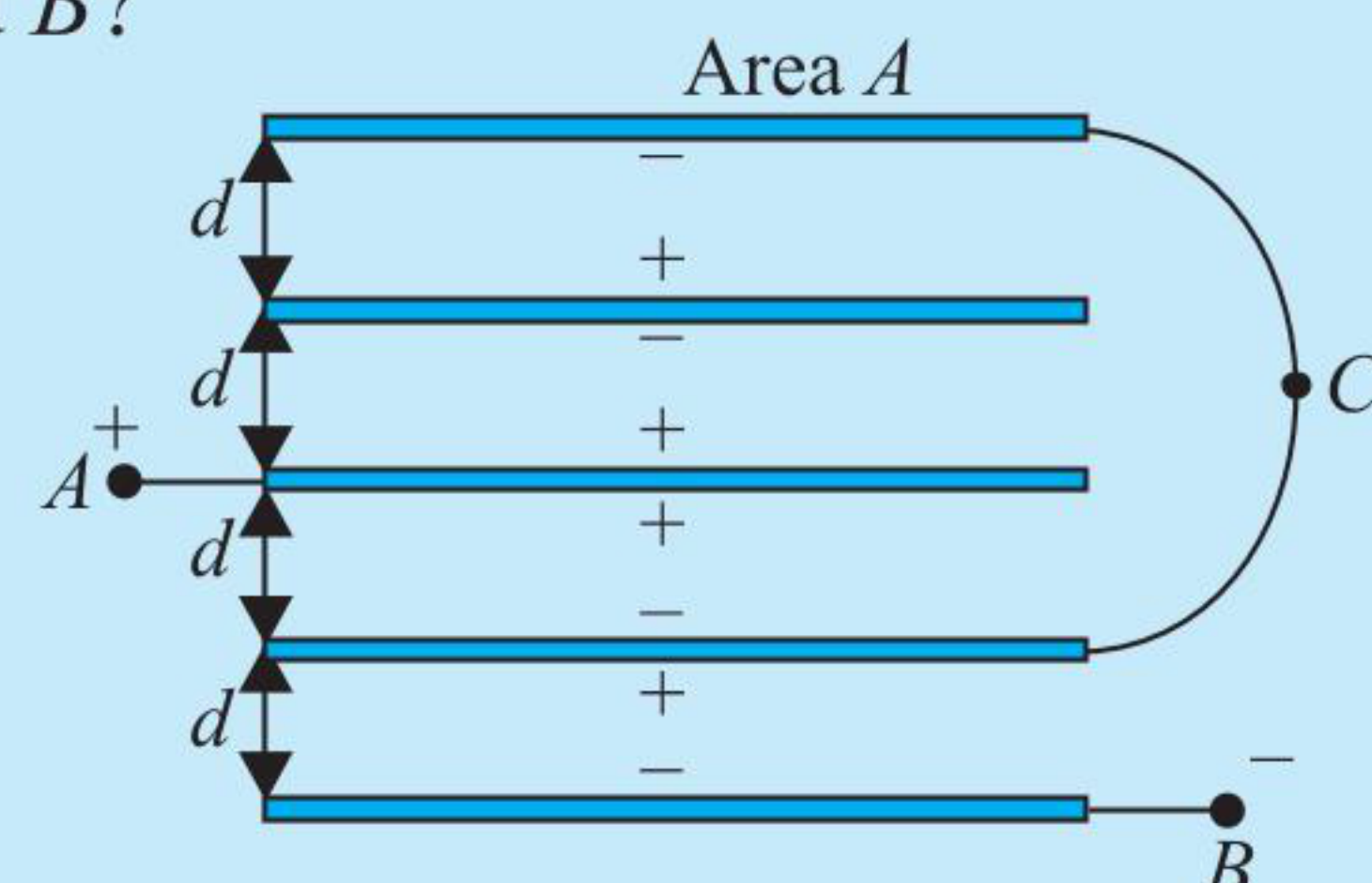
$$(Q - x) = x \text{ or } x = \frac{Q}{2} \quad \dots(v)$$

Adding Eqs. (iii) and (iv), we get

$$V = \frac{x}{C} + \frac{Q}{C} = \frac{3Q}{2C} \text{ or } \frac{Q}{C} = \frac{2}{3}V = \frac{2}{3} \frac{\epsilon_0 A}{d} = C_{eq}$$

ILLUSTRATION 4.43

Five identical plates, each having area A , are arranged as shown in figure. Find the equivalent capacity of the structure between A and B ?



Sol. Method 1: The charge distribution on different surfaces can be done by following conservation of charge and noting that facing surfaces have equal and opposite charges. Thus, the equivalent capacity can be given by

$$C_{eq} = \frac{Q}{V_A - V_B} \quad \dots(i)$$

Between plates (1) and (2),

$$(Q - x) = C(V'' - V') \quad \dots(ii)$$

Between plates (2) and (3),

$$(Q - x) = C(V - V'') \quad \dots(iii)$$

Between plates (3) and (4),

$$x = C(V - V') \quad \dots(iv)$$

Between plates (4) and (5),

$$Q = C(V' - 0) \quad \dots(v)$$

Adding Eqs. (ii) and (iii), we get

$$2(Q - x) = C(V - V') \quad \dots(vi)$$

From (iv) and (vi),

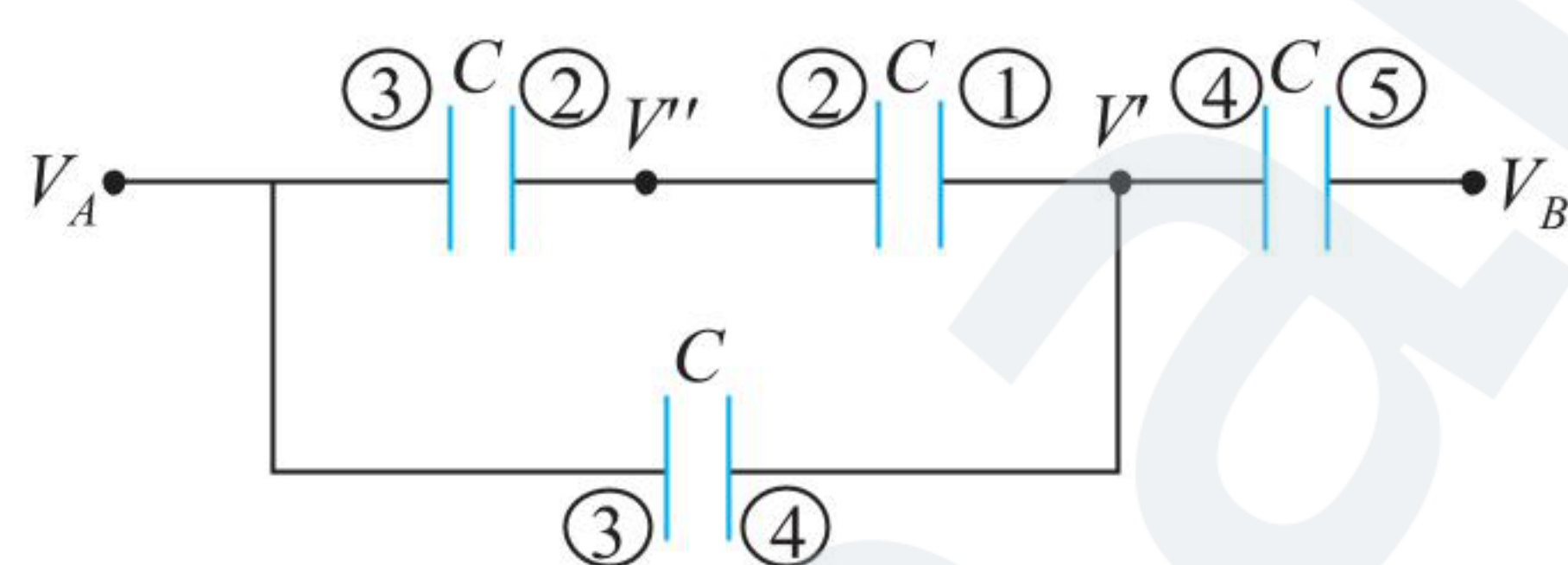
$$x = 2(Q - x) \text{ or } x = \frac{2Q}{3}$$

Adding (v) and (vi), we get

$$(V - 0) = \frac{1}{C}(Q + x) = \frac{5Q}{3C}$$

$$\text{or } \frac{Q}{(V_A - V_B)} = \frac{Q}{(V - 0)} = \frac{3}{5}C = \frac{3\epsilon_0 A}{5d}$$

Method 2: Let the potentials of A and B be V_A and V_B , respectively. Plates (1) and (2) are connected together and have a common potential, say V' . Plate (2) is isolated and has a potential, say V'' . Now we draw an equivalent circuit diagram by connecting the different plates across the assumed potential difference.



As the equivalent circuit is a simple circuit of a combination of capacitors, the equivalent capacitance can be easily calculated as follows:

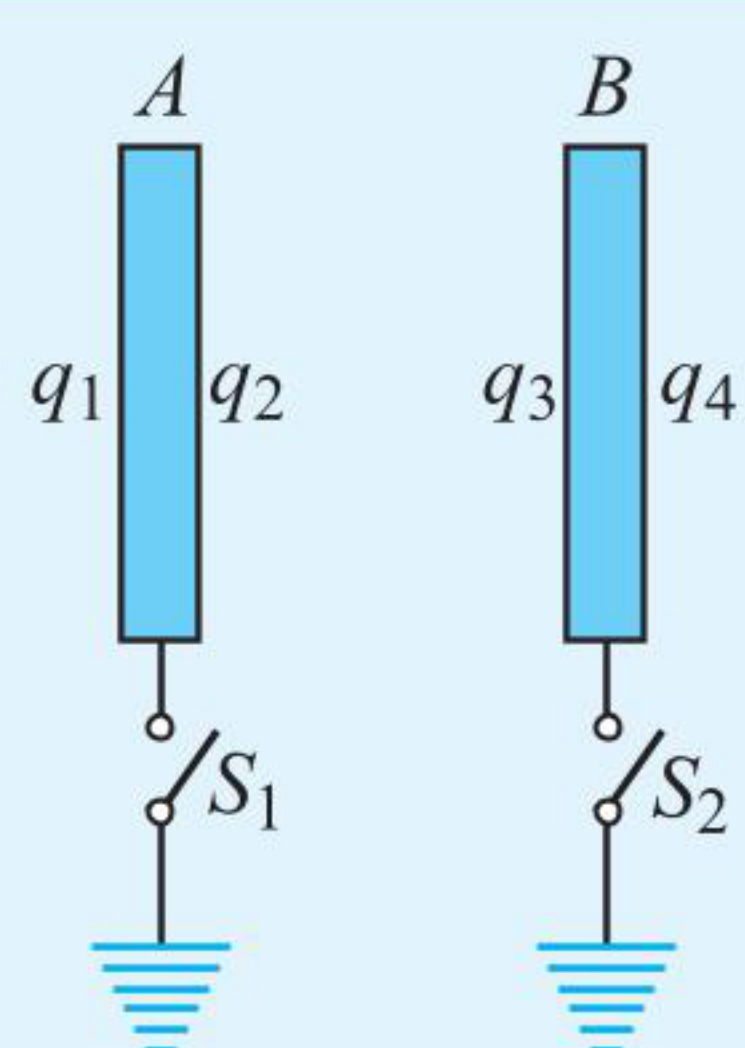
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C} \quad \text{or} \quad C_{eq} = \frac{3C}{5} = \frac{3A\epsilon_0}{5d}$$

CONCEPT APPLICATION EXERCISE 4.5

1. In figure, plate A has $100 \mu\text{C}$ charge, while plate B has $60 \mu\text{C}$ charge.

(a) When both switches are open, then

$$\begin{aligned} q_1 &= \underline{\hspace{2cm}} \\ q_2 &= \underline{\hspace{2cm}} \\ q_3 &= \underline{\hspace{2cm}} \\ q_4 &= \underline{\hspace{2cm}} \end{aligned}$$



(b) When only switch S_1 is closed, then

$$q_1 = \underline{\hspace{2cm}} \quad q_2 = \underline{\hspace{2cm}}$$

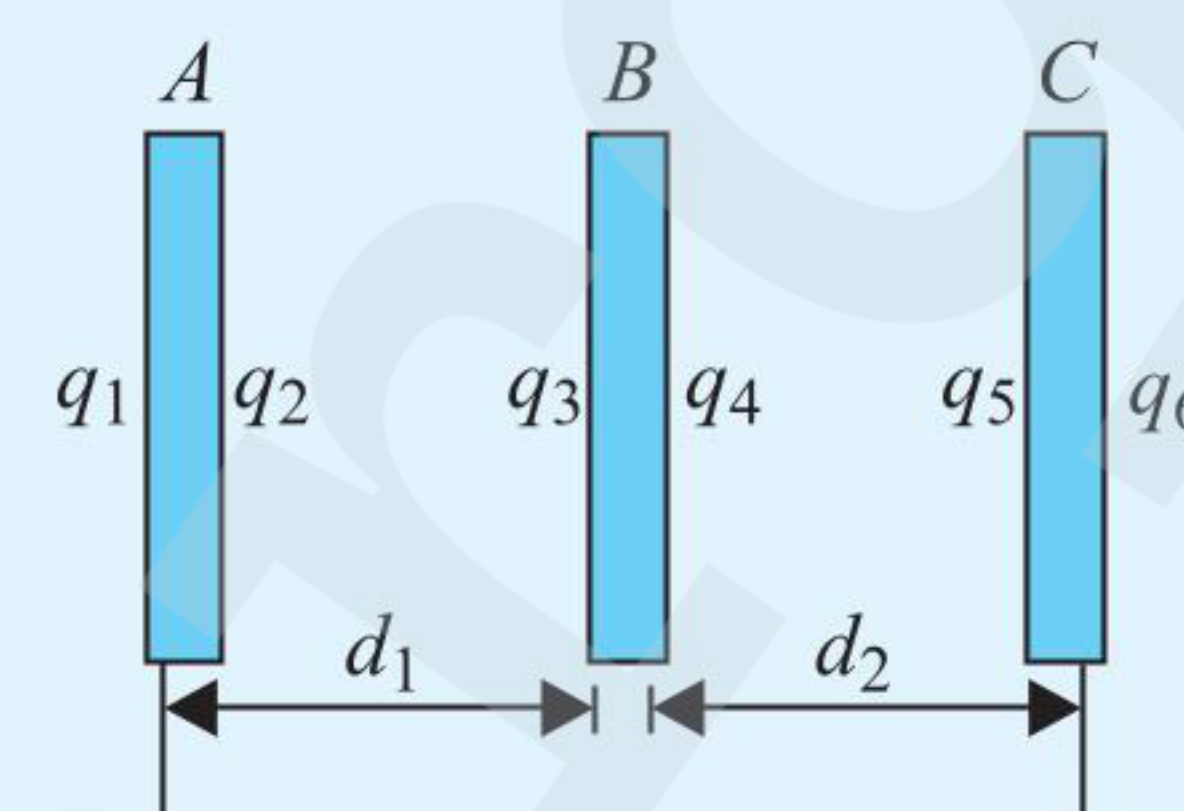
$$q_3 = \underline{\hspace{2cm}} \quad q_4 = \underline{\hspace{2cm}}$$

(c) When switch S_2 is also closed, then

$$q_1 = \underline{\hspace{2cm}} \quad q_2 = \underline{\hspace{2cm}}$$

$$q_3 = \underline{\hspace{2cm}} \quad q_4 = \underline{\hspace{2cm}}$$

2. In the arrangement shown in figure, plate B is given a charge equal to $60 \mu\text{C}$. The ratio d_1/d_2 is 2. Then

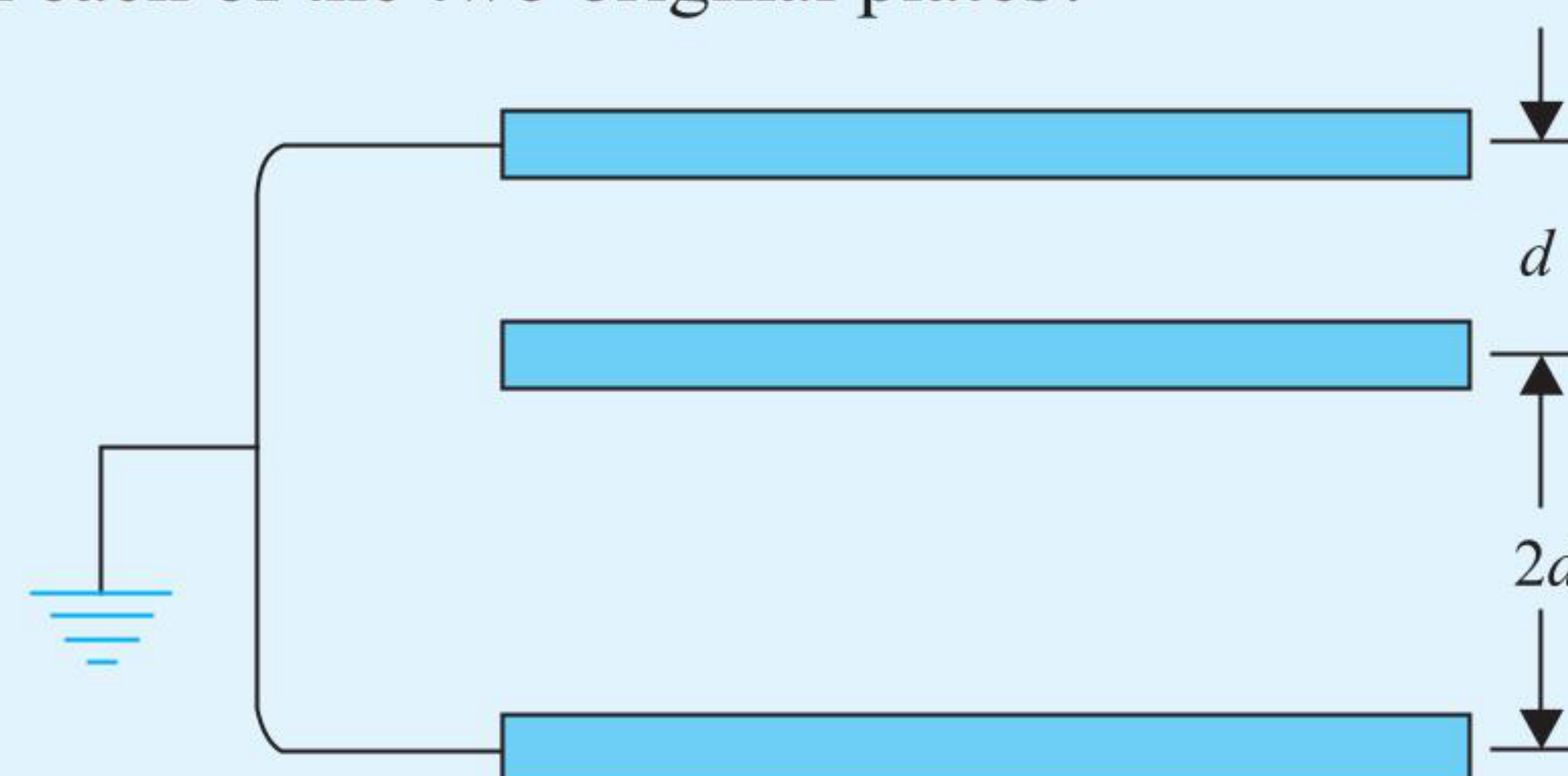


$$q_1 = \underline{\hspace{2cm}} \quad q_2 = \underline{\hspace{2cm}}$$

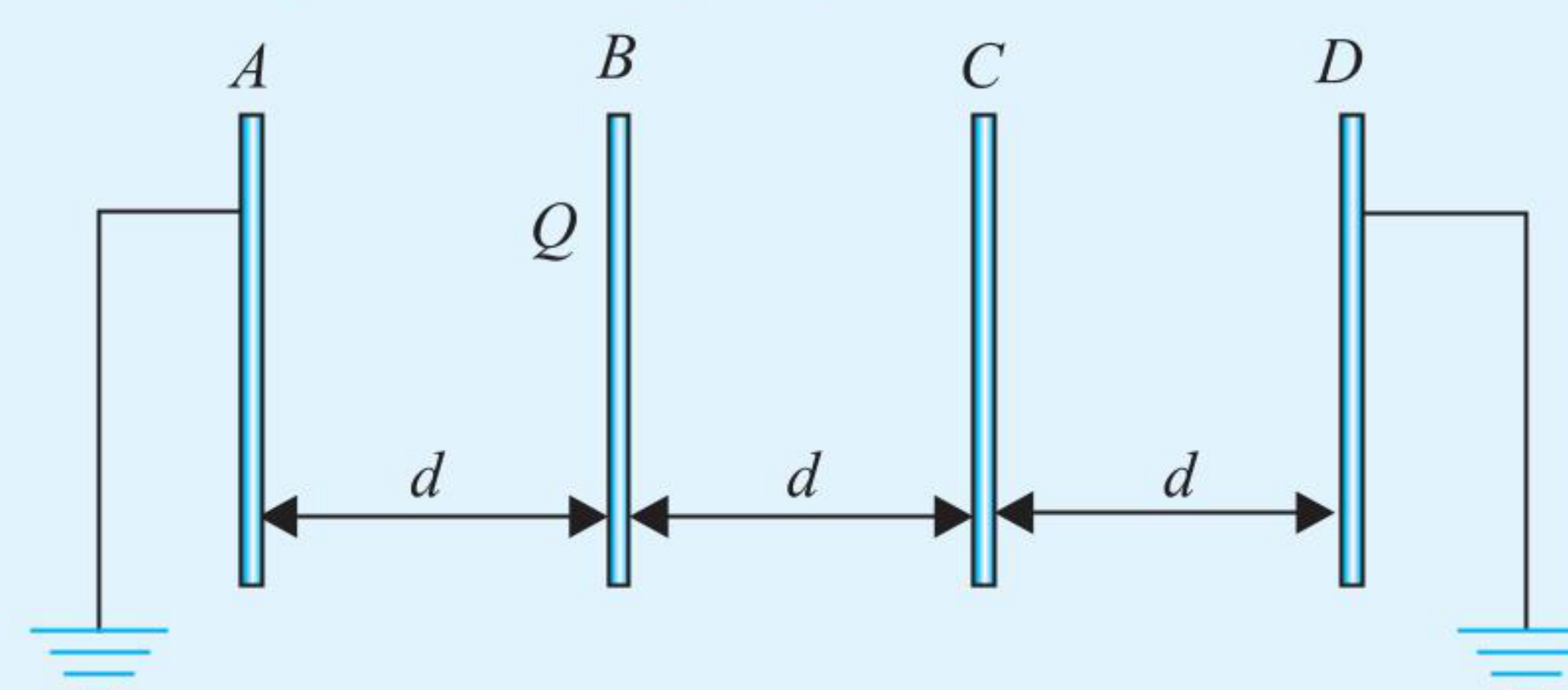
$$q_3 = \underline{\hspace{2cm}} \quad q_4 = \underline{\hspace{2cm}}$$

$$q_5 = \underline{\hspace{2cm}} \quad q_6 = \underline{\hspace{2cm}}$$

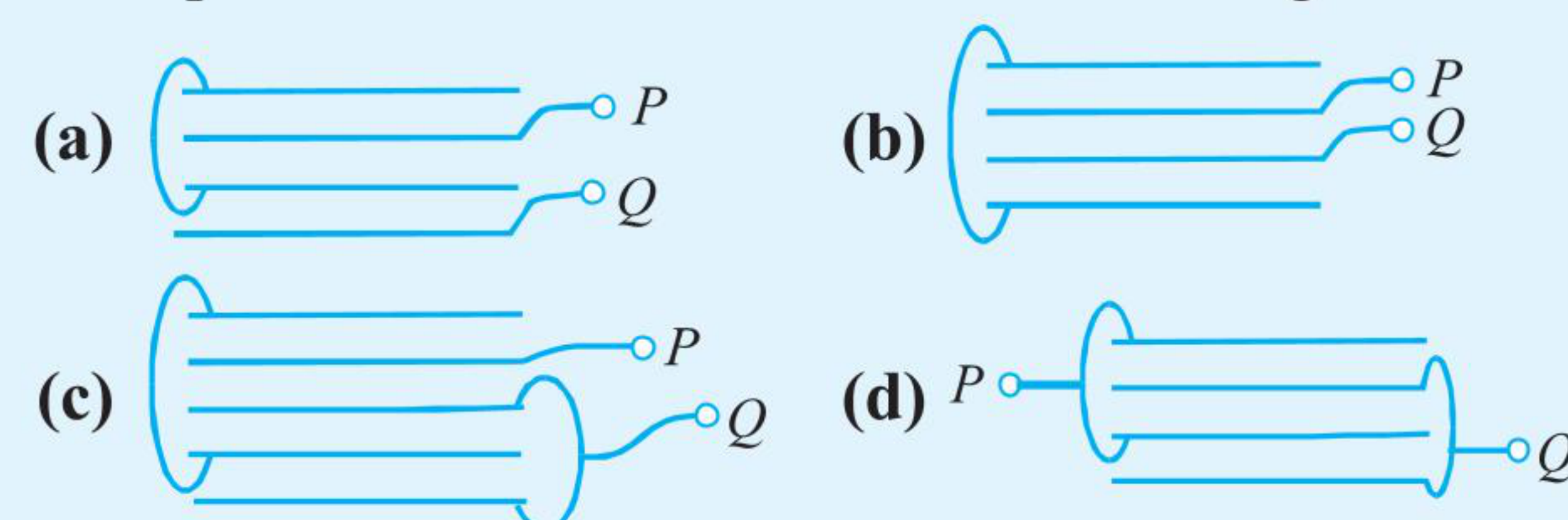
3. Two large parallel metal plates, each having area A , are oriented horizontally and separated by a distance $3d$. A grounded conducting wire joins them, and initially each plate carries no charge. Now a third identical plate carrying charge Q is inserted between the two plates, parallel to them and located a distance d from the upper plate, as shown in figure. What induced charge appears on each of the two original plates?



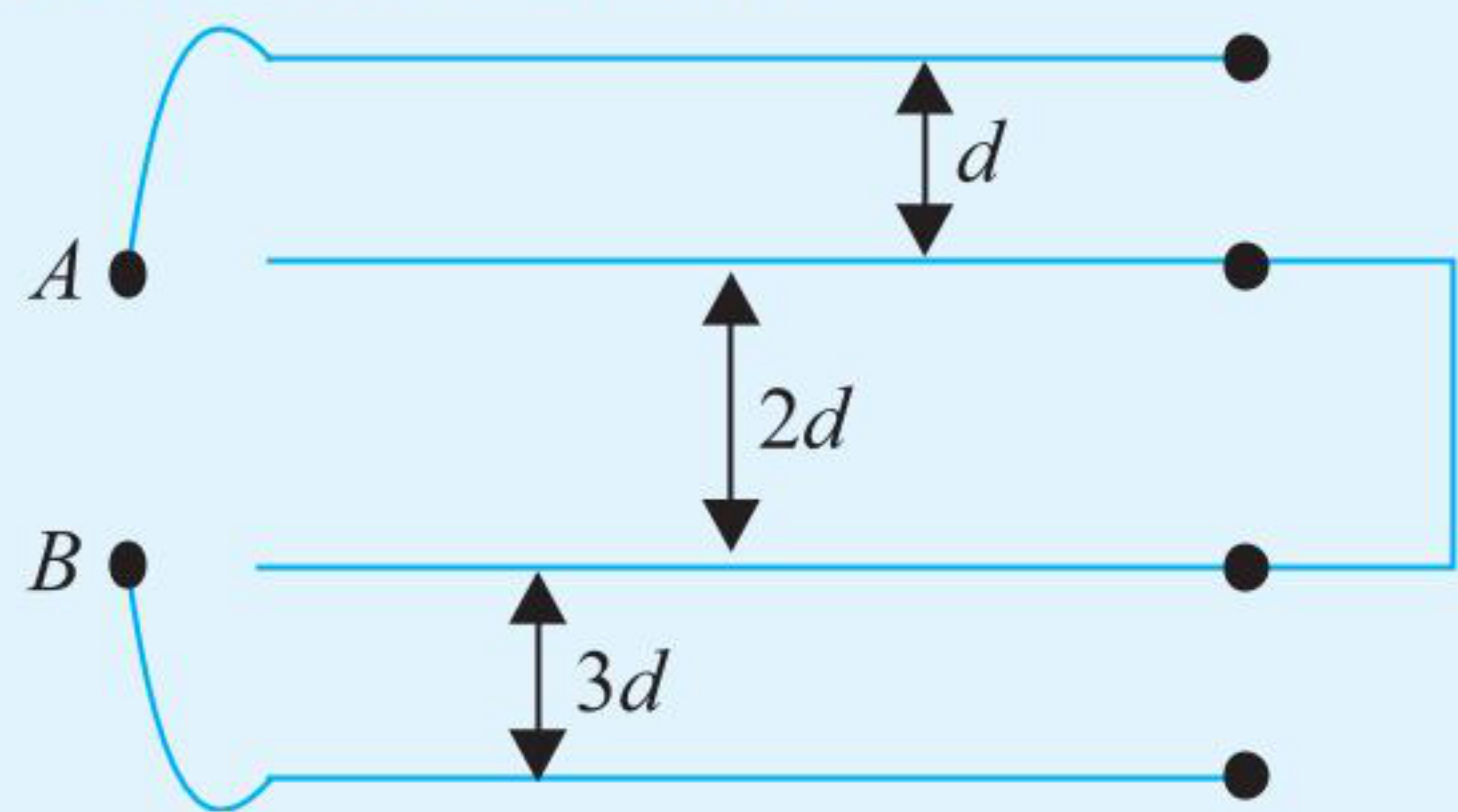
4. Four parallel large plates separated by equal distance d are arranged as shown in figure. The area of the plates is S . Find the potential difference between plates B and C if plate B is given a charge Q .



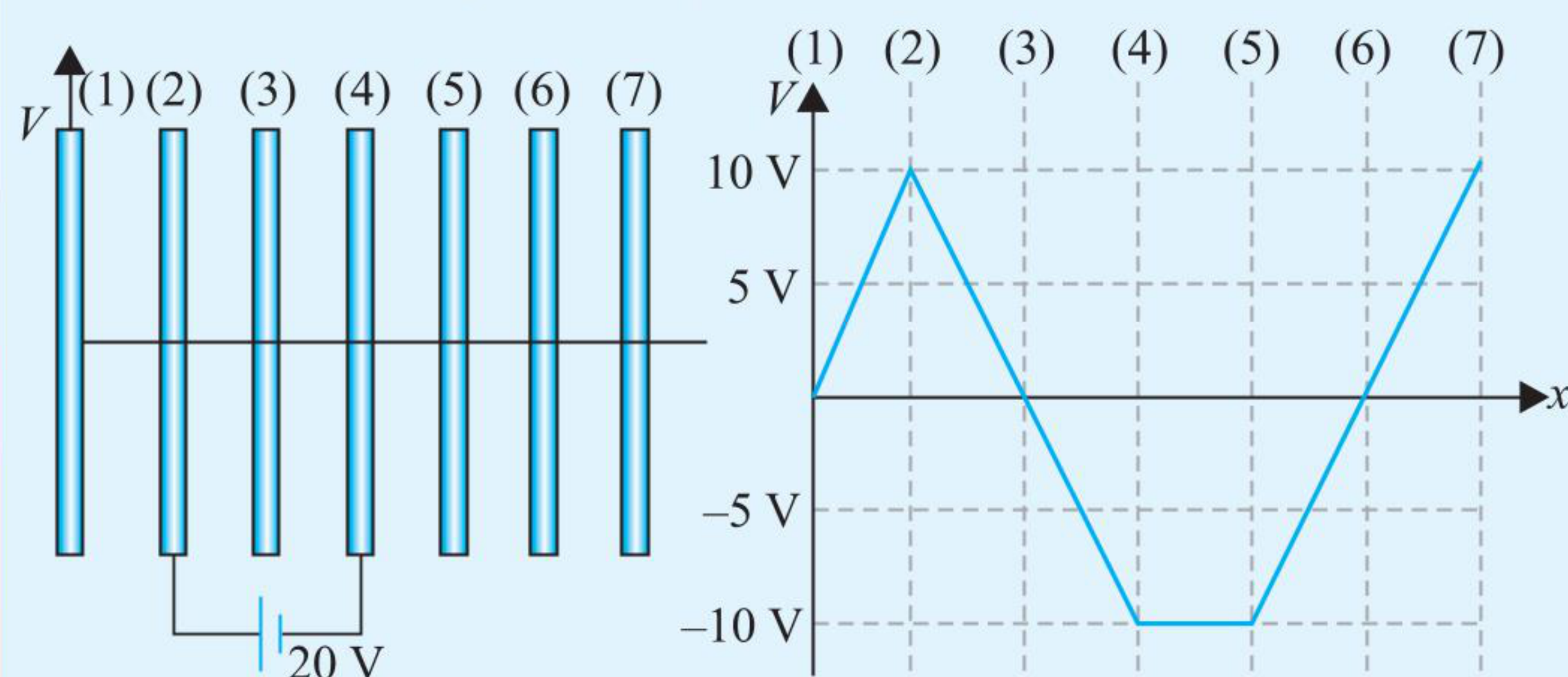
5. Identical metal plates are located in air at equal distance d from one another. The area of each plate is equal to A . Evaluate the capacitance of the system between P and Q if the plates are interconnected as shown in figure.



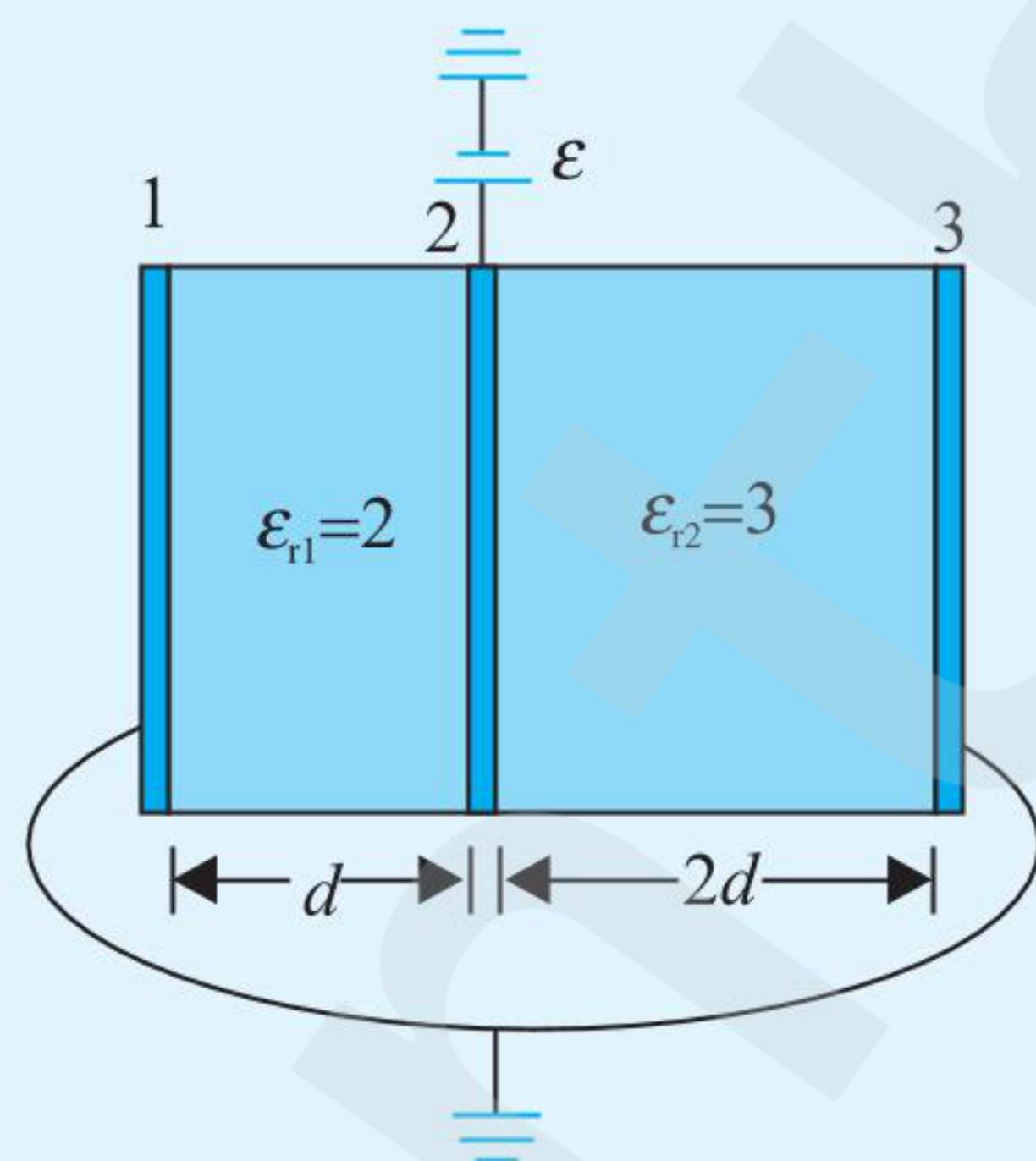
6. If the area of each plate is A (figure) and the successive separations are d , $2d$, and $3d$, then find the equivalent capacitance across A and B .



7. Figure shows an arrangement of identical metal plates placed parallel to each other. The diagram also shows the variation of potential between the plates. Using the details given in the diagram, find the equivalent capacitance connected across the battery (separation between the consecutive plates is equal to l and the cross-sectional area of each plate is A).

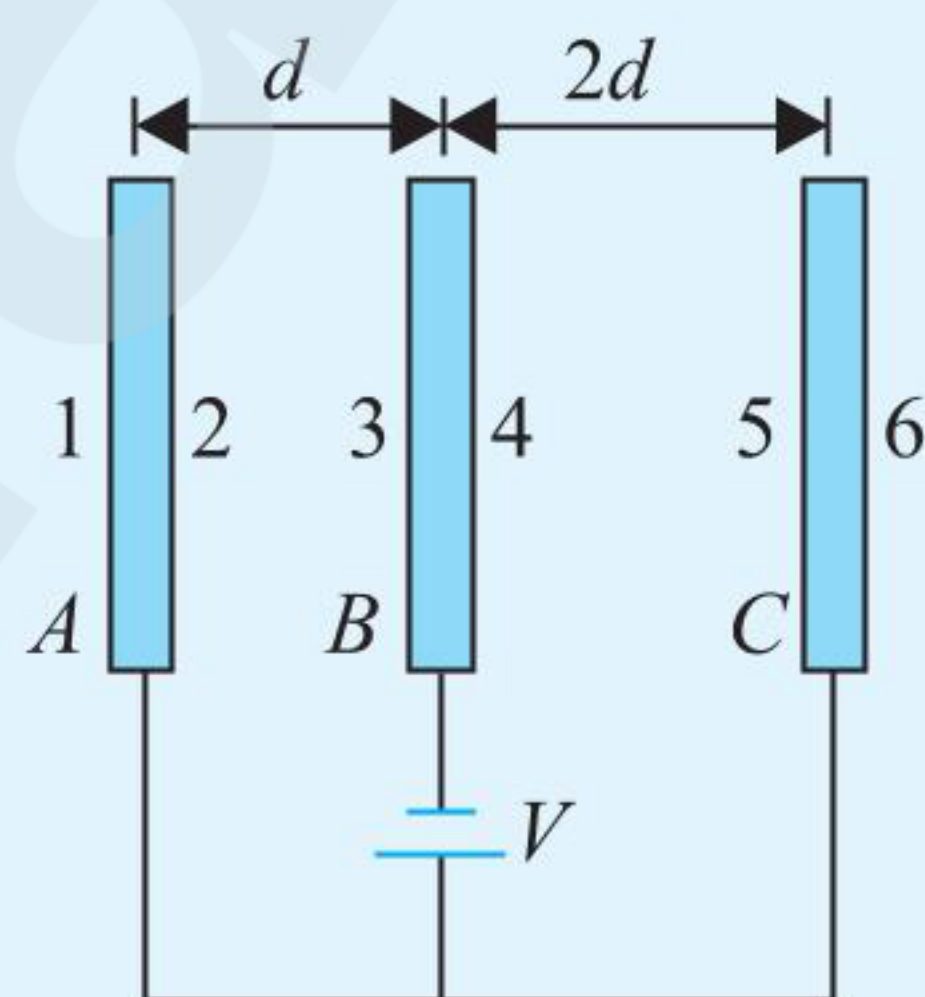


8. Two dielectric slabs of relative permittivities $\epsilon_{r1} = 2$ and $\epsilon_{r2} = 3$, thickness d and $2d$ are filled in between the grounded parallel plates each of area A . If a battery of emf ϵ is connected to the middle conducting plate, find the



- (a) capacitance of the system,
(b) charge flowing through the battery,
(c) induced charges on the plates,
(d) energy stored in the system.

9. In the given figure the Area of each plate is A . The conducting plates are connected to a battery of emf V volts. Find charges q_1 to q_6 .



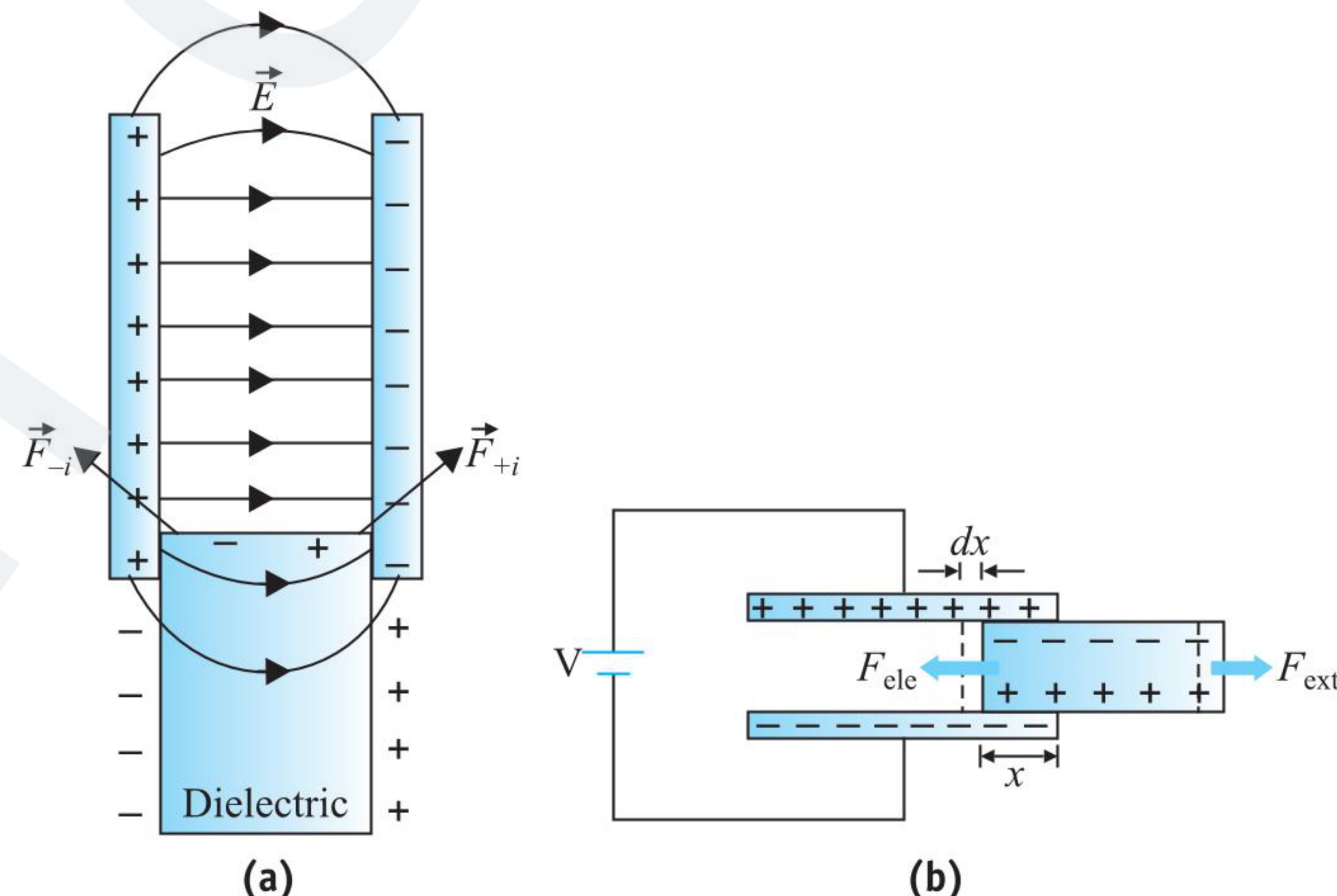
ANSWERS

1. (a) $80 \mu\text{C}$, $20 \mu\text{C}$, $-20 \mu\text{C}$, $80 \mu\text{C}$
(b) 0 , $-60 \mu\text{C}$, $60 \mu\text{C}$, 0 (c) 0 , 0 , 0 , 0
2. $q_1 = q_6 = 30 \mu\text{C}$, $q_3 = 20 \mu\text{C}$, $q_4 = 40 \mu\text{C}$ $q_2 = -20 \mu\text{C}$, $q_5 = -40 \mu\text{C}$
3. Induced charge on upper plate $= -2Q/3$
Induced charge on lower plate $= -Q/3$
4. $\frac{Qd}{3\epsilon_0 S}$ 5. (a) $\frac{2\epsilon_0 A}{3d}$ (b) $\frac{3\epsilon_0 A}{2d}$ (c) $\frac{5}{3} \frac{\epsilon_0 A}{d}$ (d) $\frac{3\epsilon_0 A}{d}$

6. $\frac{\epsilon_0 A}{4d}$ 7. $\frac{6\epsilon_0 A}{5l}$
8. (a) $\frac{7\epsilon_0 A}{2d}$ (b) $\frac{7\epsilon_0 A\epsilon}{2d}$ (c) $-\frac{2\epsilon_0 A\epsilon}{d}$, $\frac{7\epsilon_0 A\epsilon}{2d}$, $-\frac{3\epsilon_0 A\epsilon}{2d}$
(d) $\frac{7\epsilon_0 A\epsilon^2}{4d}$
9. $q_1 = q_6 = 0$, $q_2 = +\frac{\epsilon_0 AV}{d}$, $q_3 = -\frac{\epsilon_0 AV}{d}$, $q_5 = +\frac{\epsilon_0 AV}{2d}$
 $q_4 = -\frac{\epsilon_0 AV}{2d}$

FORCE ON DIELECTRIC SLAB

When a dielectric slab is placed near a charged capacitor, the fringing field at the edges of the capacitor exerts force on the negative and positive induced surface charges of the dielectric due and the slab experiences the net force towards inside the capacitor [Fig. (a)]. Let us consider a situation a capacitor of capacity C is connected to a battery of emf V . Let a dielectric slab is inserted up to distance x as shown in [Fig. (b)].



If you pull the dielectric slab by a small distance dx , the capacitance of the system changes by dC . Hence, an excess charge $dq = VdC$ flows from (or to) the supply if the system is kept connected to the supply. In this process the battery (or supply or source) does a work dW_b in sending a charge dq against the voltage V (or emf ϵ of the battery). You can call it energy supplied by the battery (sources). According to the conservation of energy, part of the energy supplied by the source is spent in changing (increasing or decreasing) the electrostatic potential energy of the system by dU , say and rest of the supplied source energy is utilised in doing mechanical work dW_{mech} , say on the system; the dielectric on which you are in tending to find the work and energy loss dH in the form of heat, light and sound, etc. Then, we can write

$$dW_b = dW_{\text{mech.}} + dU + dH \quad \dots(i)$$

where $dW_{\text{mech.}} = Fdx$; F = force on the dielectric. We can ignore the value of dH in the battery and conductors. Now we can write Eqn. (i) as, $dW_b = F \cdot dx + dU$.

Hence the force F acting on the conductor (or dielectric) given as,

$$F = \frac{dW_b}{dx} - \frac{dU}{dx} \quad \dots(ii)$$

The above expression is valid for all processes such as time varying or constant charge and voltage. We have two practical cases.

- (i) **When the battery is disconnected after charging the capacitor(or charge in capacitor remains constant):** If you plan to find the force by disconnecting the system from the battery, in this case, put $q = \text{constant}$. Then, the work done by the battery should be zero. By putting $dW_b = 0$, in Eq. (ii), we have

$$F = - \left. \frac{dU}{dx} \right|_{q = \text{constant}} \quad \dots(\text{iii})$$

- (ii) **When battery remains connected (Here V remains constant):** In this case, the battery does a work $dW_b (= Vdq)$ in sending a charge dq through a potential difference V . At the same time the change in potential energy of the system can be given as

$$dU = d \left(\frac{1}{2} CV^2 \right) = \frac{V^2}{2} dC = \frac{Vdq}{2} \quad (\because VdC = dq)$$

Then, we have, $dW_b = 2dU$

This tells us that, half of the energy supplied by the battery rises the potential energy of the system and the other half is spent in doing a mechanical work. Then, putting $dW_b = 2dU$ in Eq. (iii), we have

$$F = \frac{2dU}{dx} - \frac{dU}{dx} = \frac{dU}{dx} \text{ we have}$$

$$\text{Now we can write } F = + \left. \frac{dU}{dx} \right|_{V = \text{constant}} \quad \dots(\text{iv})$$

ILLUSTRATION 4.44

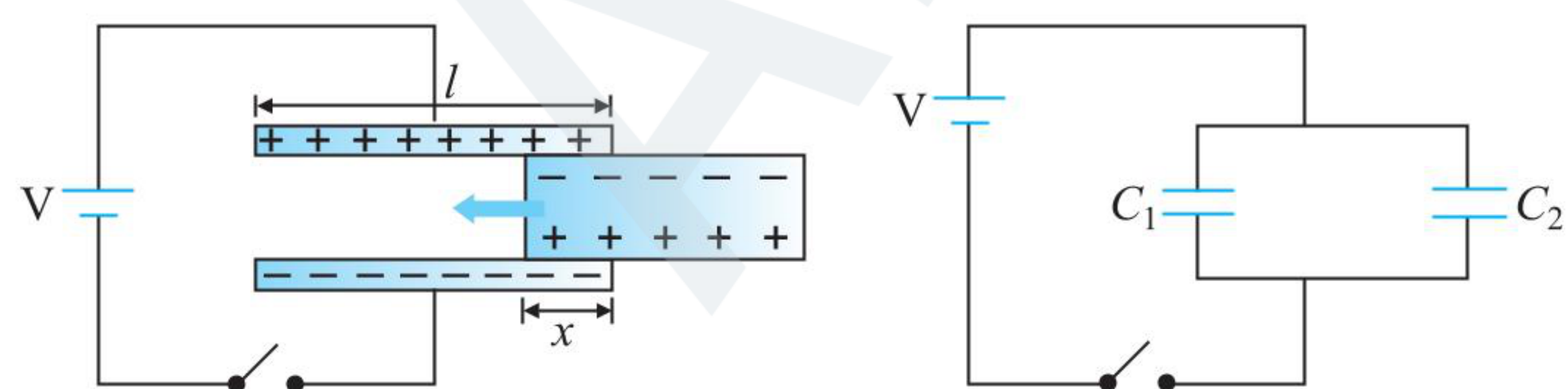
Let us consider a parallel plate of capacitor with plates of width b and length l . The distance between the plates is d . Find the force acting on the smooth dielectric slab when

- (i) When the battery is disconnected after charging the capacitor
(ii) When battery remains connected

Sol.

- (i) Force acting on the dielectric slab can be given as,

$$F = - \frac{dU}{dx}, \text{ where } U = \frac{Q^2}{2C}$$



$$\text{or } F = + \frac{Q^2}{2C^2} \frac{dC}{dx} \quad \dots(\text{i})$$

$$C_1 = \frac{\epsilon_0 b(l-x)}{d}; \quad C_2 = \frac{K\epsilon_0 bx}{d}$$

$$\text{Hence, } C = C_1 + C_2 = \frac{\epsilon_0 b}{d} (l + x(K-1)) \quad \dots(\text{ii})$$

$$\frac{dC}{dx} = \frac{\epsilon_0 b}{d} (K-1) \quad \dots(\text{iii})$$

From (i), (ii) and (iii), we get

$$F = + \frac{Q^2}{2 \left[\frac{\epsilon_0 b}{d} (l + x(K-1)) \right]^2} \frac{\epsilon_0 b}{d} (K-1)$$

$$\Rightarrow F = \frac{Q^2 d (K-1)}{2 \epsilon_0 b [l + x(K-1)]^2}$$

If the capacitor is connected to a constant voltage, put

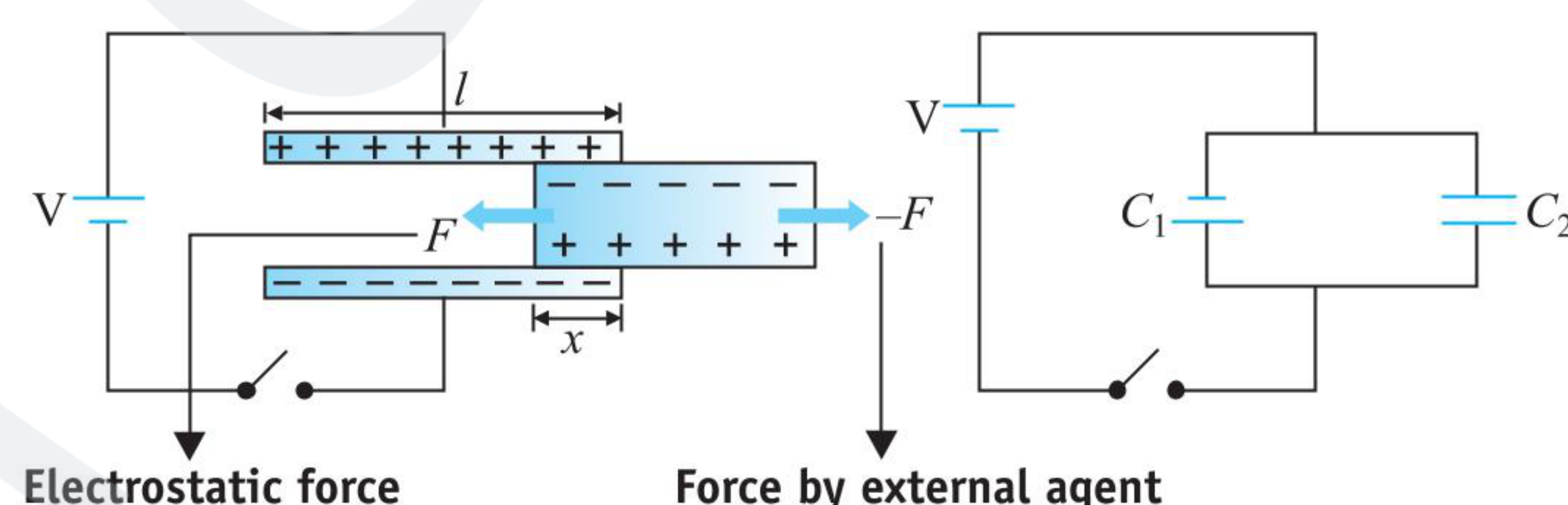
$$\frac{Q}{C} = V \text{ to obtain}$$

$$F = + (\epsilon_r - 1) \frac{\epsilon_0 l V^2}{2d}$$

The sign signifies that the force points to right.

The magnitude of the force on the dielectric is constant.

- (ii) When battery remains connected



$$F = + \frac{dU}{dx}, \text{ where } U = \frac{1}{2} CV^2$$

$$\text{or } F = V^2 \frac{dC}{dx} \quad \dots(\text{i})$$

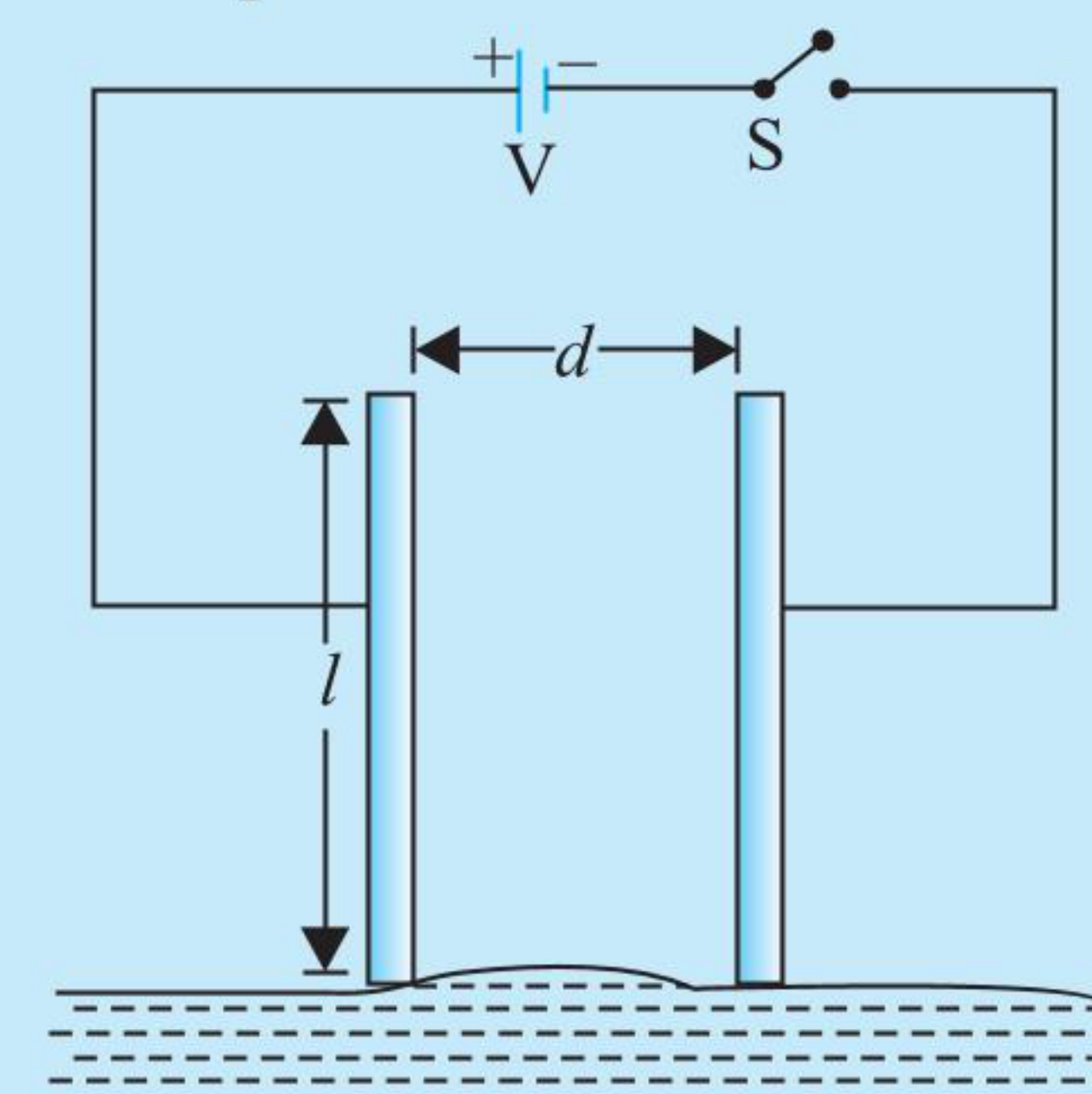
$$\text{Here, } C = C_1 + C_2 = \frac{\epsilon_0 b}{d} (l + x(K-1)) \quad \dots(\text{ii})$$

$$\text{and } \frac{dC}{dx} = \frac{\epsilon_0 b}{d} (K-1) \quad \dots(\text{iii})$$

$$\text{Hence } F = (K-1) \frac{\epsilon_0 b V^2}{2d}$$

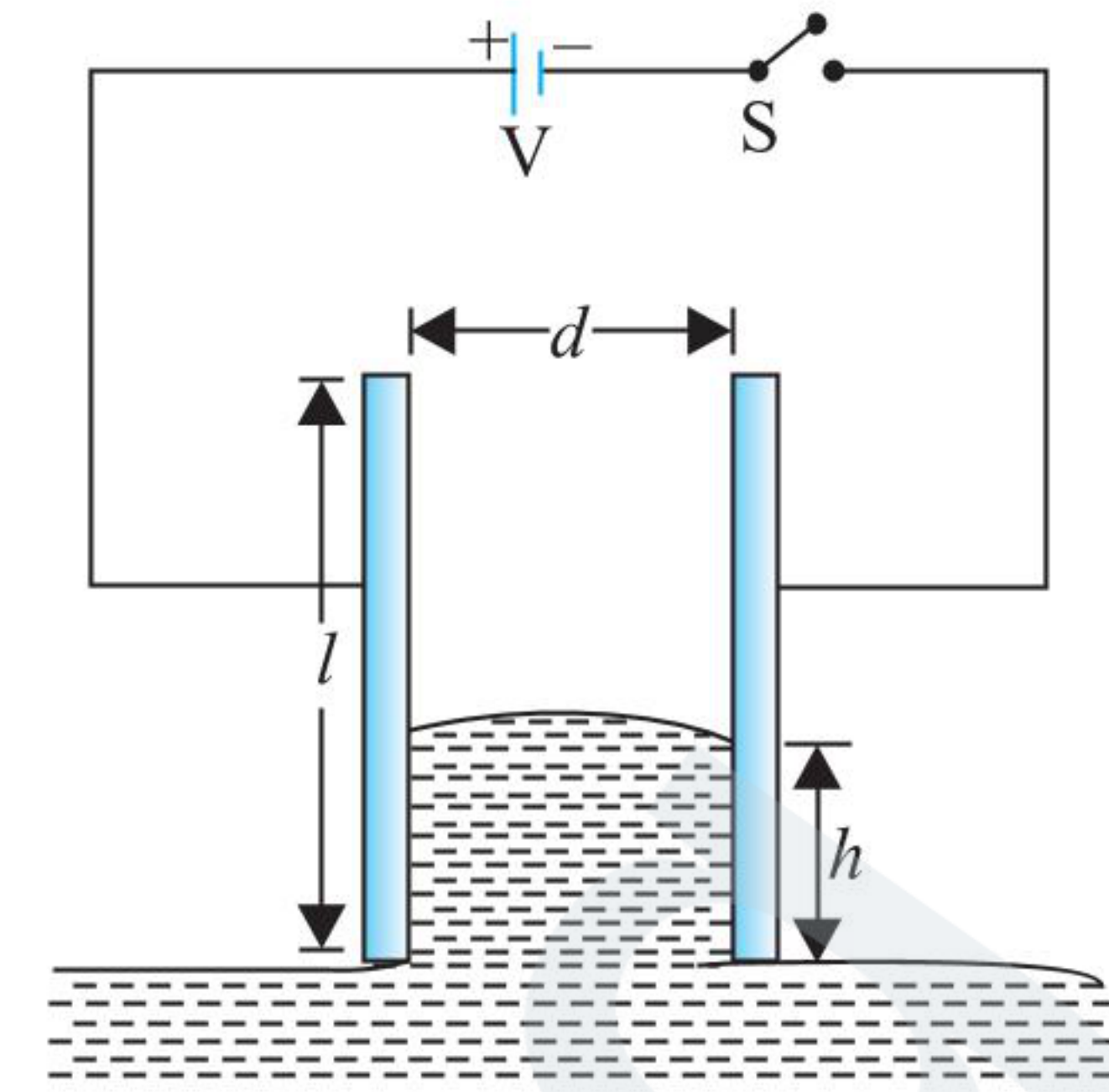
ILLUSTRATION 4.45

Figure shows a parallel plate capacitor of plate area $A = lb$ with separation d connected to a battery via a switch S . Capacitor plates are kept vertical and touched on the surface of a liquid of density ρ as shown. If S is closed find the height between plates to which the liquid level will rise.



Sol. Here the liquid acts as dielectric. At the time when the switch is closed, due to the upward force on dielectric(liquid) because of polarization it starts raising up between the plates and upto a level where the upward force on liquid balances its weight. If it is raised up to a height h as shown in figure and battery is connected to the capacitor we use result for the force on dielectric which we proved in theory. At equilibrium we use

$$(K-1) \frac{\epsilon_0 b V^2}{2d} = h b d \rho g \Rightarrow h = \frac{\epsilon_0 V^2 (K-1)}{2d^2 \rho g}$$

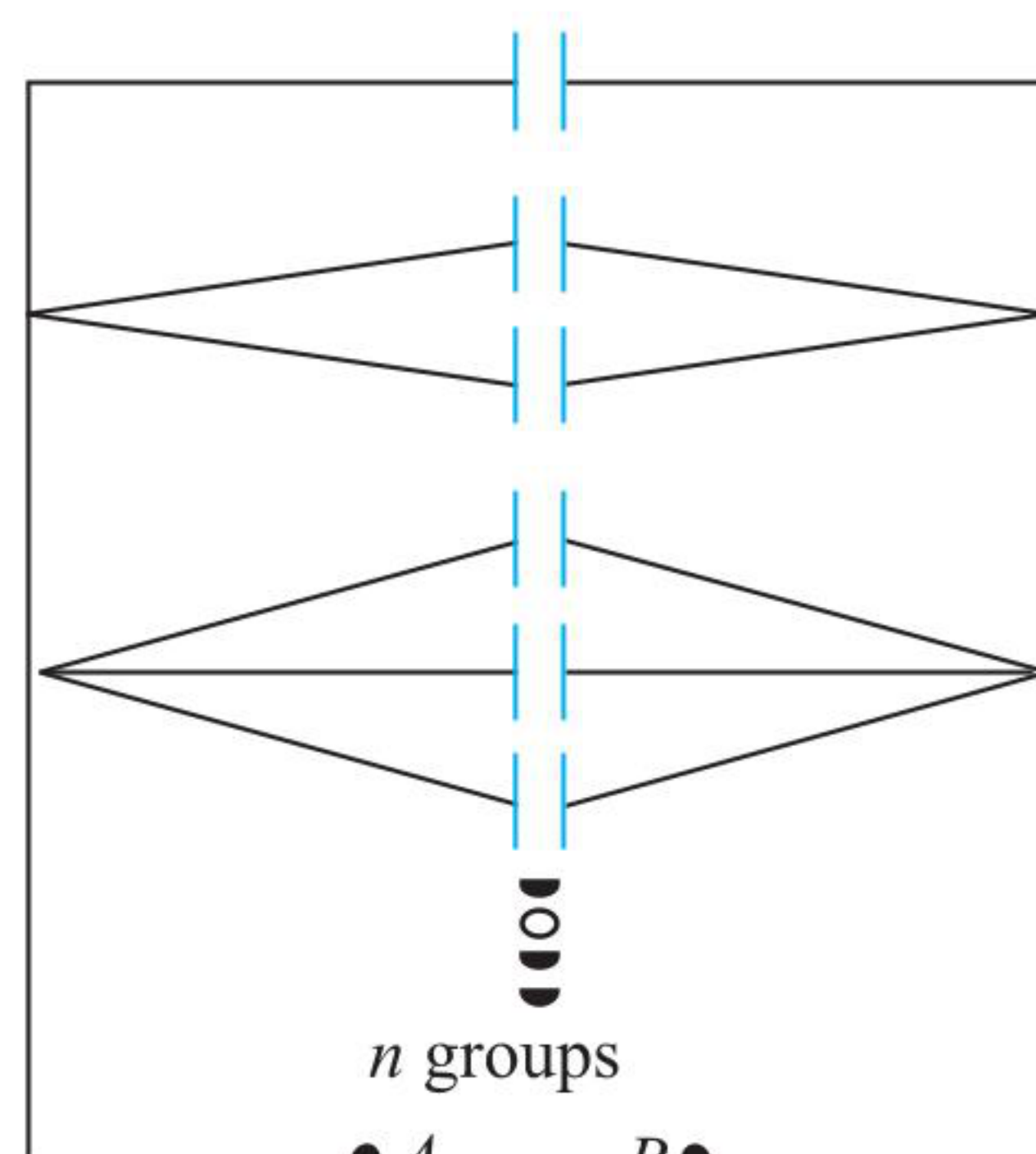


Exercises

Single Correct Answer Type

1. A number of capacitors, each of equal capacitance C , are arranged as shown in figure. The equivalent capacitance between A and B is

- (1) n^2C
 (2) $(2n + 1)C$
 (3) $\frac{(n-1)n}{2}C$
 (4) $\frac{(n+1)n}{2}C$

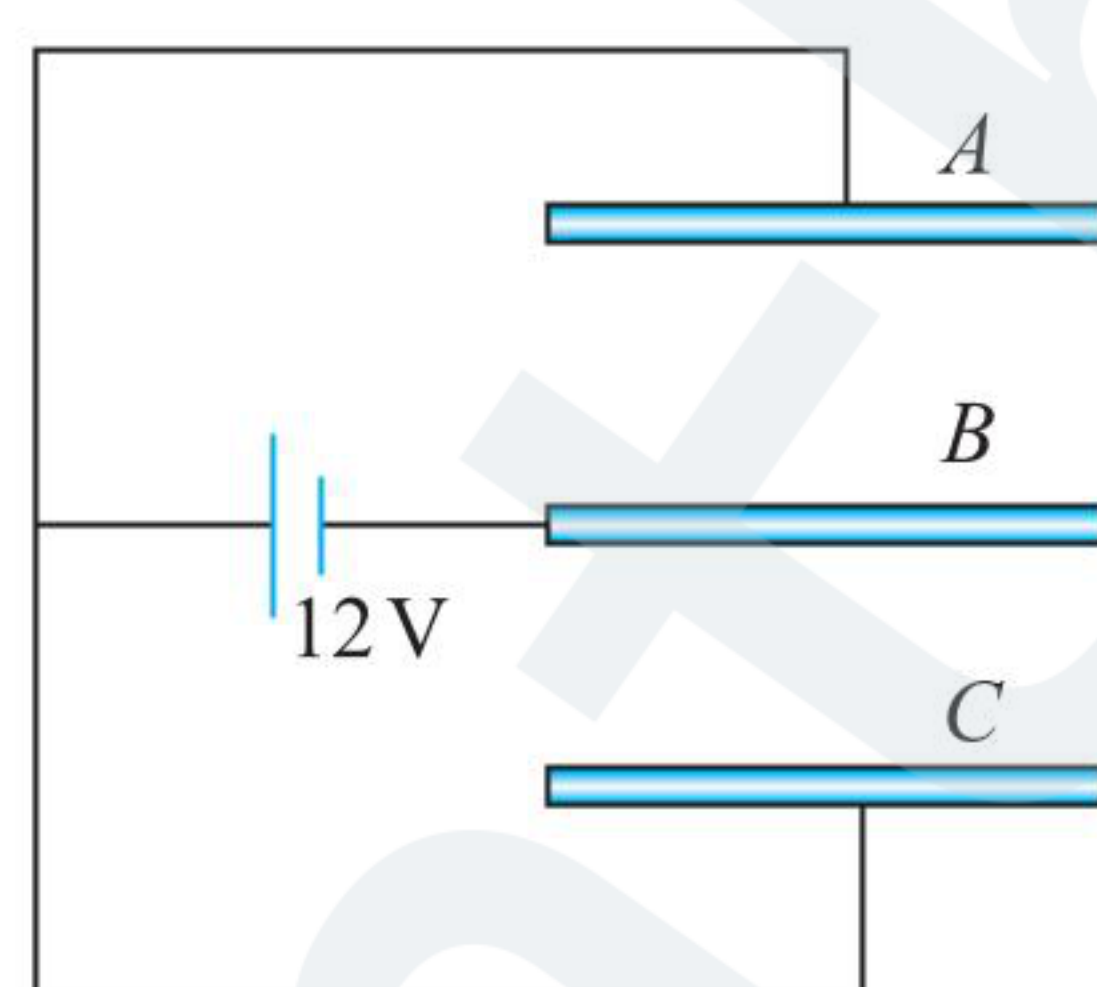


2. The plates of a parallel plate capacitor are charged up to 100 V. Now, after removing the battery, a 2 mm thick plate is inserted between the plates. Then, to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. The dielectric constant of the plate is

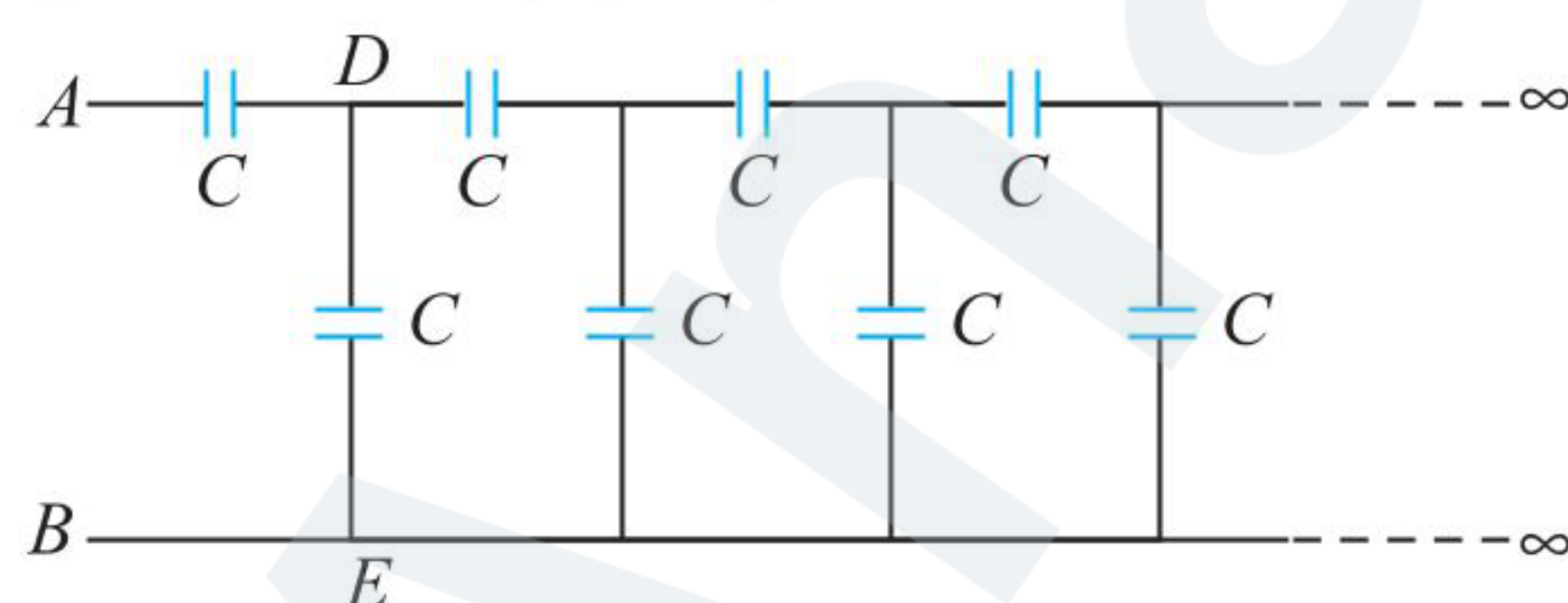
- (1) 5 (2) 1.25
 (3) 4 (4) 2.5

3. Three plates A , B , and C each of area 50 cm^2 have separation 3 mm between A and B and 3 mm between B and C . The energy stored when the plates are fully charged is

- (1) $6 \times 10^{-9} \text{ J}$
 (2) $3.12 \times 10^{-9} \text{ J}$
 (3) $2.12 \times 10^{-9} \text{ J}$
 (4) none of these

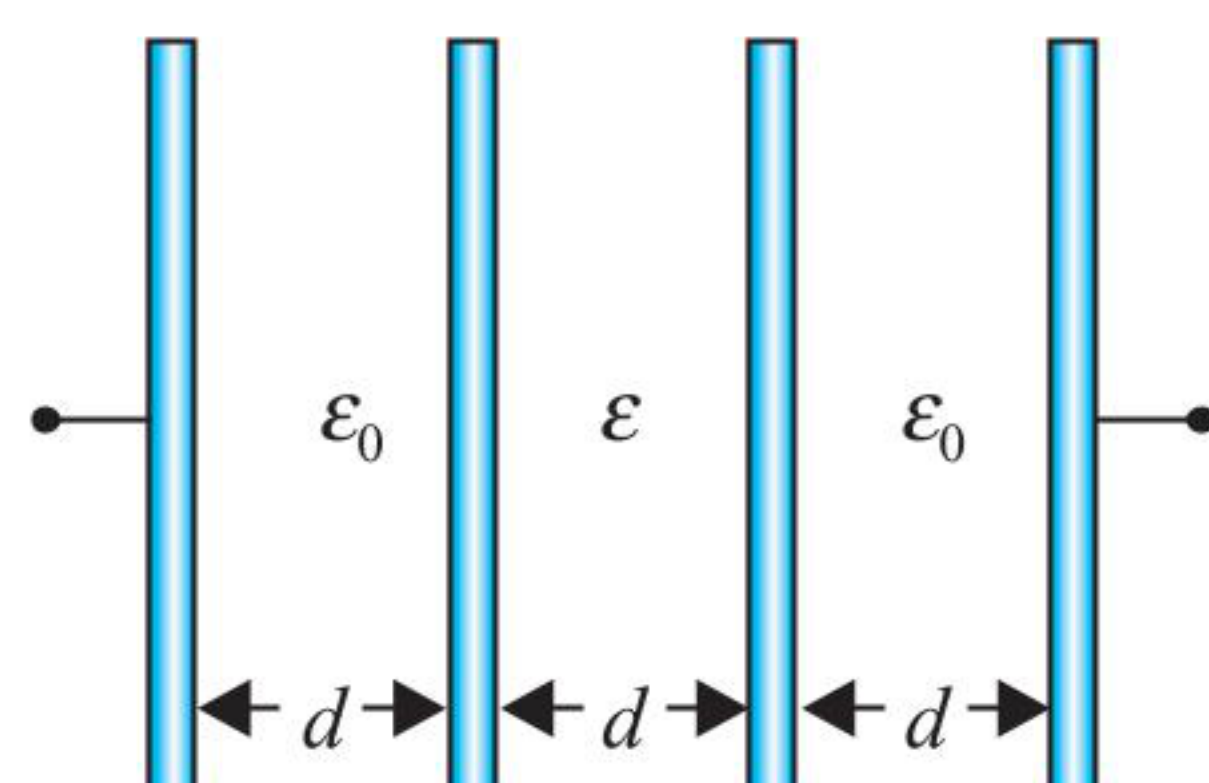


4. The capacitance of an infinite circuit formed by the repetition of the same link consisting of two identical capacitors, each with capacitance C (figure), is



- (1) zero (2) $\frac{\sqrt{5}-1}{2}C$
 (3) $\frac{\sqrt{5}+1}{2}C$ (4) infinite

5. For the configuration of media of permittivities ϵ_0 , ϵ , and ϵ_0 between parallel plates each of area A , as shown in figure, the equivalent capacitance is



- (1) $\epsilon_0 A/d$ (2) $\epsilon \epsilon_0 A/d$
 (3) $\frac{\epsilon \epsilon_0 A}{d(\epsilon + \epsilon_0)}$ (4) $\frac{\epsilon \epsilon_0 A}{(2\epsilon + \epsilon_0)d}$

6. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed, and the inner sphere is given a charge of $2.5 \mu\text{C}$. The space between the concentric spheres is filled with a liquid of dielectric constant 32. Determine the potential of the inner sphere.

- (1) 400 V (2) 450 V
 (3) 500 V (4) 300 V

7. A parallel plate capacitor has plates of area A and separation d and is charged to a potential difference V . The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. What is the work required to separate the plates?

- (1) $2\epsilon_0 AV^2/d$ (2) $\epsilon_0 AV^2/d$
 (3) $3\epsilon_0 AV^2/2d$ (4) $\epsilon_0 AV^2/2d$

8. A parallel plate capacitor is charged and then disconnected from the source of potential difference. If the plates of the condenser are then moved farther apart by the use of insulated handle, which of the following is true?

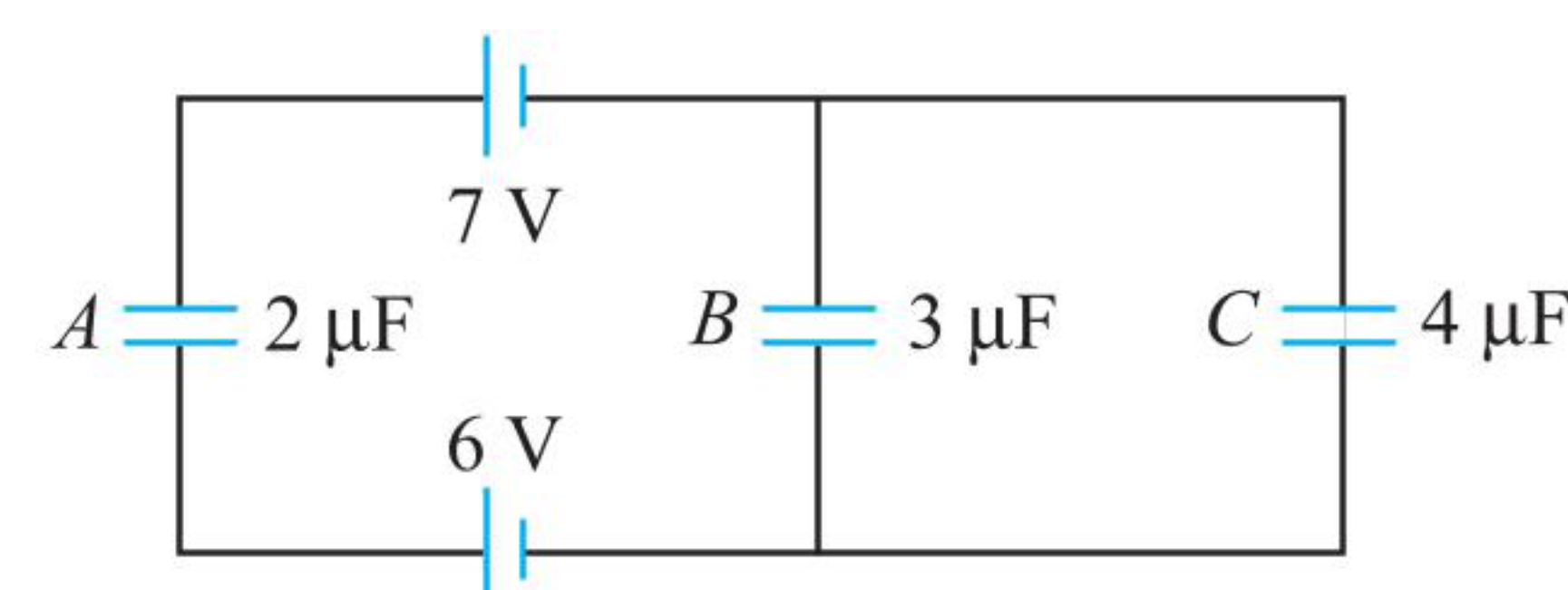
- (1) The charge on the capacitor increases.
 (2) The charge on the capacitor decreases.
 (3) The capacitance of the capacitor increases.
 (4) The potential difference across the plates increases.

9. For section AB of a circuit shown in figure, $C_1 = 1 \mu\text{F}$, $C_2 = 2 \mu\text{F}$, $E = 10 \text{ V}$, and the potential difference $V_A - V_B = -10 \text{ V}$. Charge on capacitor C_1 is



- (1) $0 \mu\text{C}$ (2) $20/3 \mu\text{C}$
 (3) $-40/3 \mu\text{C}$ (4) none of these

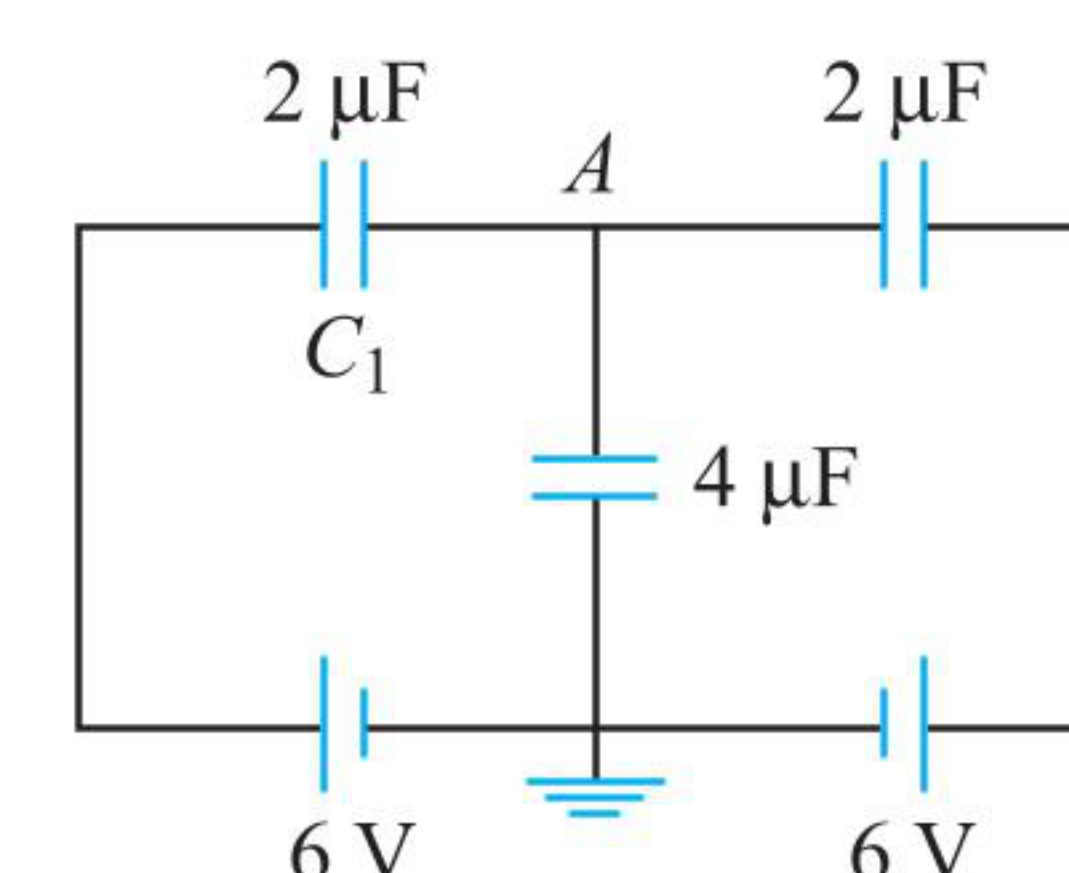
10. Three capacitors A , B , and C are connected in a circuit as shown in figure. What is the charge in μC on the capacitor B ?



- (1) $1/3$ (2) $2/3$
 (3) 1 (4) $4/3$

11. Three capacitors are connected as shown in figure. Then, the charge on capacitor C_1 is

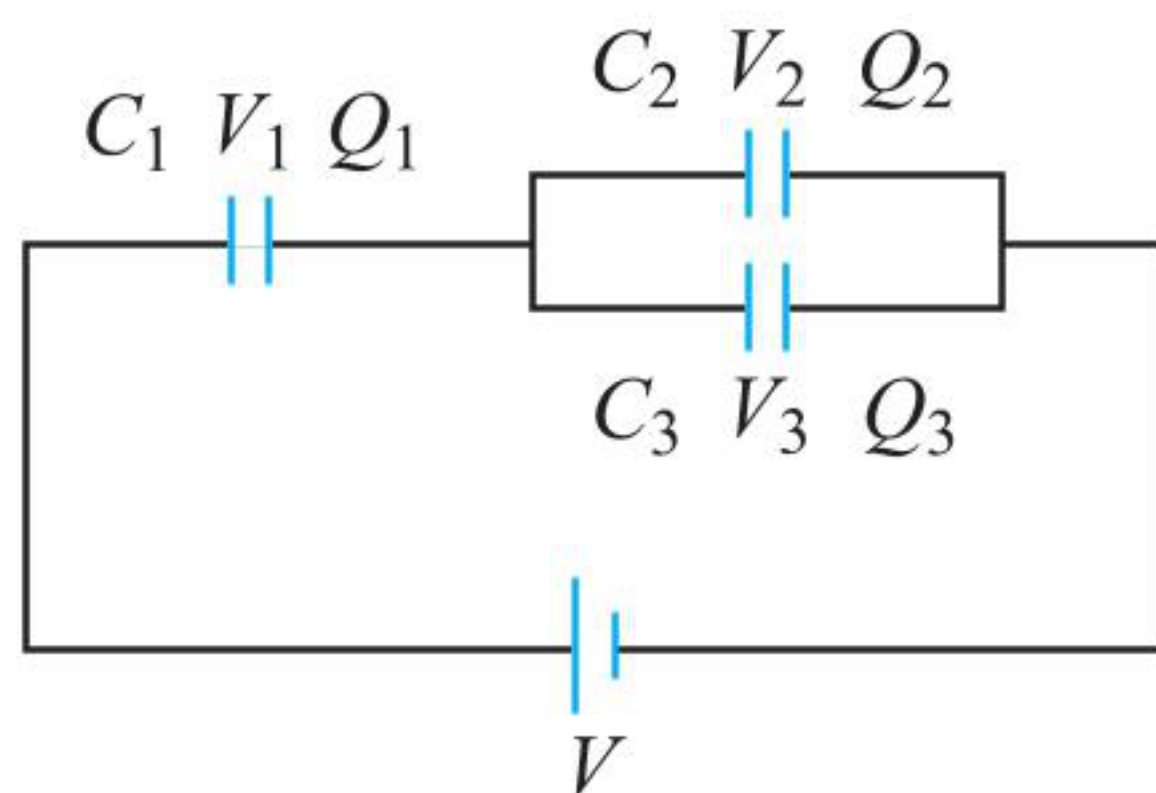
- (1) $6 \mu\text{C}$
 (2) $12 \mu\text{C}$
 (3) $18 \mu\text{C}$
 (4) $24 \mu\text{C}$



12. A capacitor of capacitance $C_1 = 1 \mu\text{F}$ charged up to a voltage $V = 110 \text{ V}$ is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing capacitances $C_2 = 2 \mu\text{F}$ and $C_3 = 3 \mu\text{F}$. Then, the amount of charge that will flow through the connecting wires is

(1) $40 \mu\text{C}$ (2) $50 \mu\text{C}$
(3) $60 \mu\text{C}$ (4) $110 \mu\text{C}$

13. In figure, three capacitors C_1 , C_2 , and C_3 are joined to a battery. With symbols having their usual meaning, the correct conditions will be



(1) $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V_3 + V$
(2) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2 + V_3$
(3) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$
(4) $Q_2 = Q_3$ and $V_2 = V_3$

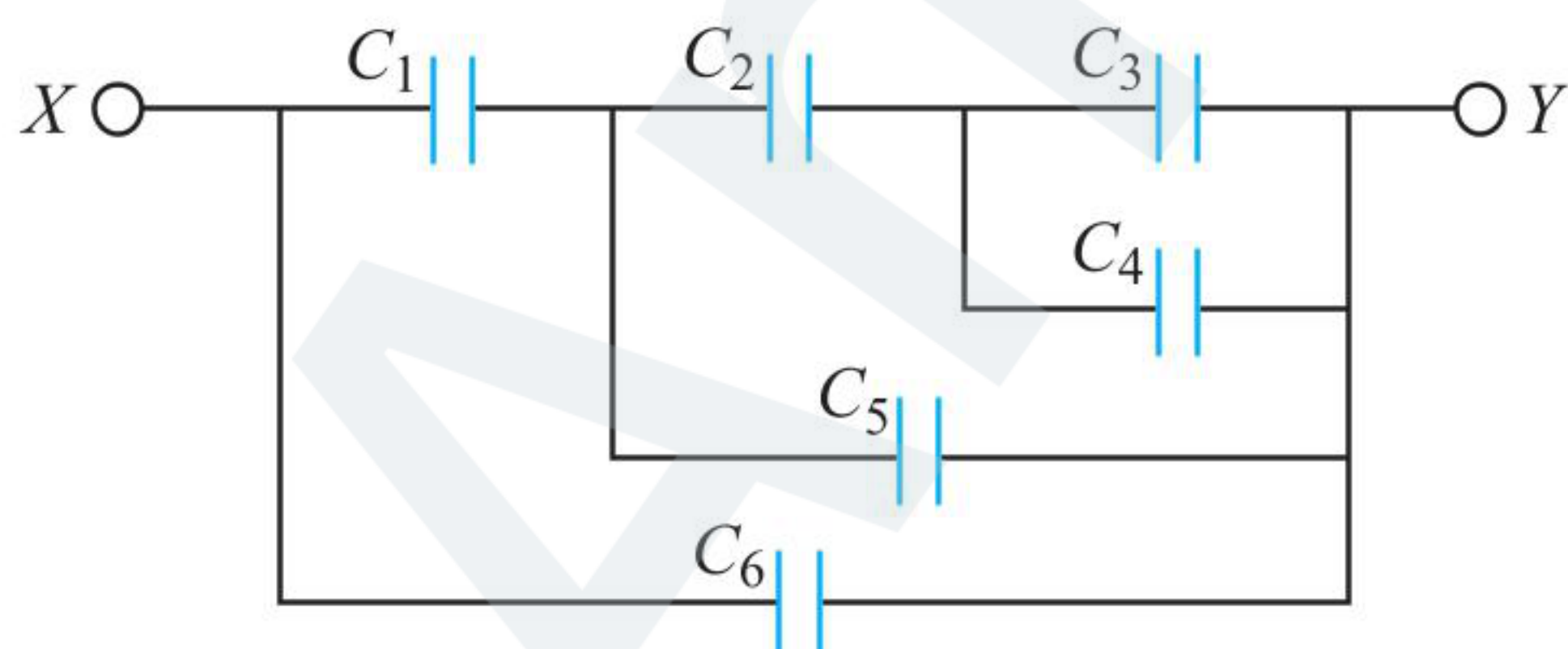
14. An uncharged parallel plate capacitor having a dielectric of dielectric constant K is connected to a similar air cored parallel plate capacitor charged to a potential V_0 . The two share the charge, and the common potential becomes V . The dielectric constant K is

(1) $\frac{V_0}{V} - 1$ (2) $\frac{V_0}{V} + 1$
(3) $\frac{V}{V_0} - 1$ (4) $\frac{V}{V_0} + 1$

15. Two identical parallel plate capacitors are connected in series and then joined in series with a battery of 100 V . A slab of dielectric constant $K = 3$ is inserted between the plates of the first capacitor. Then, the potential difference across the capacitors will be, respectively,

(1) $25 \text{ V}, 75 \text{ V}$ (2) $75 \text{ V}, 25 \text{ V}$
(3) $20 \text{ V}, 80 \text{ V}$ (4) $50 \text{ V}, 50 \text{ V}$

16. In the given network of capacitors as shown in figure, given that $C_1 = C_2 = C_3 = 400 \text{ pF}$ and $C_4 = C_5 = C_6 = 200 \text{ pF}$. The effective capacitance of the circuit between X and Y is



(1) 810 pF (2) 205 pF
(3) 600 pF (4) 410 pF

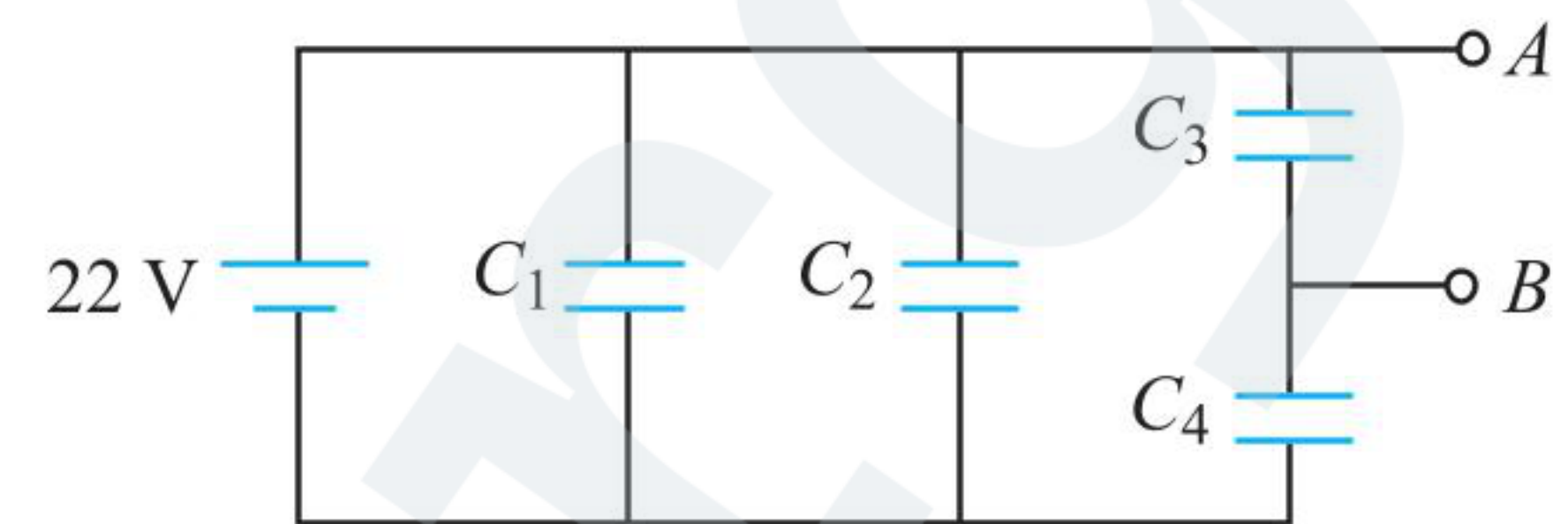
17. The work done in increasing the potential of a capacitor from V volt to $2V$ volt is W . Then, the work done in increasing the potential of the same capacitor from $2V$ volt to $4V$ volt will be

(1) W (2) $2W$
(3) $4W$ (4) $8W$

18. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2 mm . The capacitor is charged by connecting it to a 400 V supply. Then the energy density of the energy stored (in Jm^{-3}) in the capacitor is (take $\epsilon_0 = 8.8 \times 10^{-12} \text{ Fm}^{-1}$)

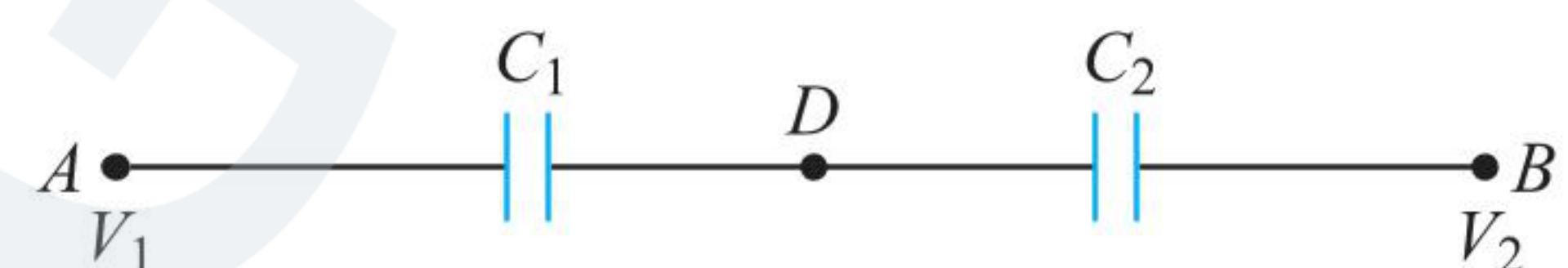
(1) 0.113 (2) 0.117
(3) 0.152 (4) none of these

19. In figure, given $C_1 = 3 \mu\text{F}$, $C_2 = 5 \mu\text{F}$, $C_3 = 9 \mu\text{F}$, and $C_4 = 13 \mu\text{F}$. What is the potential difference between points A and B ?



(1) 13 V (2) 9 V
(3) 0 V (4) 11 V

20. Two condensers C_1 and C_2 in a circuit are joined as shown in figure. The potential of point A is V_1 and that of B is V_2 . The potential of point D will be

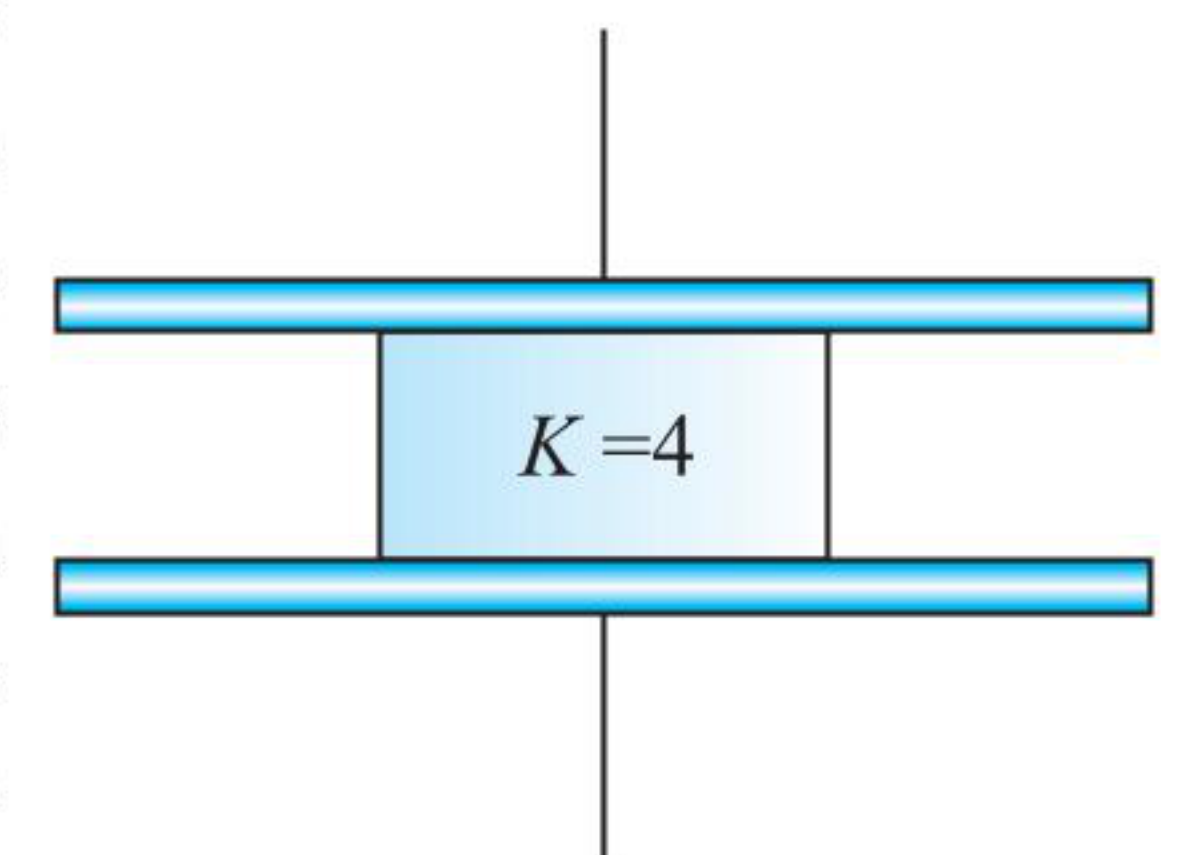


(1) $\frac{1}{2}(V_1 + V_2)$ (2) $\frac{C_1 V_2 + C_2 V_1}{C_1 + C_2}$
(3) $\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$ (4) $\frac{C_2 V_1 - C_1 V_2}{C_1 + C_2}$

21. A capacitor is charged to store an energy U . The charging battery is disconnected. An identical capacitor is now connected to the first capacitor in parallel. The energy in each capacitor is now

(1) $3U/2$ (2) U
(3) $U/4$ (4) $U/2$

22. Consider a parallel plate capacitor of capacity $10 \mu\text{F}$ with air filled in the gap between the plates. Now, one-half of the space between the plates is filled with a dielectric of dielectric constant 4 as shown in figure. The capacity of the capacitor changes to

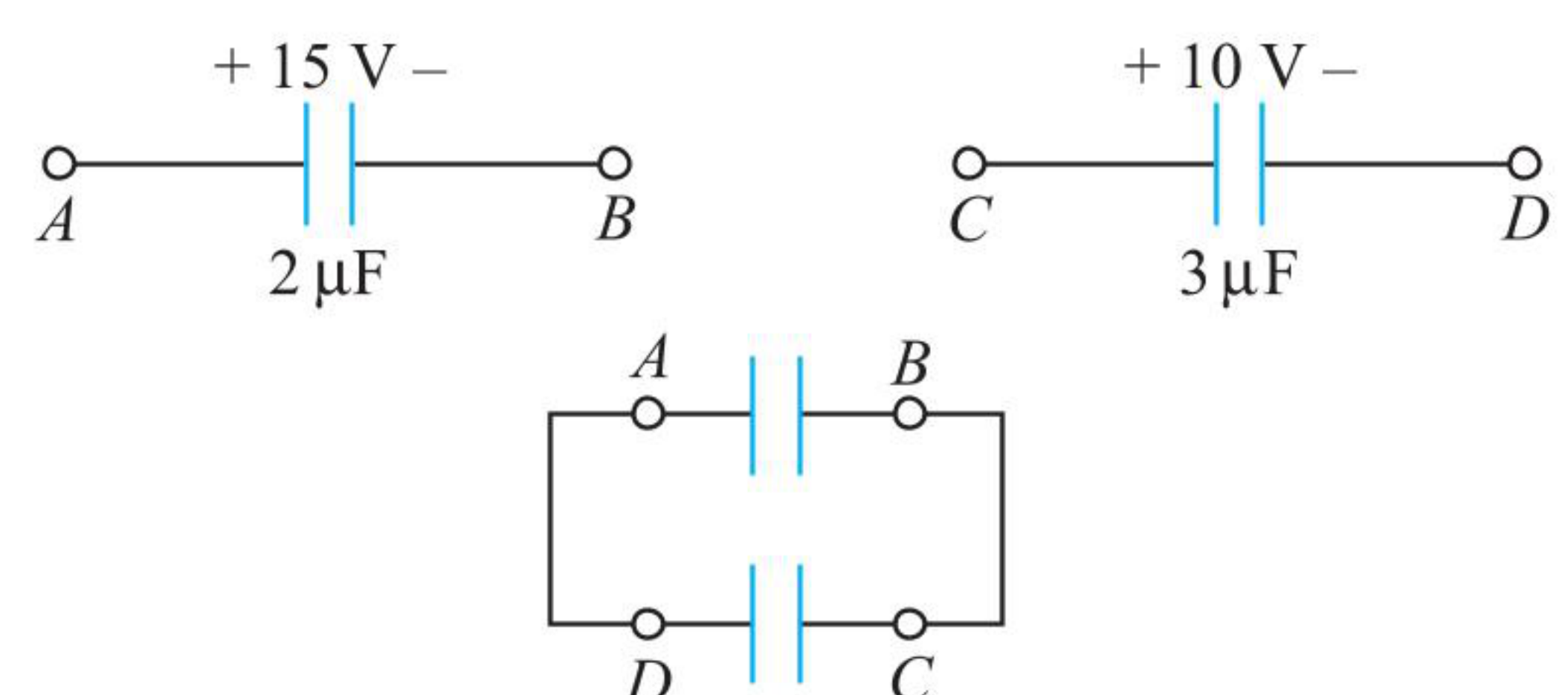


(1) $25 \mu\text{F}$ (2) $20 \mu\text{F}$
(3) $40 \mu\text{F}$ (4) $5 \mu\text{F}$

23. A $2 \mu\text{F}$ capacitor is charged to 100 V , and then its plates are connected by a conducting wire. The heat produced is

(1) 0.001 J (2) 0.01 J
(3) 0.1 J (4) 1 J

24. In figure, the initial status of capacitors and their connections is shown. Which of the following is incorrect about this circuit?

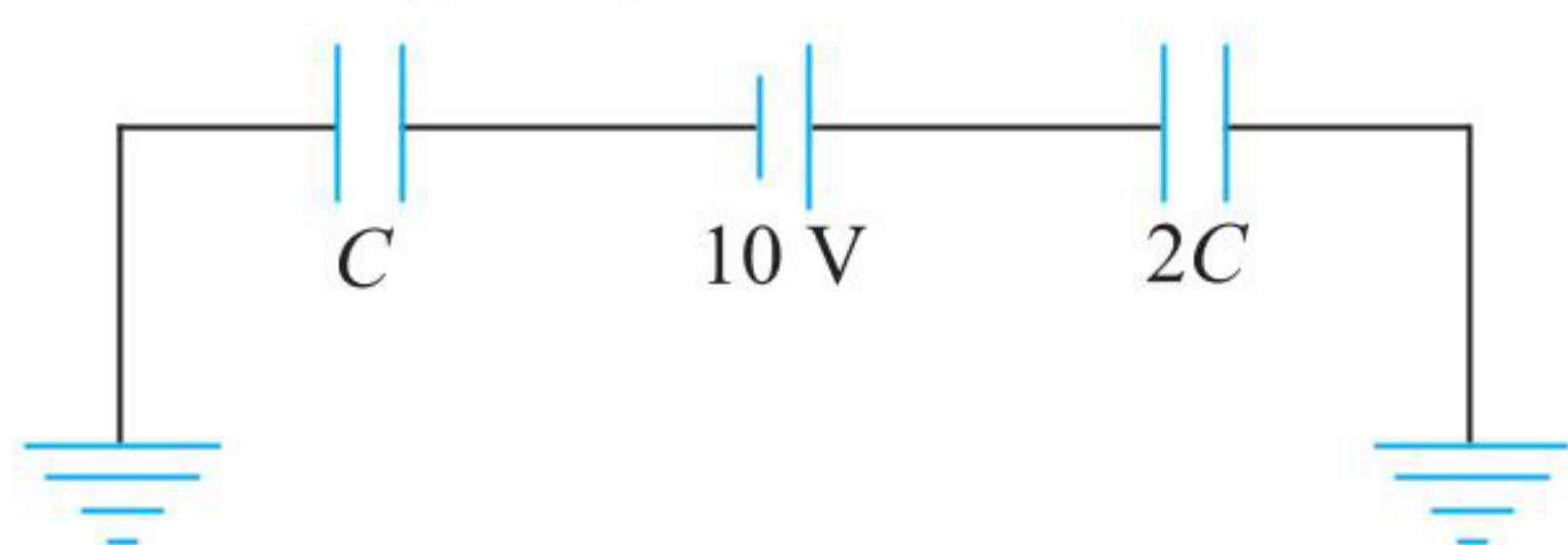


- (1) Final charge on each capacitor will be zero.
- (2) Final total electrical energy of the capacitor will be zero.
- (3) Total charge flowing from A to D is $30 \mu\text{C}$.
- (4) Total charge flowing from A to D is $-30 \mu\text{C}$.

25. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0 , discharged and charged again; the process being repeated n times. Due to this, the potential of the larger capacitor is decreased to V . The value of C is

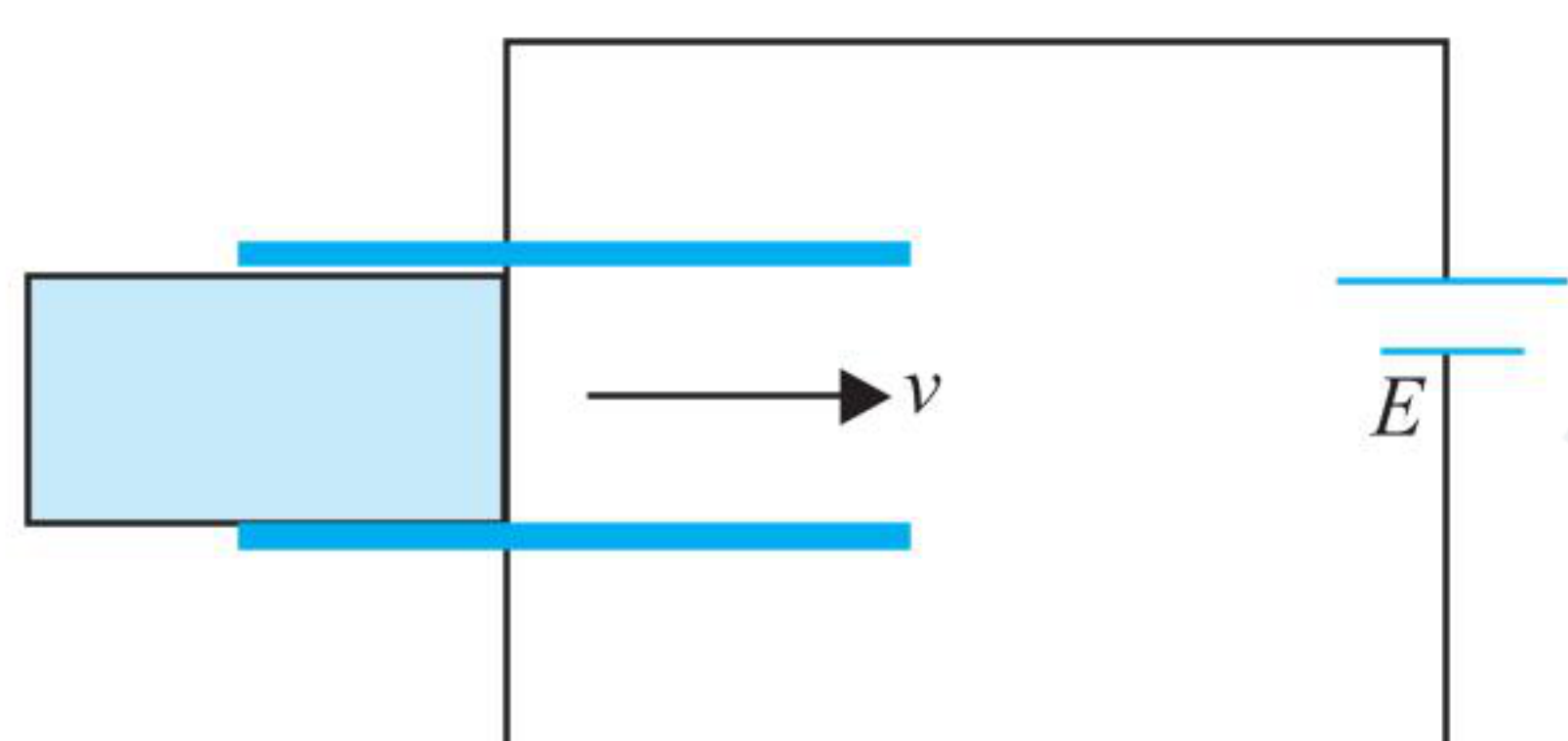
- (1) $C_0 \left(\frac{V_0}{V} \right)^{1/n}$
- (2) $C_0 \left[\left(\frac{V_0}{V} \right)^{1/n} - 1 \right]$
- (3) $C_0 \left[\left(\frac{V}{V_0} \right) - 1 \right]^n$
- (4) $C_0 \left[\left(\frac{V}{V_0} \right)^n + 1 \right]$

26. In the circuit shown in figure, $C = 6 \mu\text{F}$. The charge stored in the capacitor of capacity C is

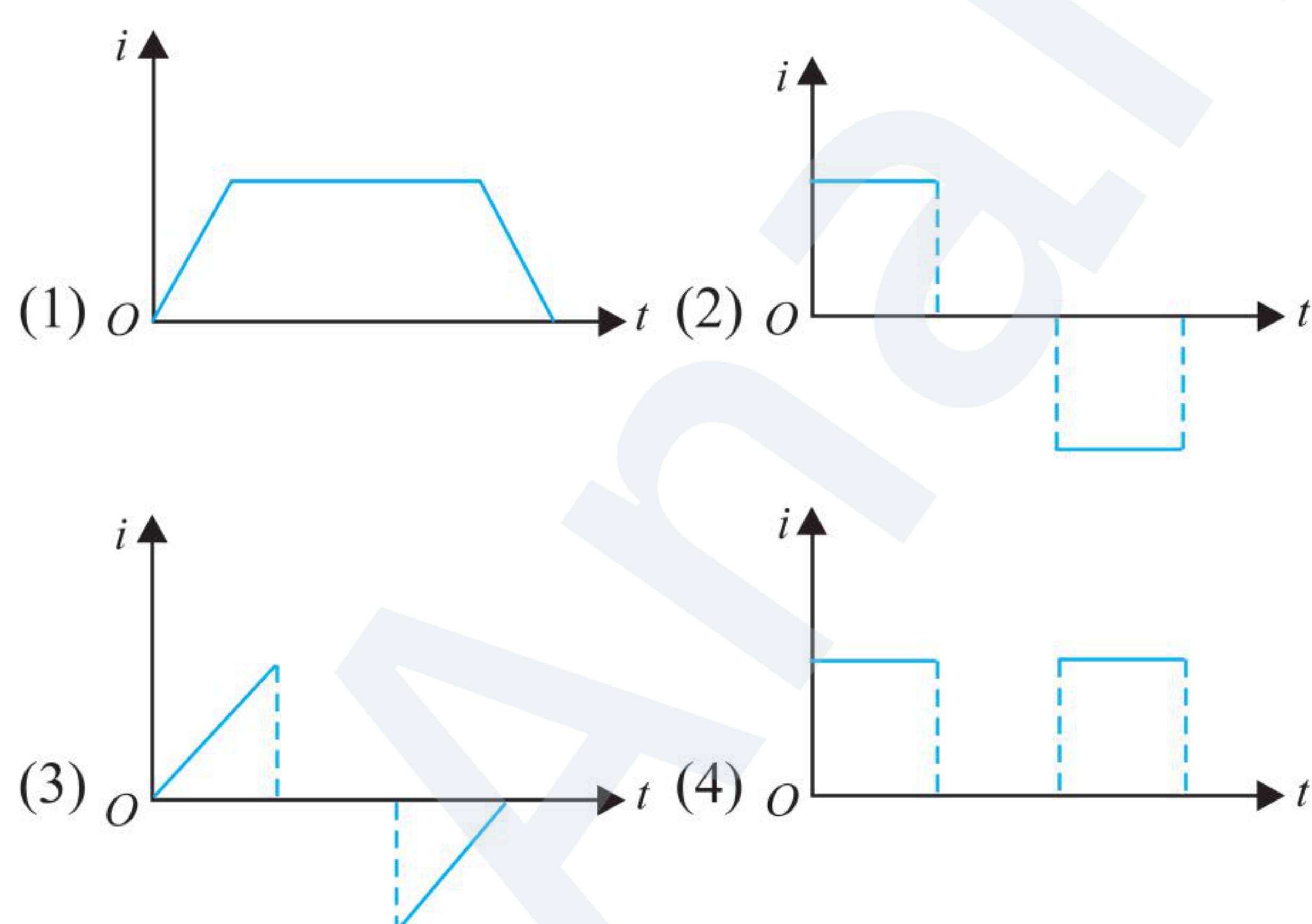


- (1) zero
- (2) $90 \mu\text{C}$
- (3) $40 \mu\text{C}$
- (4) $60 \mu\text{C}$

27. A dielectric slab of area A and thickness d is inserted between the plates of a capacitor of area $2A$ with constant speed v as shown in figure. Distance between the plates is d .



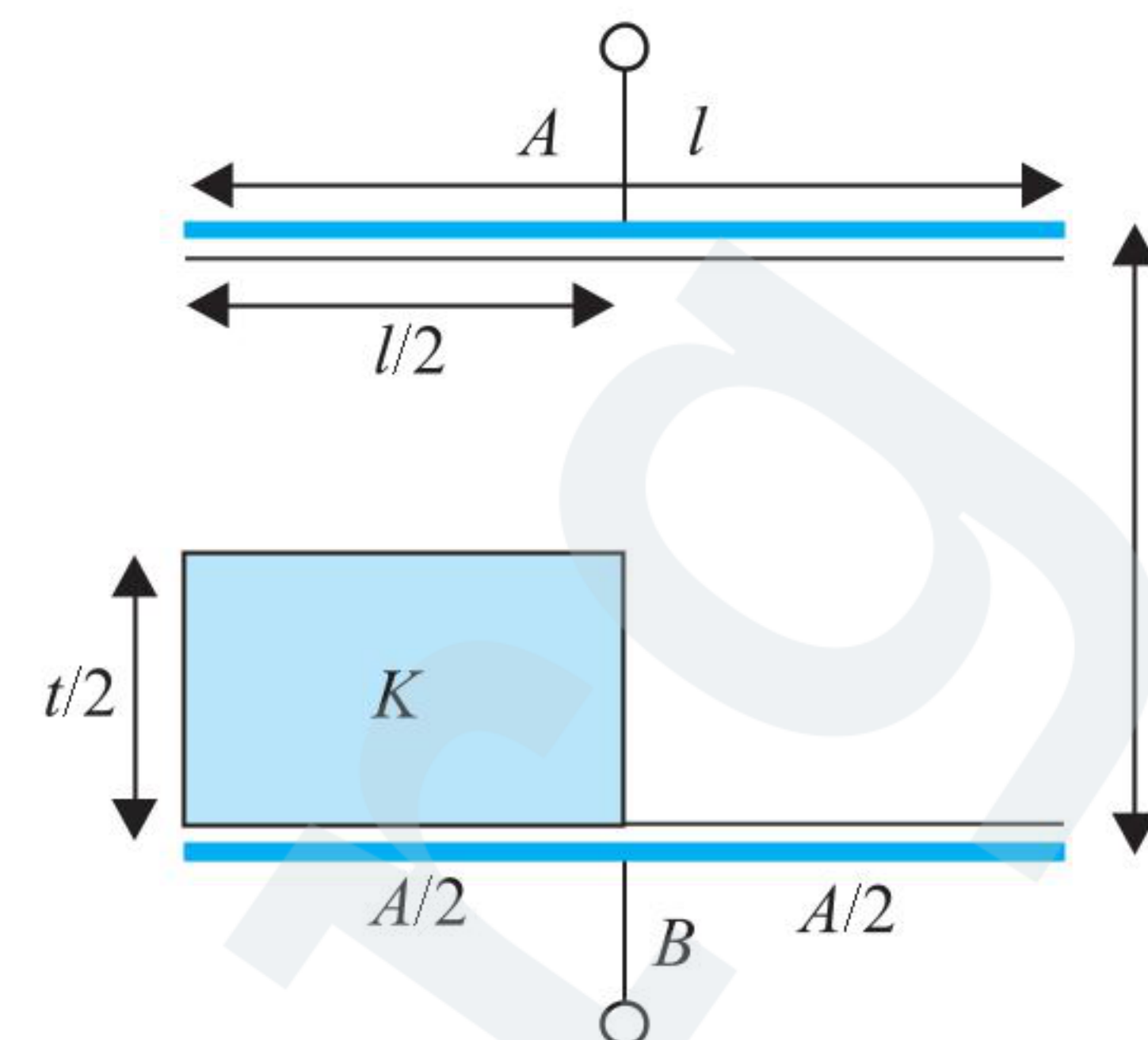
The capacitor is connected to a battery of emf E . The current in the circuit varies with time as



28. A photographic flash unit consists of a xenon-filled tube. It gives a flash of average power 2000 W for 0.04 s . The flash is due to discharge of a fully charged capacitor of $40 \mu\text{F}$. The voltage to which it is charged before a flash is given by the unit is

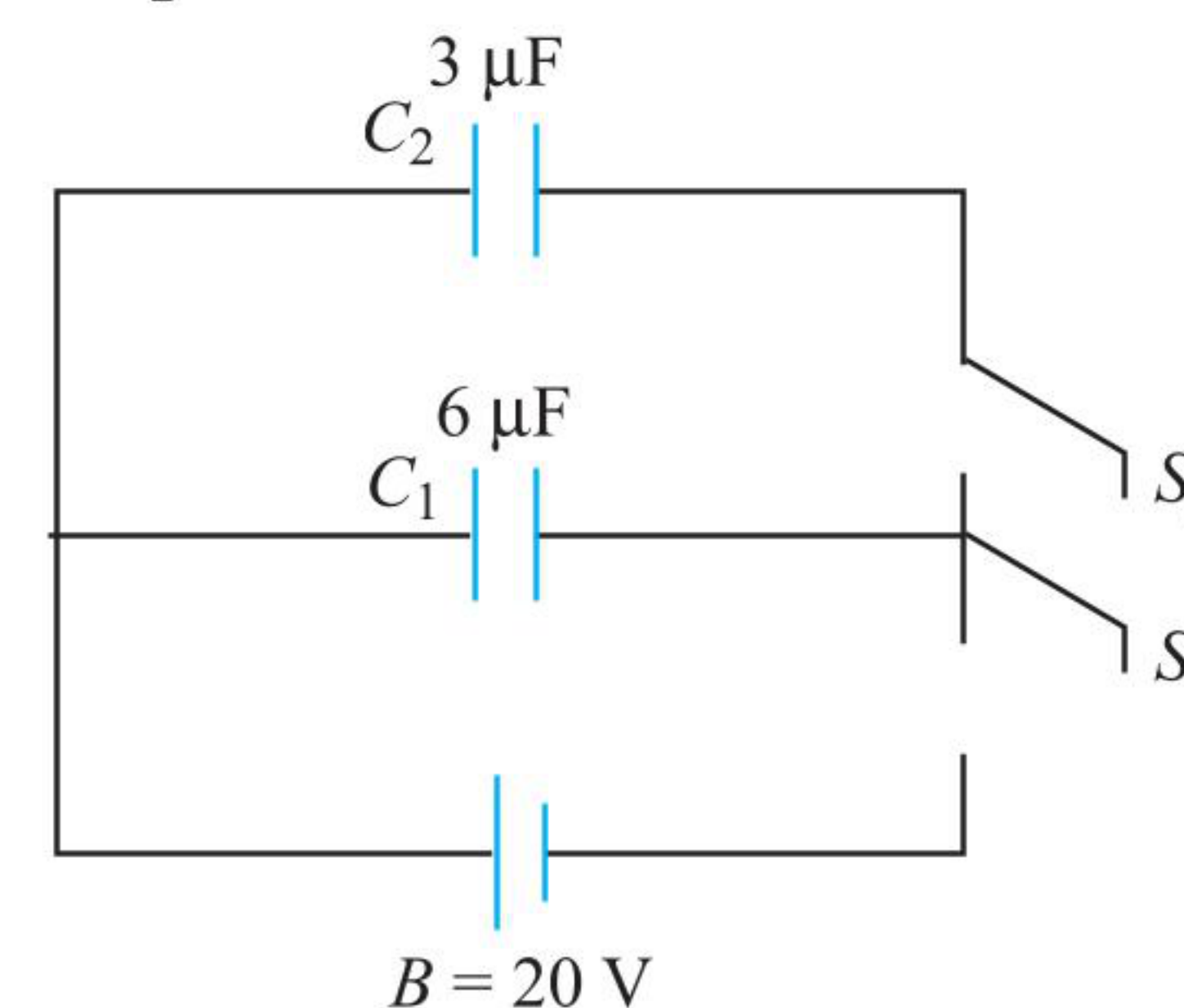
- (1) 1500 V
- (2) 2000 V
- (3) 2500 V
- (4) 3000 V

29. Two square plates ($l \times l$) and dielectric $\left(\frac{l}{2} \times \frac{t}{2} \times l \right)$ are arranged as shown in figure. Find the equivalent capacitance of the structure.



- (1) $\frac{\epsilon_0 A}{t} \left(\frac{3K+1}{2(K+1)} \right)$
- (2) $\frac{2\epsilon_0 A}{t} \left(\frac{K+1}{K+3} \right)$
- (3) $\frac{\epsilon_0 A}{t} \left(\frac{K+1}{K+3} \right)$
- (4) $\frac{\epsilon_0 A}{t} \left(\frac{2K+1}{2K+3} \right)$

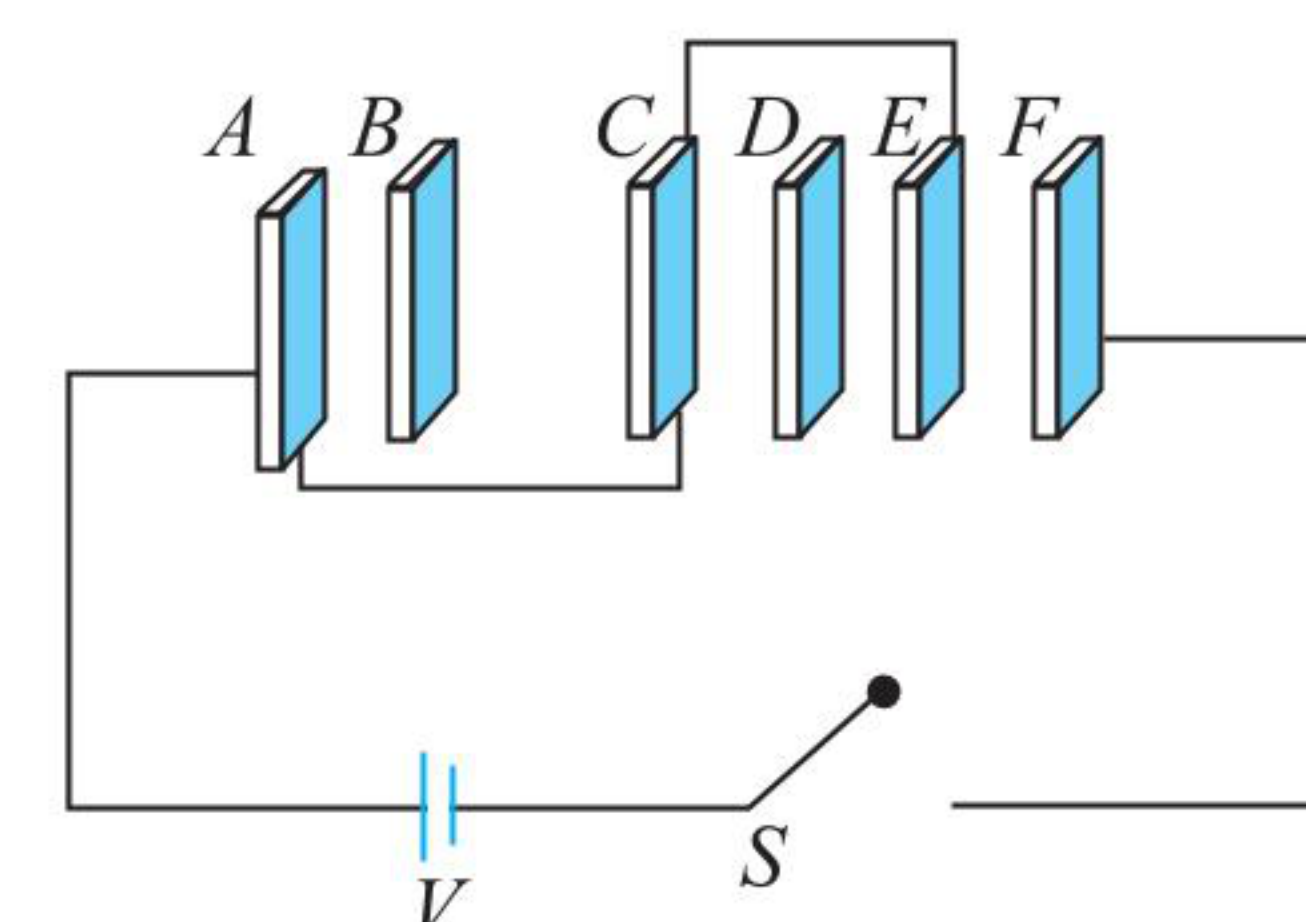
30. In the circuit shown in figure, $C_1 = 6 \mu\text{F}$, $C_2 = 3 \mu\text{F}$, and battery $B = 20 \text{ V}$. The switch S_1 is first closed. It is then opened, and S_2 is closed. What is the final charge on C_2 ?



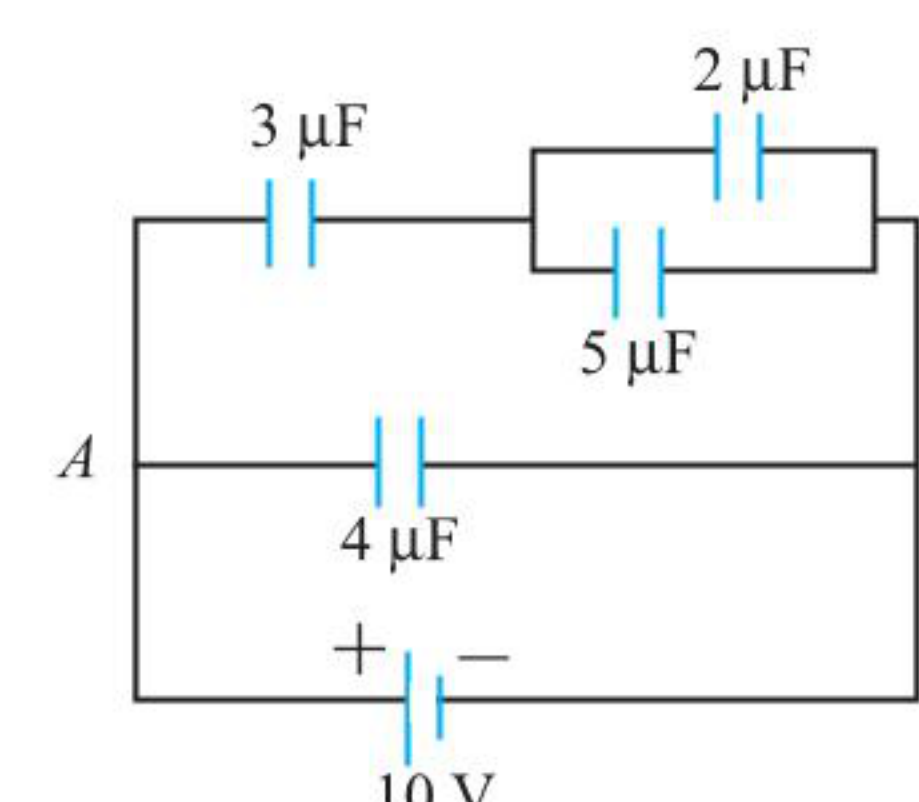
- (1) $120 \mu\text{C}$
- (2) $80 \mu\text{C}$
- (3) $40 \mu\text{C}$
- (4) $20 \mu\text{C}$

31. A, B, C, D, E , and F are conducting plates each of area A , and any two consecutive plates are separated by a distance d . The net energy stored in the system after the switch S is closed is

- (1) $\frac{3\epsilon_0 A}{2d} V^2$
- (2) $\frac{5\epsilon_0 A}{12d} V^2$
- (3) $\frac{\epsilon_0 A}{2d} V^2$
- (4) $\frac{\epsilon_0 A}{d} V^2$

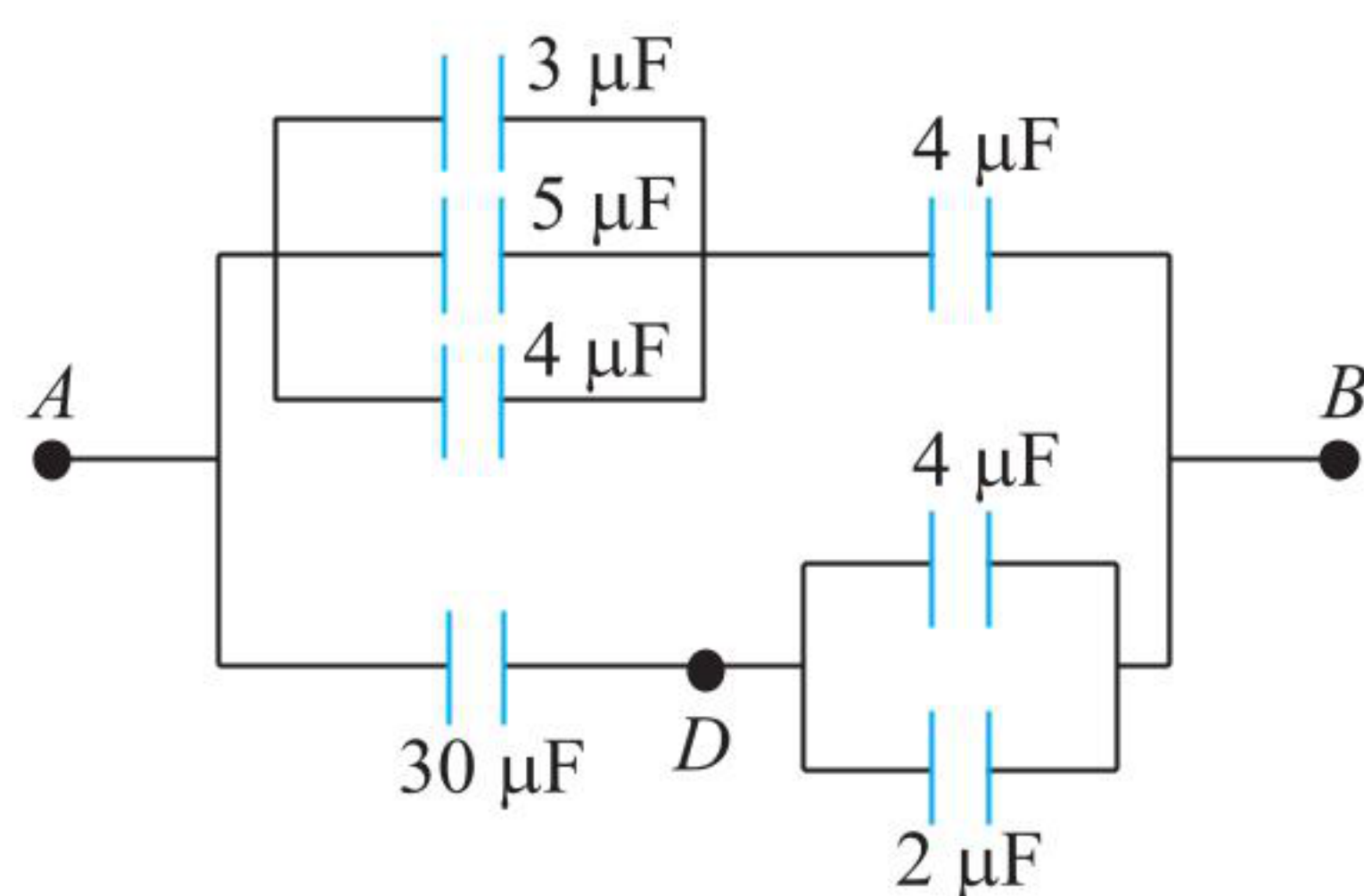


32. In the circuit shown in figure, the charge on the $5 \mu\text{F}$ capacitor will be



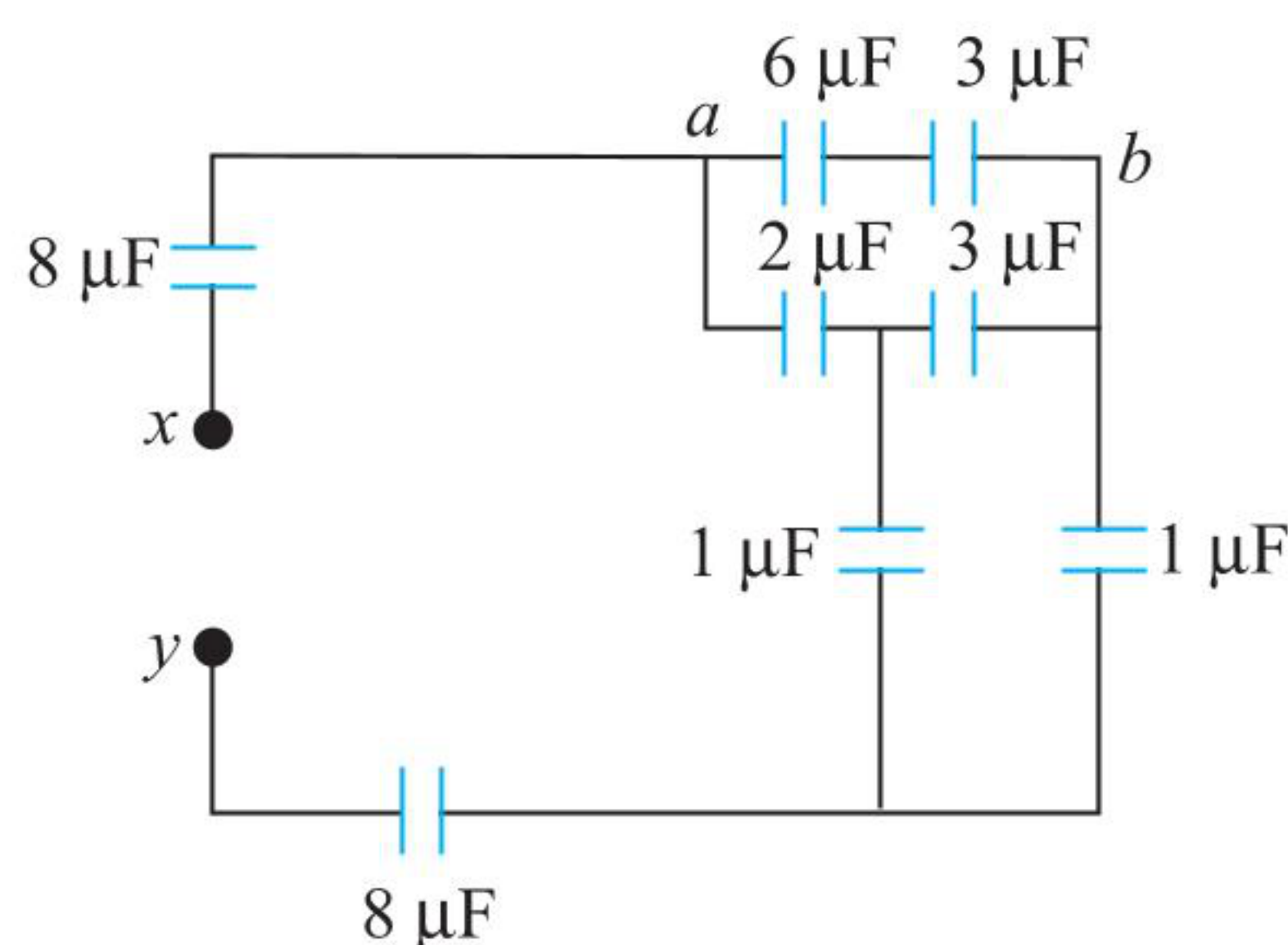
- (1) $4.5 \mu\text{C}$ (2) $9 \mu\text{C}$
 (3) $15 \mu\text{C}$ (4) none of these

33. Several capacitors are connected as shown in figure. If the charge on the $5 \mu\text{F}$ capacitor is $120 \mu\text{C}$, the potential between points A and D is



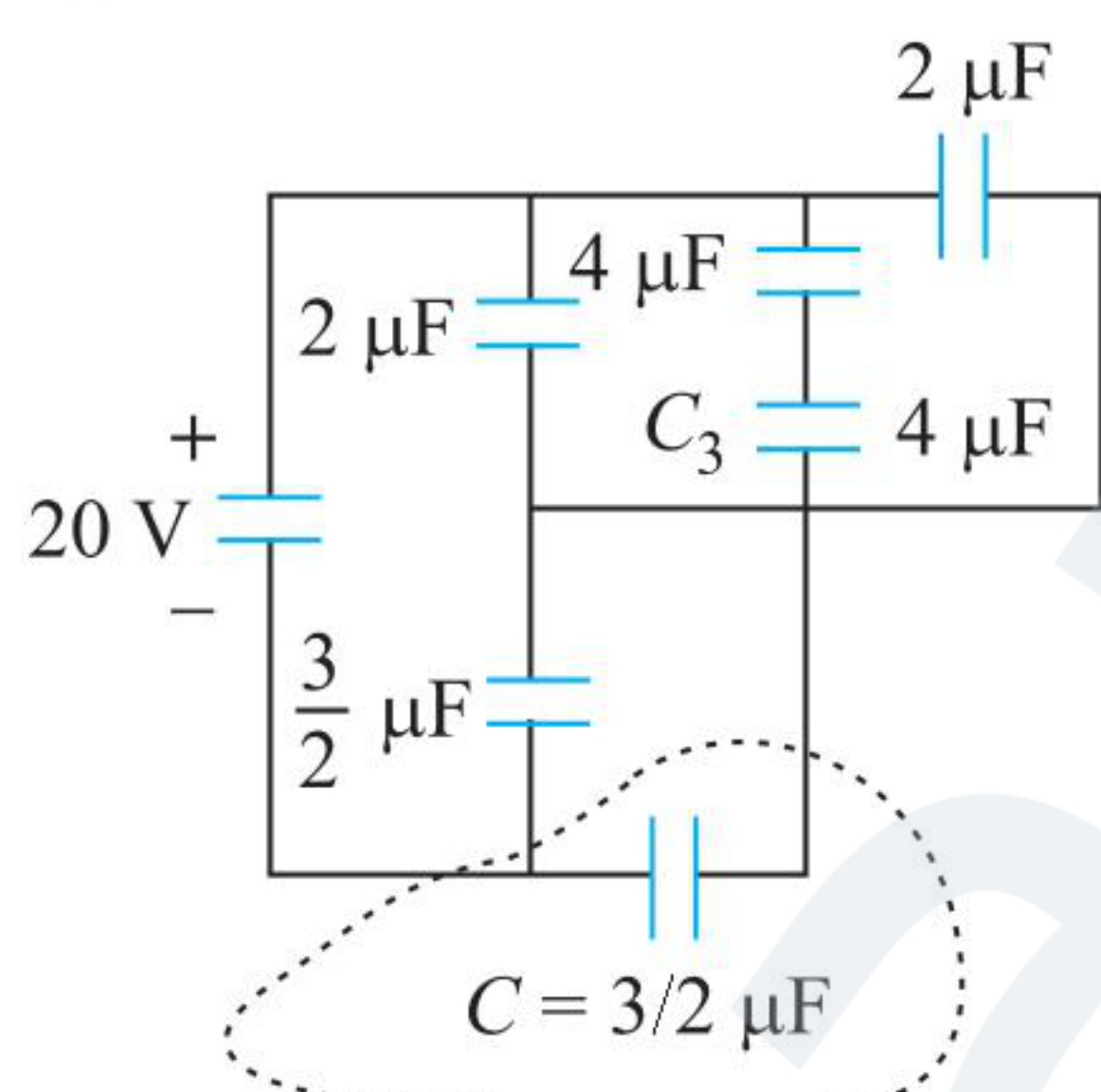
- (1) 16 V (2) 32 V
 (3) 64 V (4) none of these

34. In the circuit shown, the effective capacitance between points X and Y is



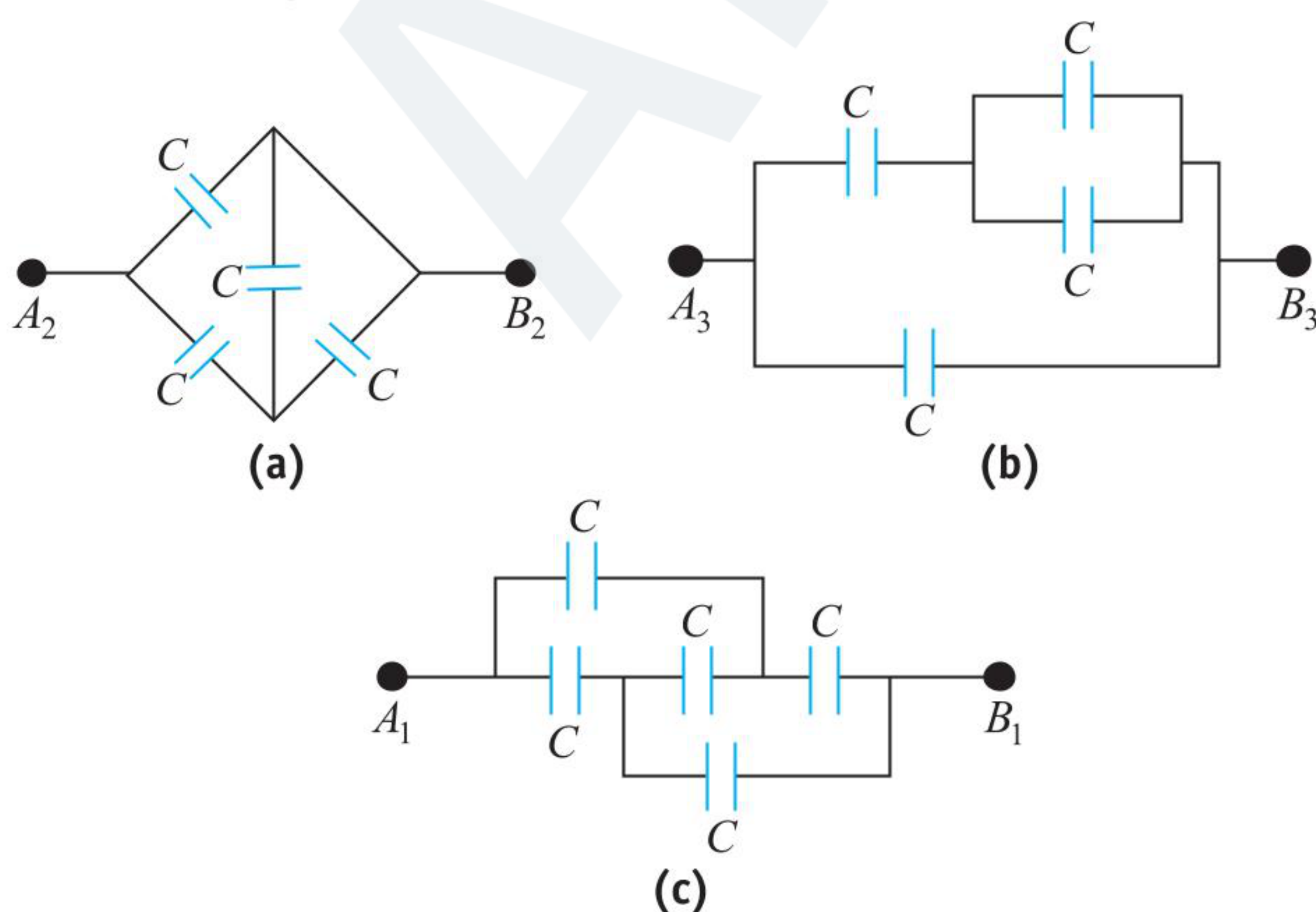
- (1) $3.33 \mu\text{F}$ (2) $1 \mu\text{F}$
 (3) $0.44 \mu\text{F}$ (4) none of these

35. In figure, the battery has a potential difference of 20 V. The charge in the capacitor marked as C is



- (1) $20 \mu\text{C}$ (2) $40 \mu\text{C}$
 (3) $10 \mu\text{C}$ (4) none of these

36. In figure, identical capacitors are connected in the following three configurations.

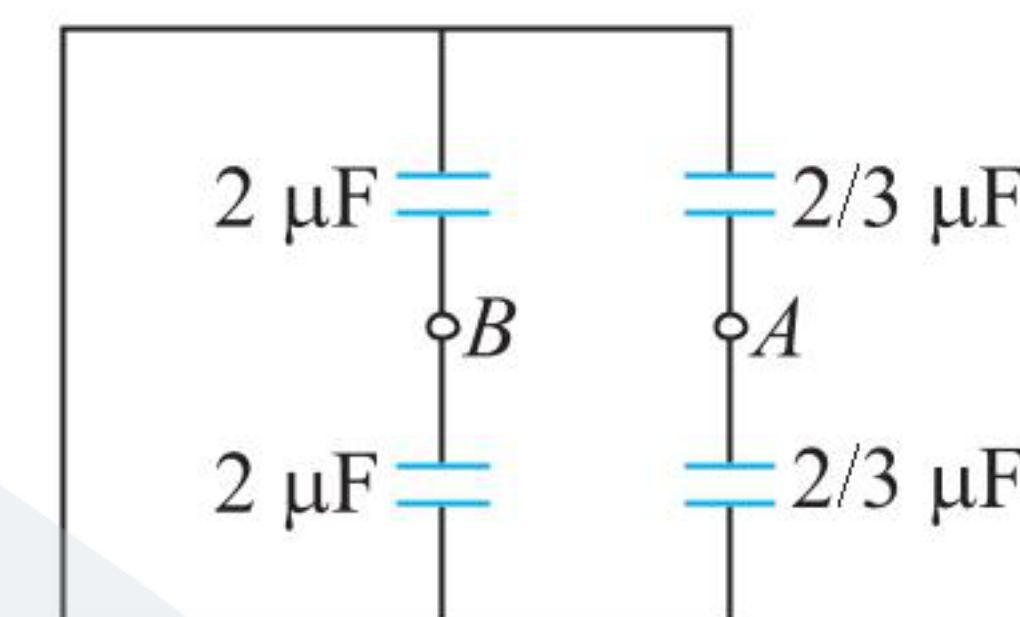


The ratio of the total capacitances in (i), (ii), and (iii), respectively, is

- (1) 3 : 5 : 5 (2) 3 : 3 : 5
 (3) 5 : 4 : 4 (4) 5 : 5 : 3

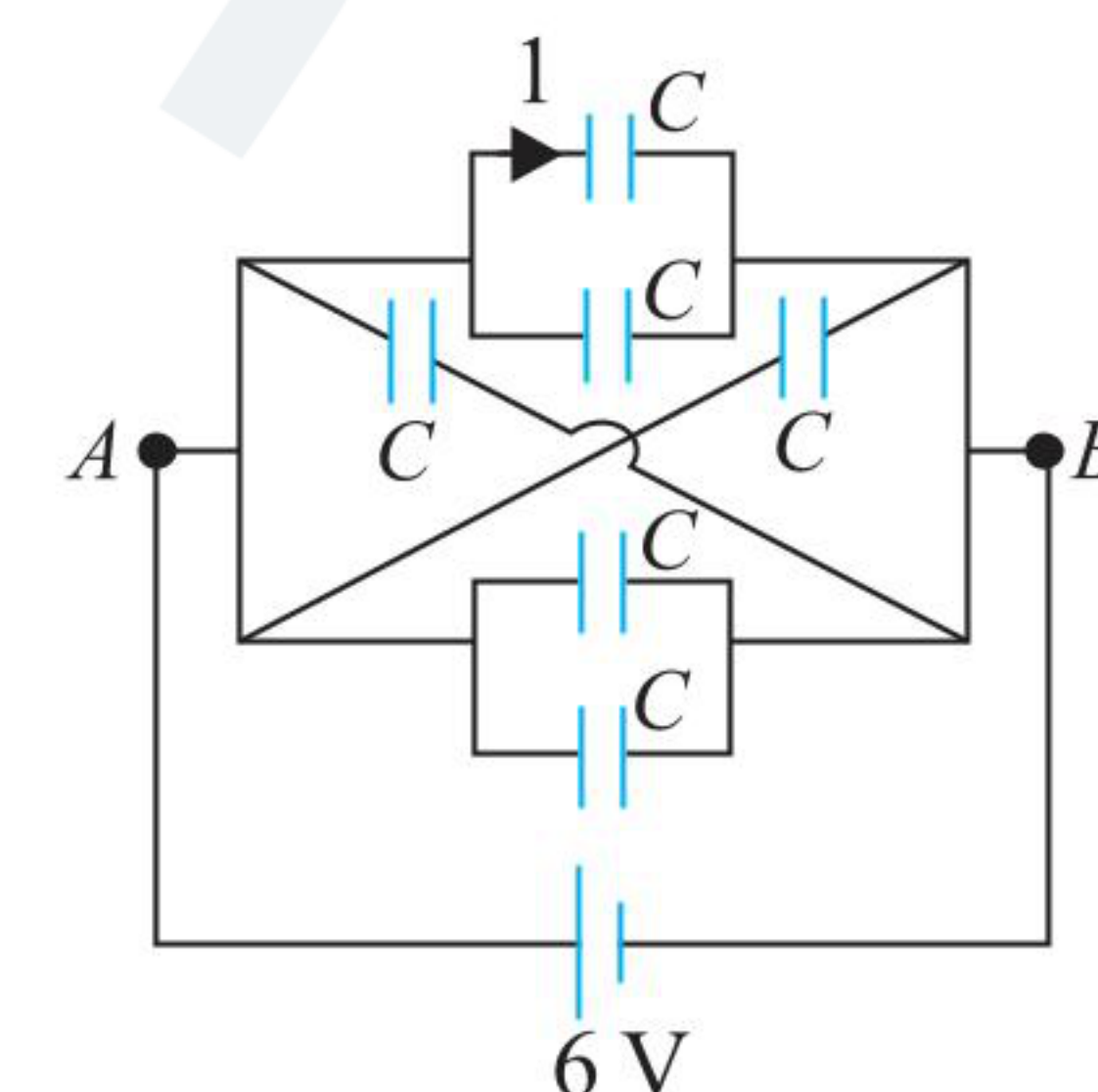
37. The equivalent capacitance of the circuit across the terminals A and B is equal to

- (1) $0.5 \mu\text{F}$
 (2) $2 \mu\text{F}$
 (3) $1 \mu\text{F}$
 (4) none of these



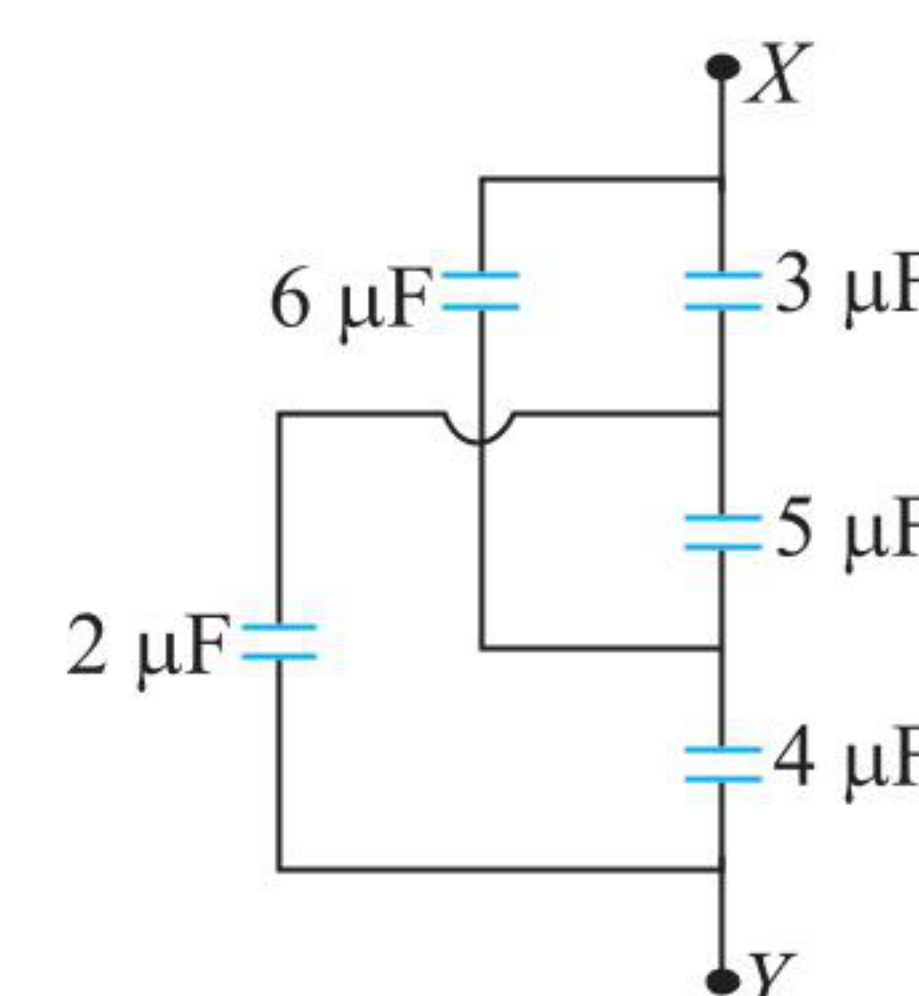
38. Six capacitors each of capacitance $1 \mu\text{F}$ are connected as shown in figure. Find the charge flowing in direction 1 as shown in the figure will be

- (1) $12 \mu\text{C}$
 (2) $6 \mu\text{C}$
 (3) $3 \mu\text{C}$
 (4) none of these

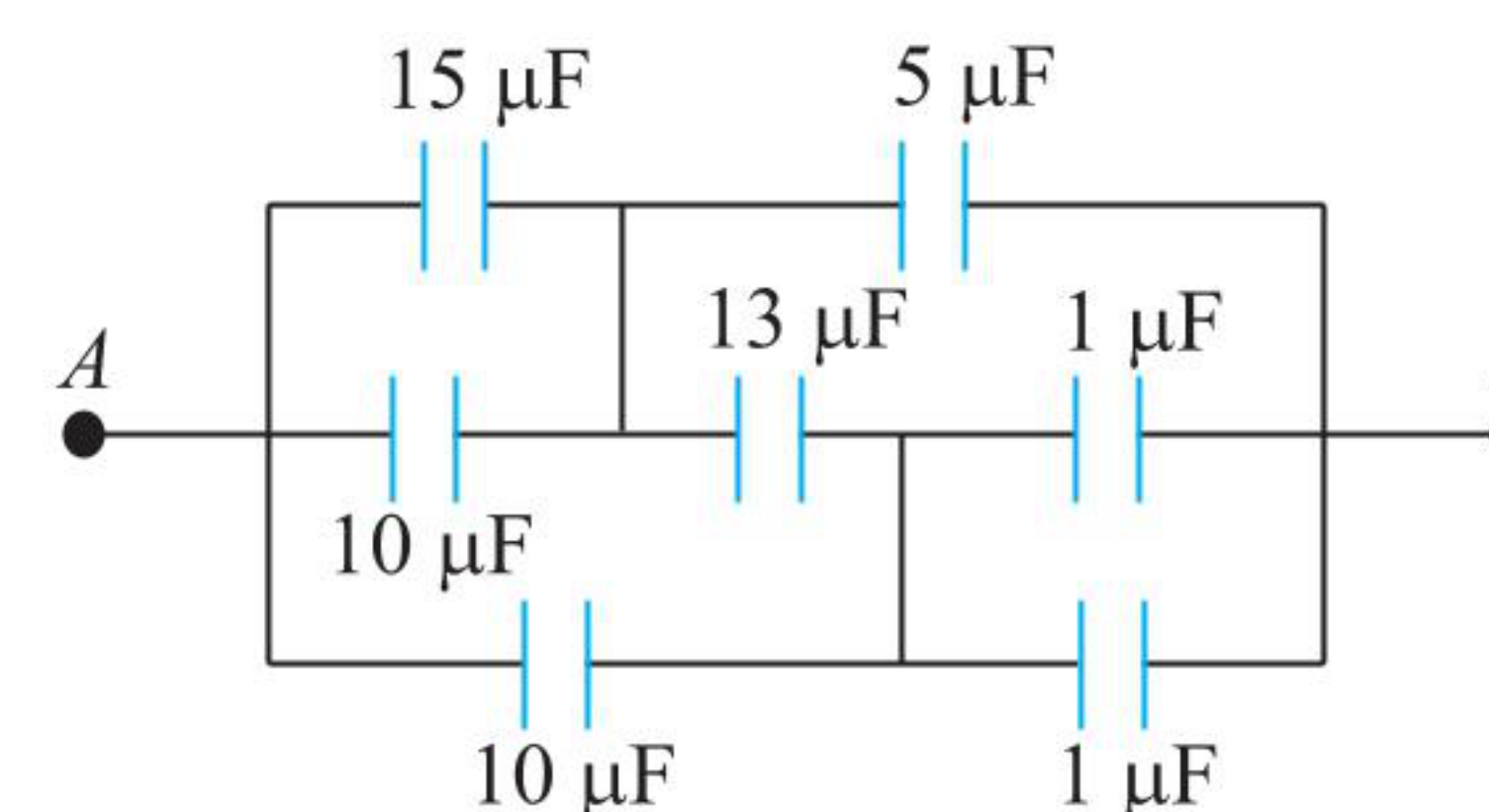


39. The equivalent capacitance between points X and Y in figure is

- (1) $\frac{6}{5} \mu\text{F}$
 (2) $4 \mu\text{F}$
 (3) $\frac{18}{5} \mu\text{F}$
 (4) none of these



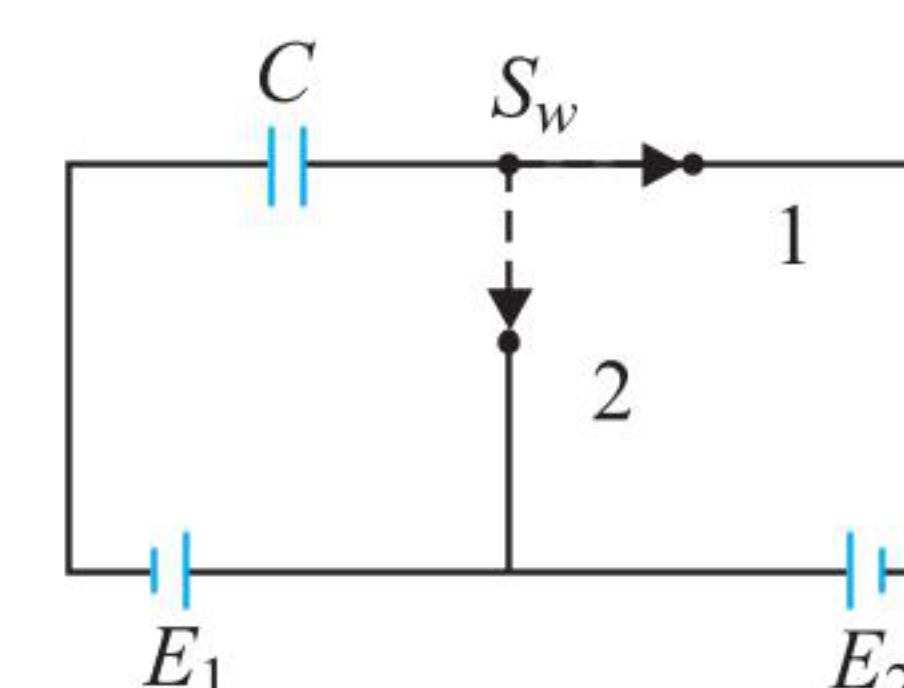
40. Find the equivalent capacitance across A and B .



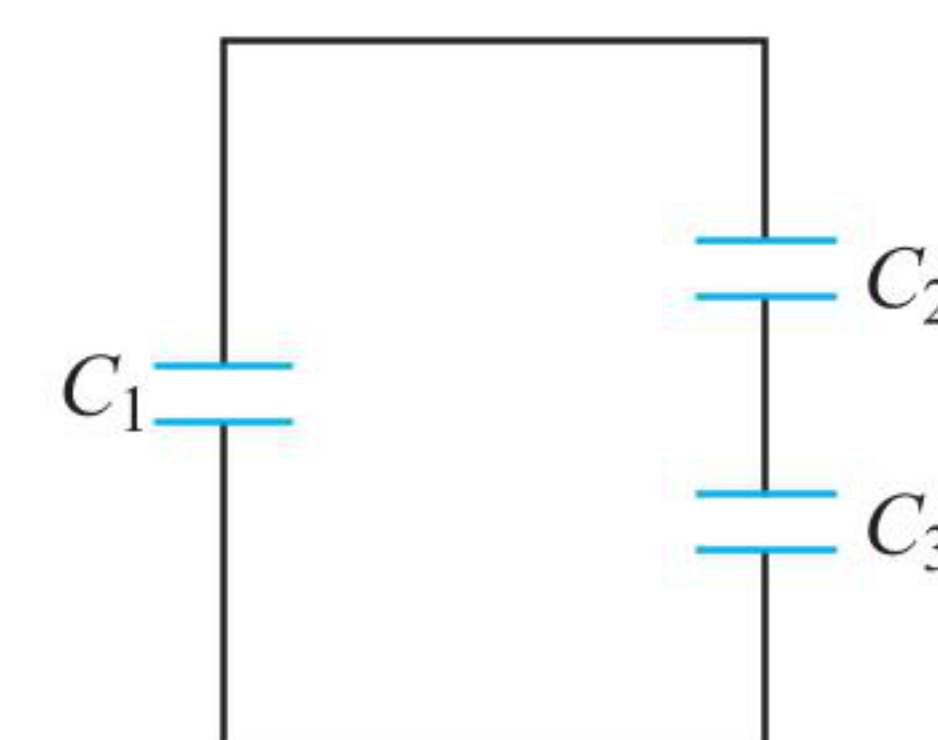
- (1) $\frac{35}{6} \mu\text{F}$ (2) $\frac{25}{6} \mu\text{F}$
 (3) $15 \mu\text{F}$ (4) none of these

41. In the given circuit diagram (figure), switch S_w is shifted from position 1 to position 2. Then

- (1) a charge of amount CE_2 will be supplied to battery E_1
 (2) heat generated in the circuit is $CE_2^2/2$
 (3) a charge of amount CE_2 will be supplied by battery E_1
 (4) heat generated in the circuit is $CE_1E_2/2$



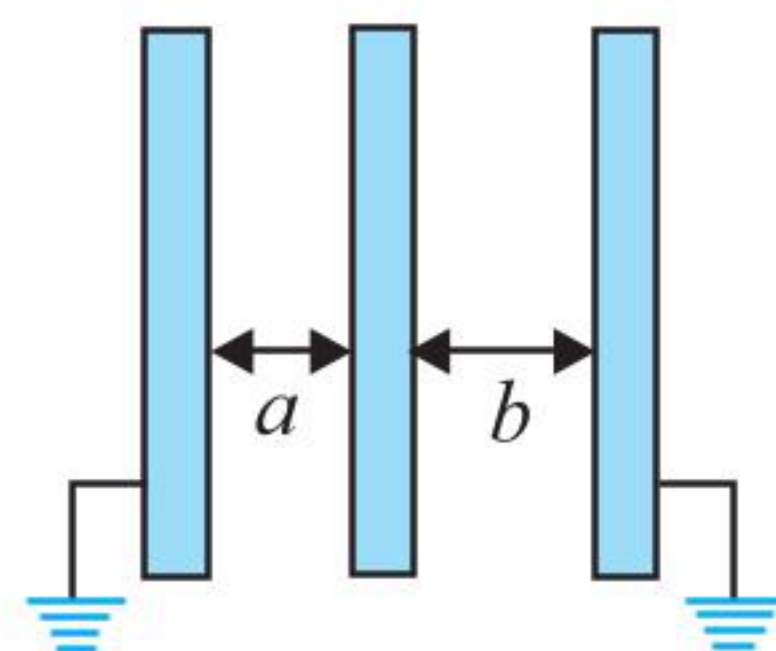
42. Capacitor C_1 is connected to a battery and charged till the magnitude of the charge on each plate is q_0 . Then, the battery is disconnected and C_1 is connected to two other uncharged capacitors C_2 and C_3 as shown (figure). Final charges on the capacitors (q_1 , q_2 , and q_3) are related by



- (1) $q_0 = q_1 + q_2 + q_3$ (2) $q_1 + q_2 + q_3 = 0$
 (3) $q_0 = q_3 + q_2, q_1 = 0$ (4) $q_0 = q_1 + q_2, q_2 = q_3$

43. Three identical metallic plates are kept parallel to one another at separations a and b . The outer plates are connected to the ground and the middle plate is given charge Q (figure). Then, the charge on the right side of middle plate is

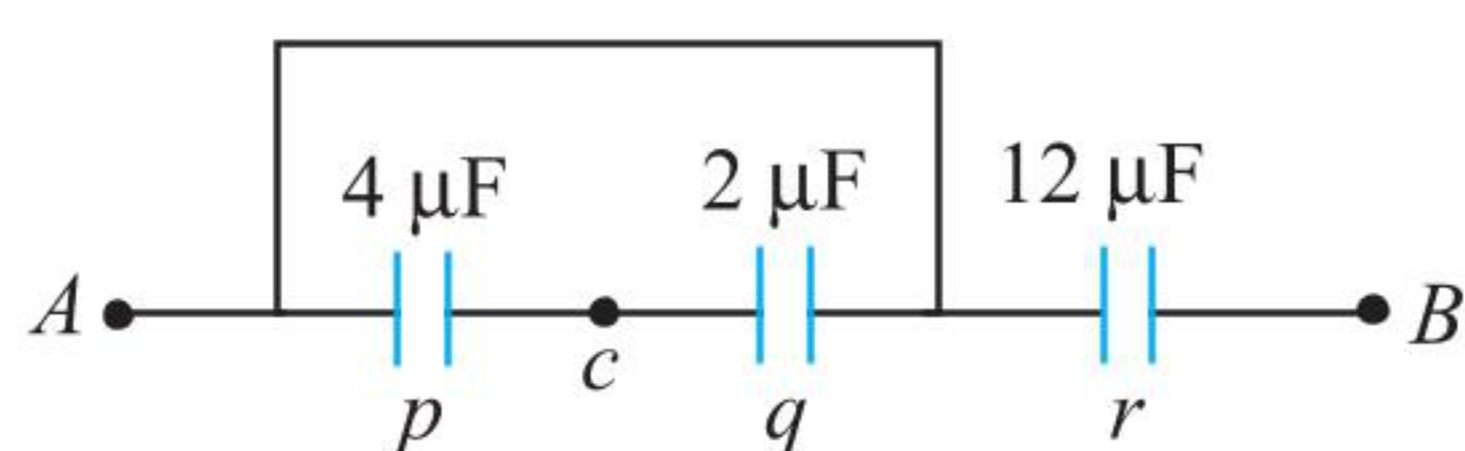
- (1) $Q/2$ (2) $-\frac{Qb}{a+b}$
 (3) $\frac{Qb}{a+b}$ (4) $\frac{Qa}{a+b}$



44. A $16 \mu\text{F}$ capacitor, initially charged to 5 V , is started charging at $t = 0$ by a source at the rate of $40t \mu\text{Cs}^{-1}$. How long will it take to raise its potential to 10 V ?

- (1) 1 s (2) 2 s
 (3) 3 s (4) none of these

45. Find the equivalent capacitance between C and B .

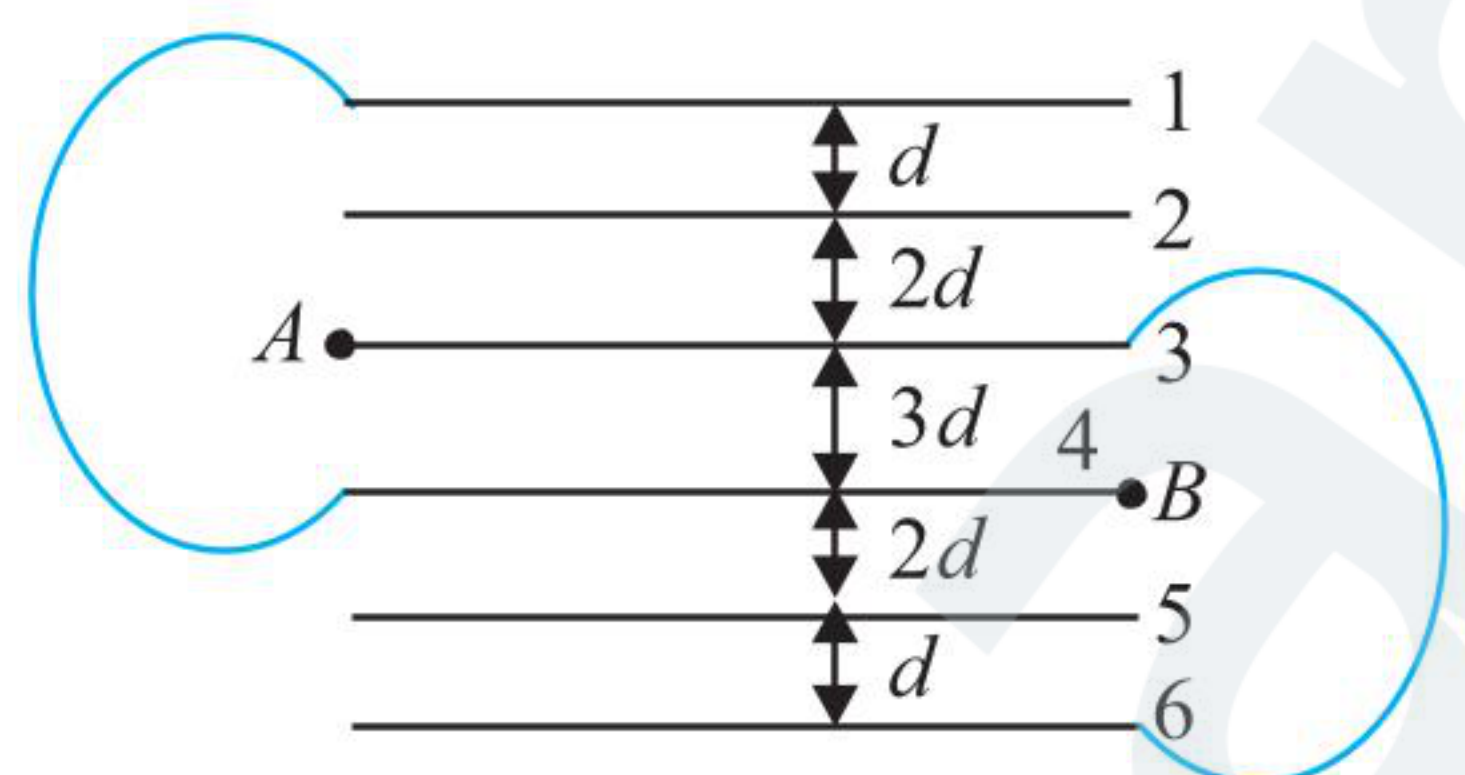


- (1) $6/5 \mu\text{F}$ (2) $5/6 \mu\text{F}$
 (3) $4 \mu\text{F}$ (4) none of these

46. A parallel plate capacitor of capacitance $10 \mu\text{F}$ is connected across a battery of emf 5 mV . Now, the space between the plates of the capacitor is filled with a dielectric material of dielectric constant $K = 5$. Then, the charge that will flow through the battery till equilibrium is reached is

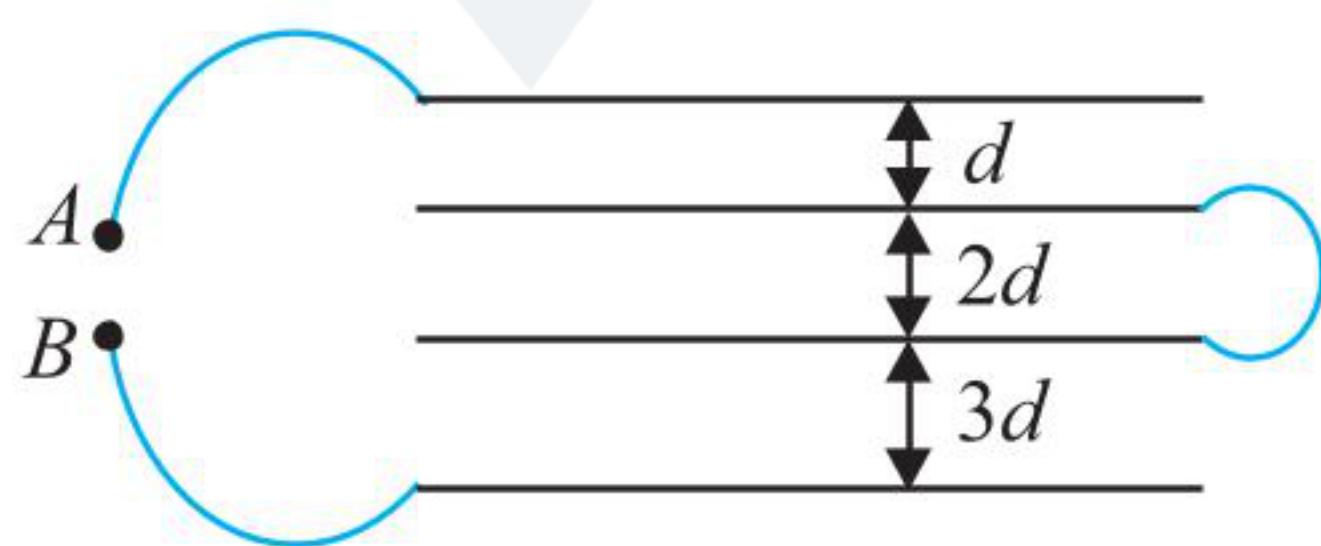
- (1) $250 \mu\text{C}$ (2) 250 nC
 (3) 200 nC (4) $200 \mu\text{C}$

47. Six plates of equal area A and plate separation as shown (figure) are arranged. The equivalent capacitance between A and B is



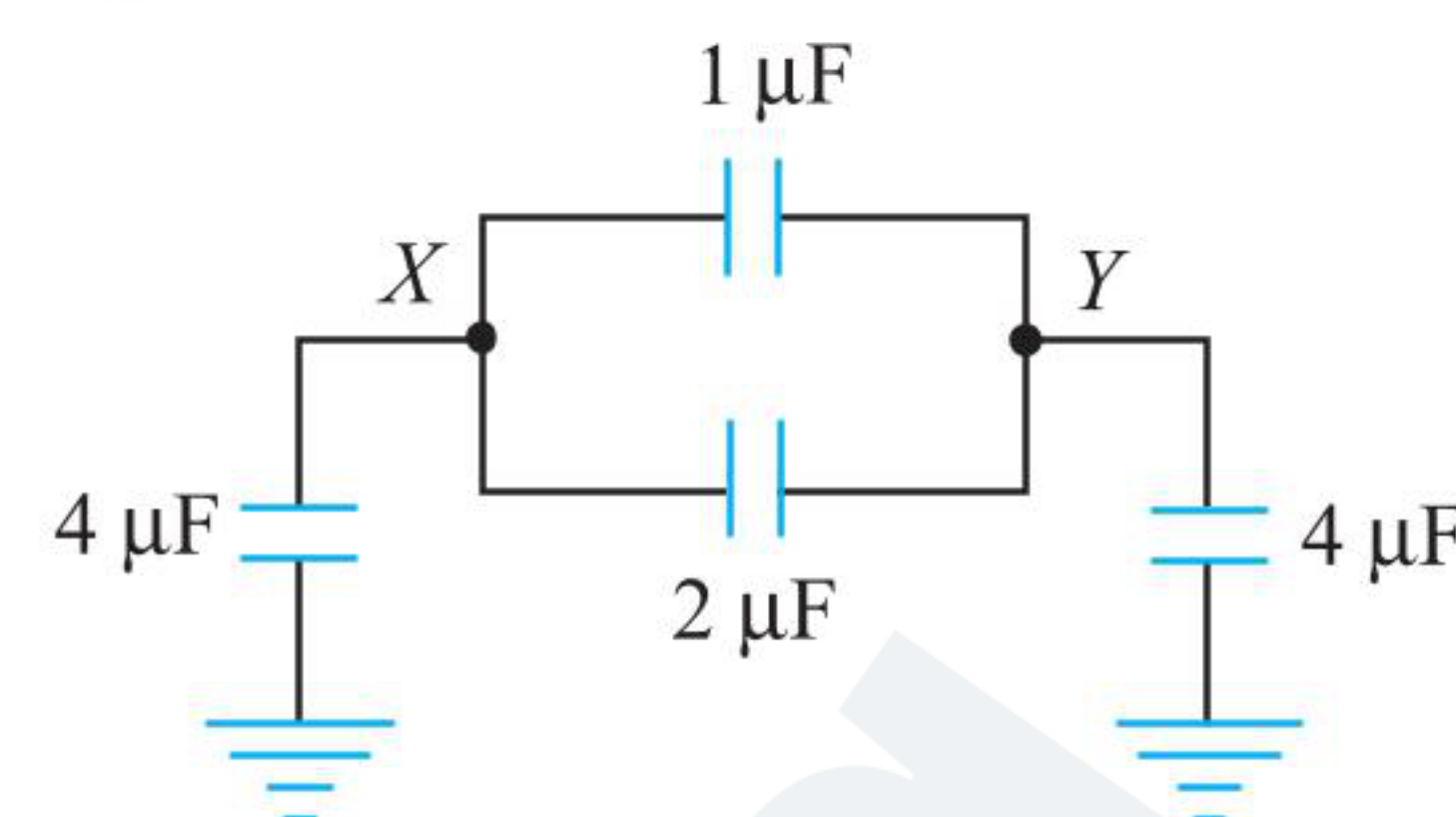
- (1) $\frac{\epsilon_0 A}{d}$ (2) $\frac{2\epsilon_0 A}{d}$
 (3) $\frac{3\epsilon_0 A}{d}$ (4) $\frac{\epsilon_0 A}{4d}$

48. If area of each plate is A and the successive separations are d , $2d$, and $3d$, then the equivalent capacitance across A and B is



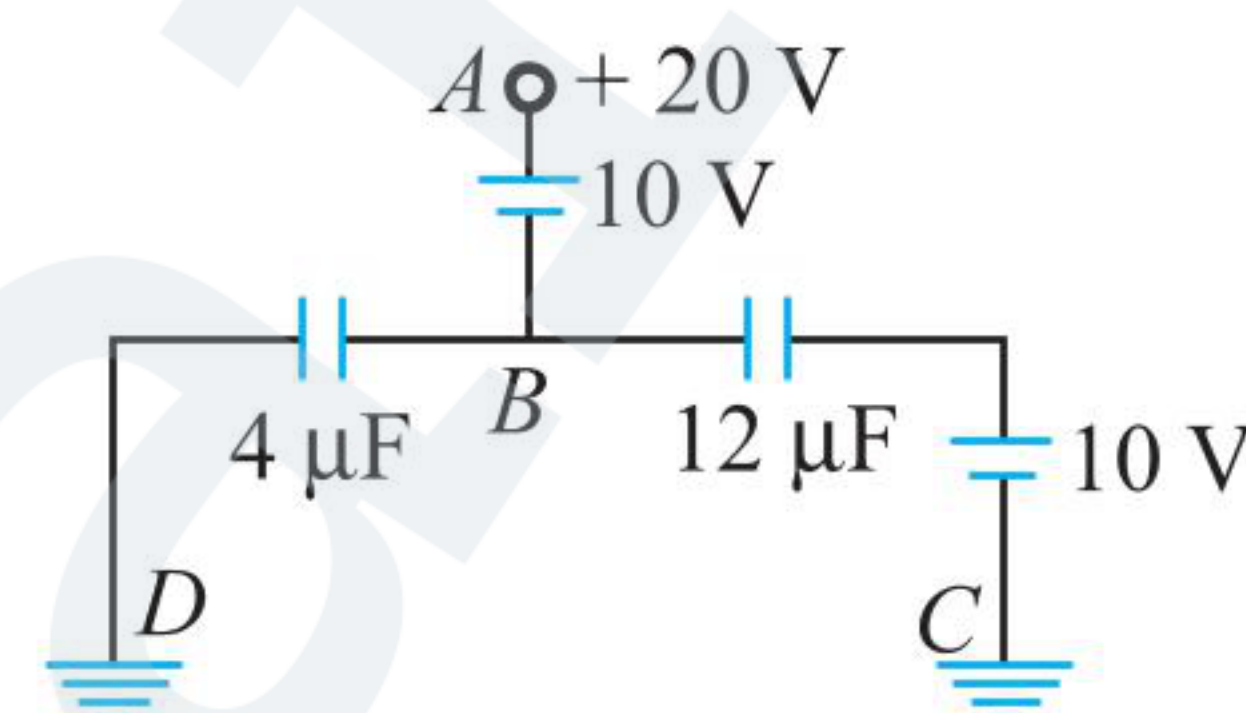
- (1) $\frac{\epsilon_0 A}{6d}$ (2) $\frac{\epsilon_0 A}{4d}$
 (3) $\frac{3\epsilon_0 A}{4d}$ (4) $\frac{\epsilon_0 A}{3d}$

49. In the circuit shown (figure), the equivalent capacitance between the points X and Y is



- (1) $2 \mu\text{F}$ (2) $3 \mu\text{F}$
 (3) $4 \mu\text{F}$ (4) $5 \mu\text{F}$

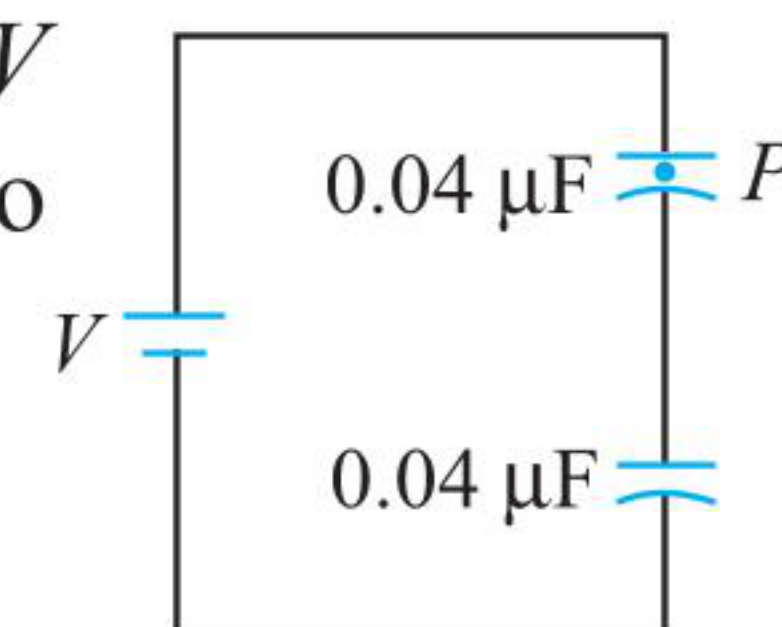
50. For the arrangement shown in figure, identify the correct statement.



- (1) The charge on the $12 \mu\text{F}$ capacitor is zero.
 (2) The charge on the $12 \mu\text{F}$ capacitor is $30 \mu\text{C}$.
 (3) The charge on the $4 \mu\text{F}$ capacitor is $30 \mu\text{C}$.
 (4) None of these.

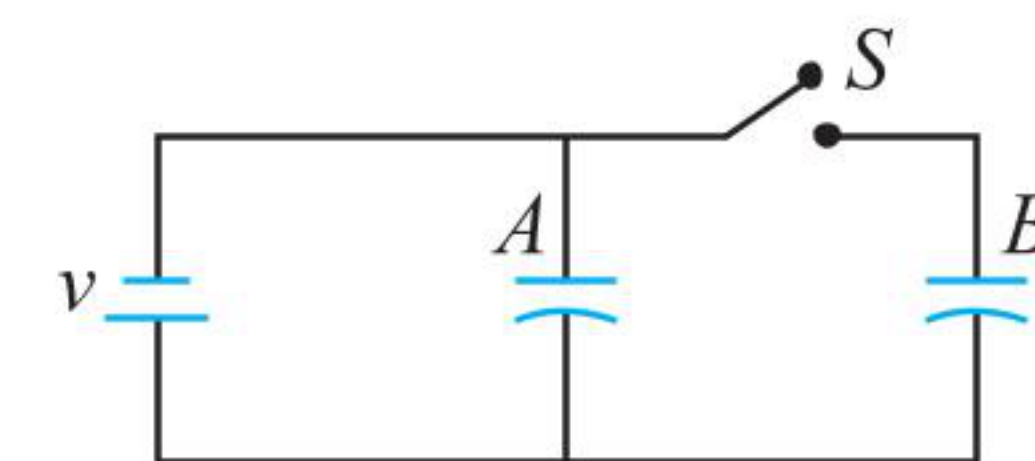
51. The particle P shown in the figure has a mass of 10 mg and a charge of $-0.01 \mu\text{C}$. Each plate has a surface area 10^{-2} m^2 on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium?

- (1) 43 mV (2) 35 mV
 (3) 50 mV (4) 55 mV



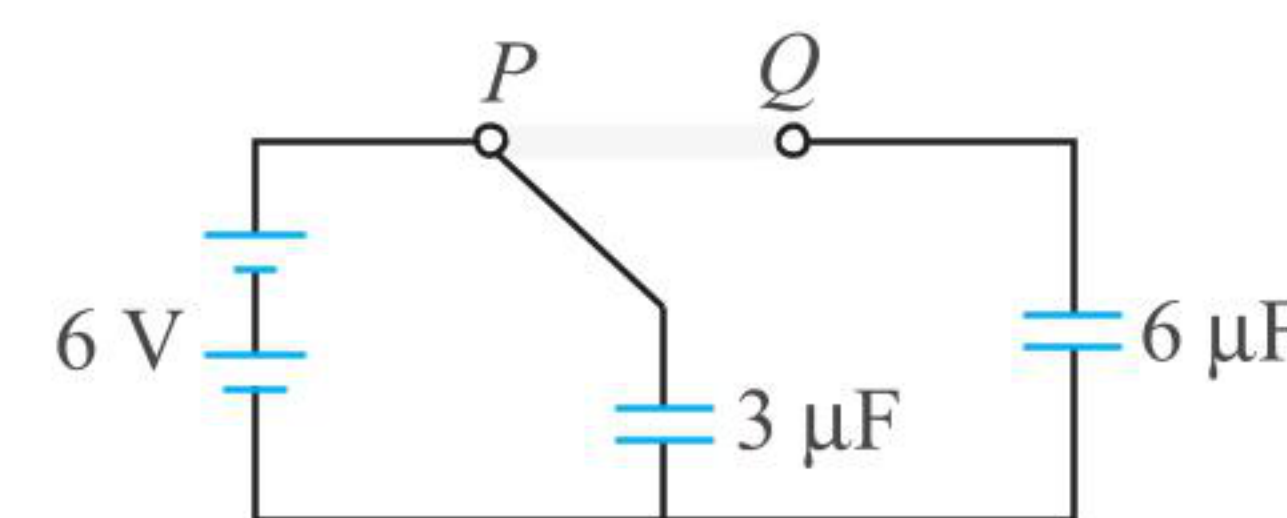
52. Figure shows two identical parallel plate capacitors connected to a battery through a switch S . Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.

- (1) $9 : 16$ (2) $5 : 9$
 (3) $2 : 3$ (4) $3 : 5$



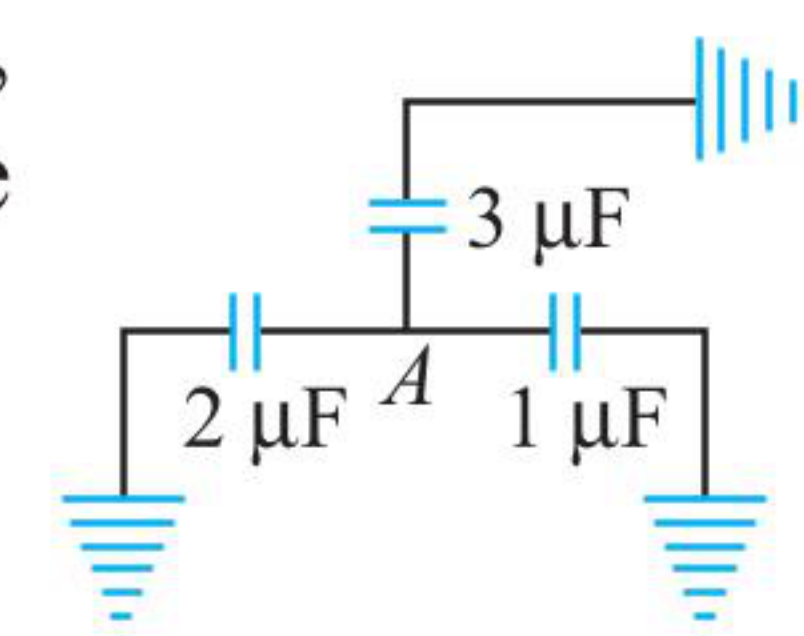
53. In the circuit shown, a capacitor of capacitance $3 \mu\text{F}$ is charged from a battery of emf 6 V with switch connected to terminal P . The switch is now connected to Q . This charges the $6 \mu\text{F}$ capacitor from the $3 \mu\text{F}$ one. What is the new potential difference across combination?

- (1) 1 V (2) 2 V
 (3) 4 V (4) 6 V

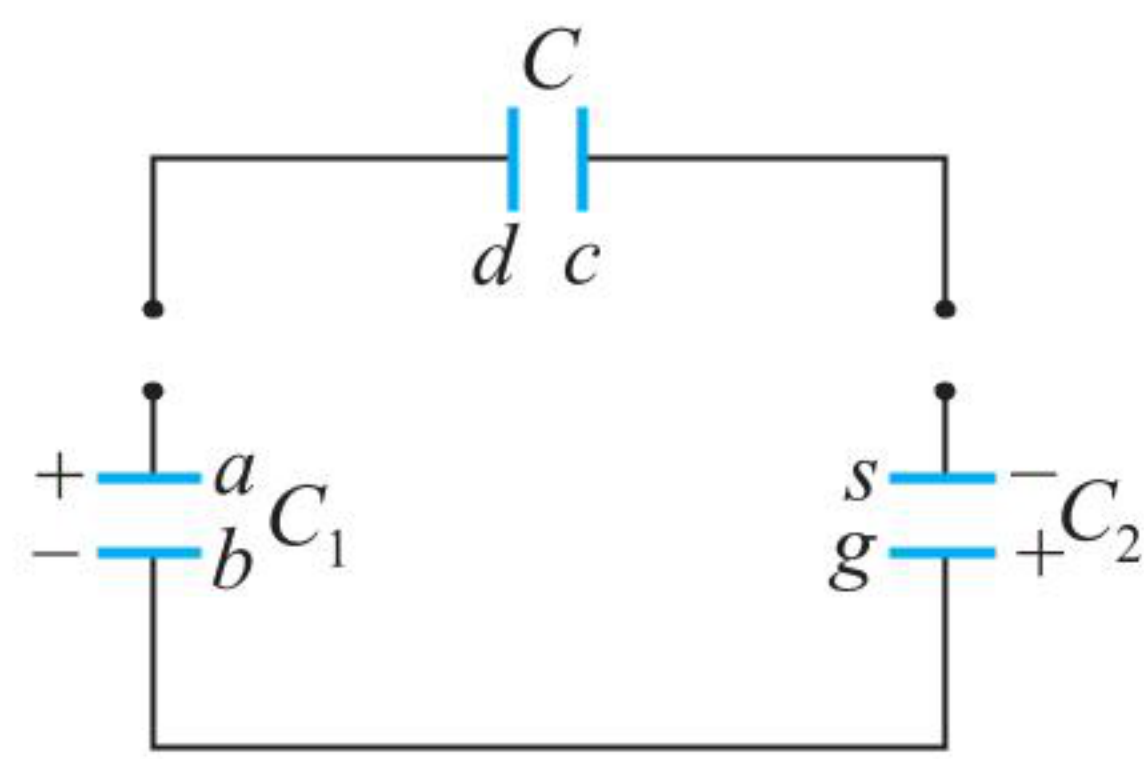


54. In the given arrangement of capacitors, $6 \mu\text{C}$ charge is added to point A . Find the charge on upper capacitor.

- (1) $3 \mu\text{C}$ (2) $1 \mu\text{C}$
 (3) $2 \mu\text{C}$ (4) $6 \mu\text{C}$

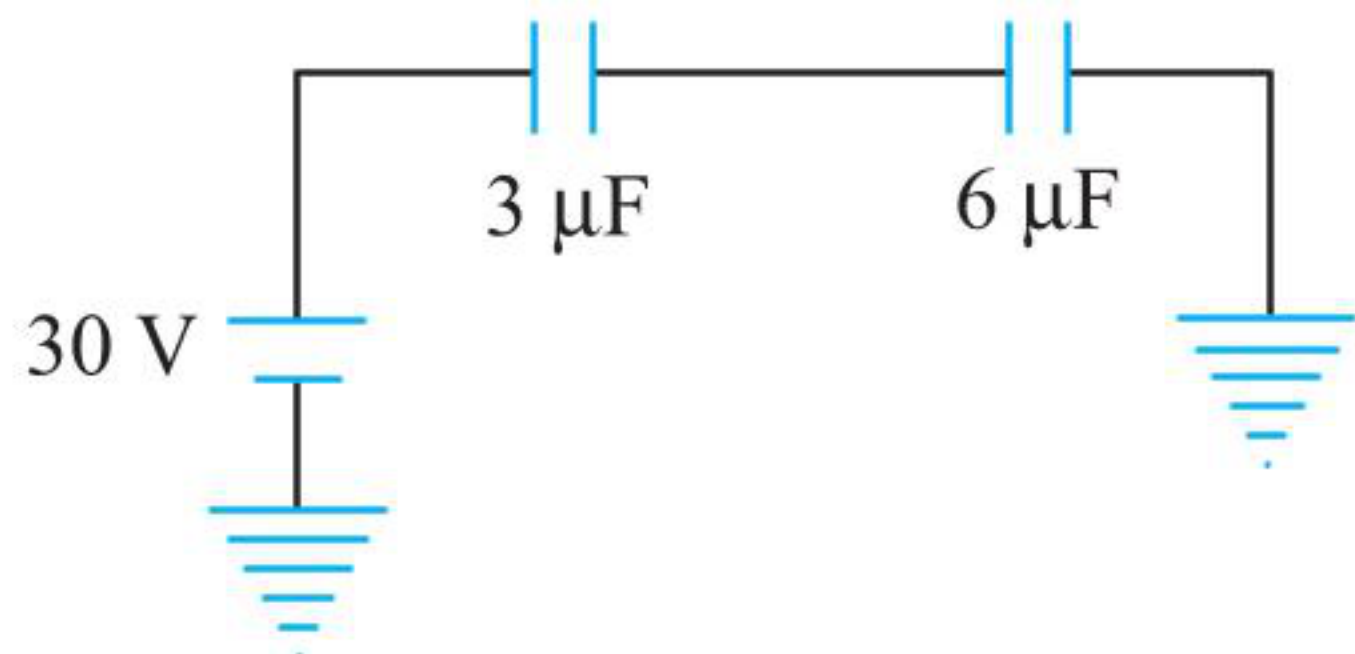


55. Two capacitors A and B with capacities C_1 and C_2 are charged to potential difference of V_1 and V_2 , respectively. The plates of the capacitors are connected as shown in the figure. The upper plate of A is positive and that of B is negative. An uncharged capacitor of capacitance C and falls on the free ends to complete circuit, then



- (1) the final charge on each capacitor are same to each other
 (2) the final sum of charge on plates a and d is $C_1 V_1$
 (3) the final sum of charge on plates b and g is $C_2 V_2 - C_1 V_1$
 (4) both (2) and (3) are correct

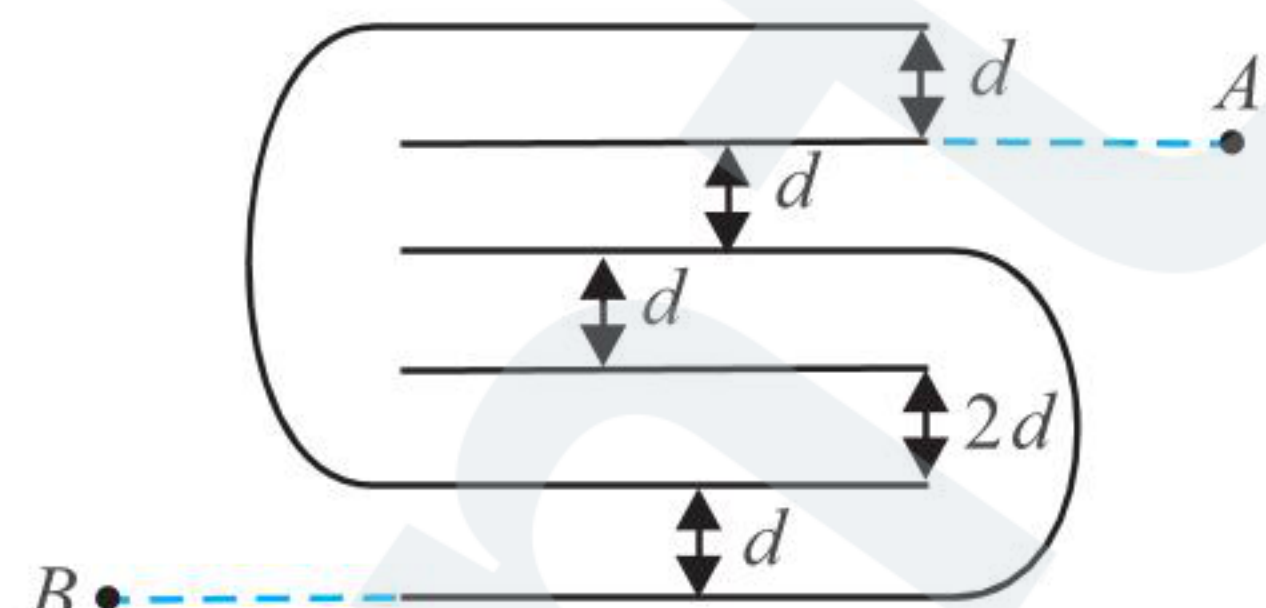
56. In the circuit shown in figure, the charge stored on a capacitor of capacitance $3 \mu\text{F}$ is



- (1) zero (2) $40 \mu\text{C}$
 (3) $60 \mu\text{C}$ (4) $90 \mu\text{C}$
57. A parallel plate capacitor is connected to a battery. The plates are pulled apart with uniform speed. If x is the separation between the plates, then the rate of change of electrostatic energy of the capacitor is proportional to

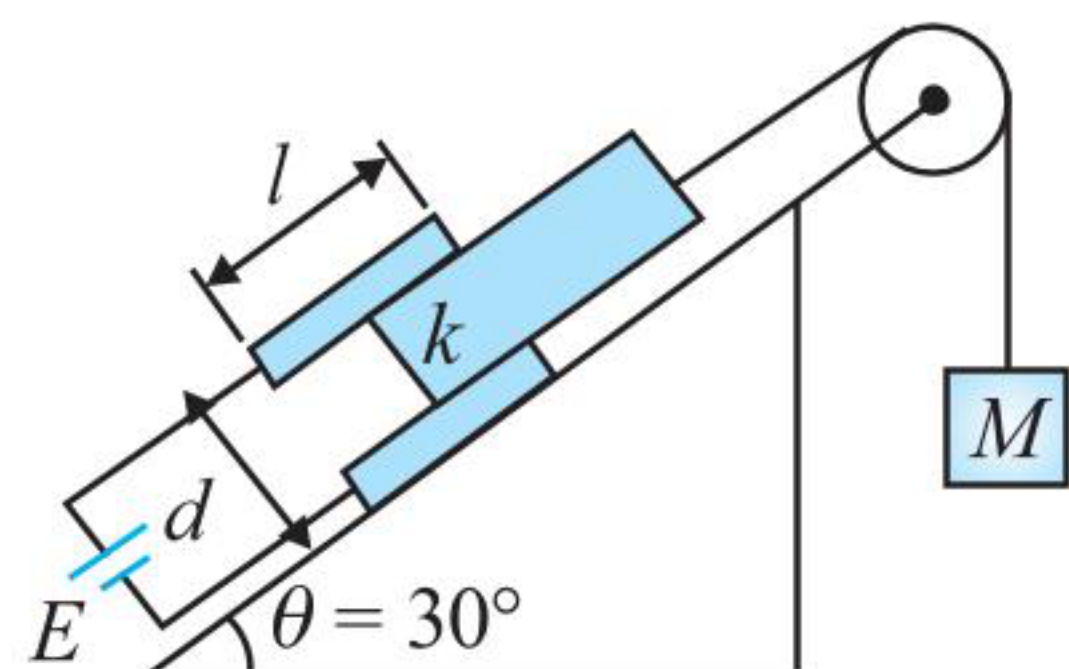
- (1) x^2 (2) x
 (3) $\frac{1}{x}$ (4) $\frac{1}{x^2}$

58. Find the equivalent capacitance between A and B . [Assume each conducting plate is having same dimensions and neglect the thickness of the plate, $\epsilon_0 A/d = 7 \mu\text{F}$, where A is area of plates and $A \gg d$].



- (1) $7 \mu\text{F}$ (2) $11 \mu\text{F}$
 (3) $12 \mu\text{F}$ (4) $15 \mu\text{F}$
59. If the plates of a parallel plate capacitor are not equal in area, then
- (1) quantity of charge on the plates will be the same, but nature of charge will differ
 (2) quantity of charge as well as nature of charge on the plates will be different
 (3) quantity of charge on the plates will be different, but nature of charge will be the same
 (4) quantity of charge as well as nature of charge will be the same

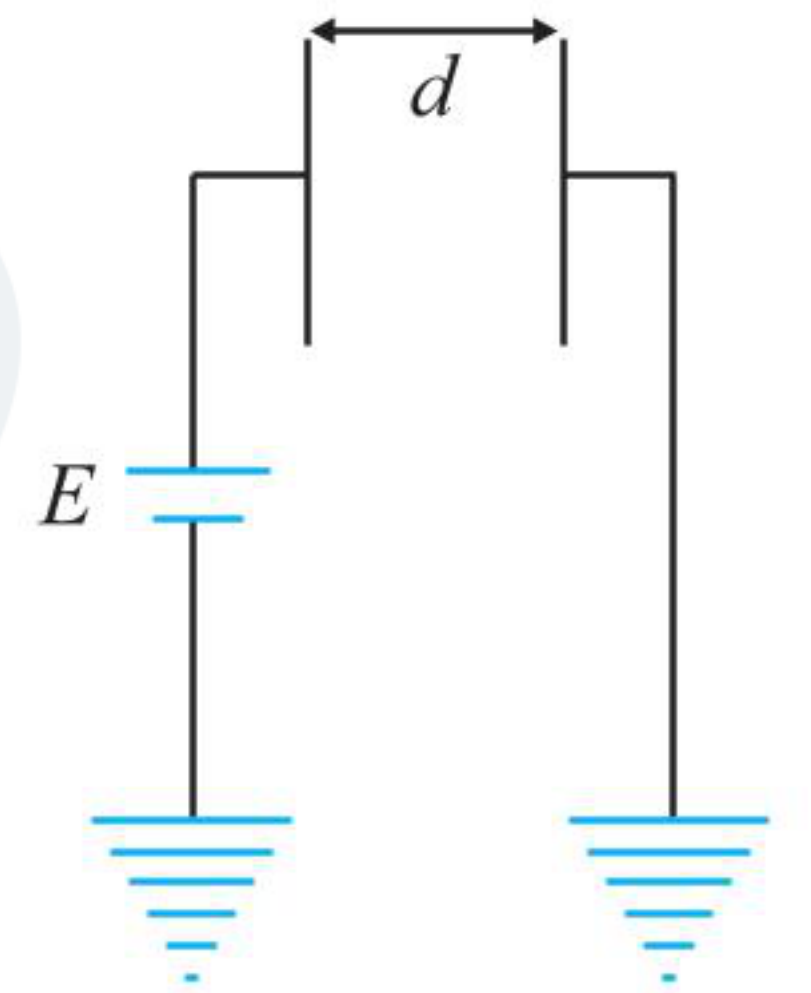
60. The capacitor plates are fixed on an inclined plane and connected to a battery of emf E . The capacitor plates have plate area a , length l , and the distance between them is d . A dielectric slab of mass m and dielectric constant K is inserted into the capacitor and tied to mass M by a massless string as shown in the figure. Find the



value of M for which the slab will stay in equilibrium. There is no friction between slab and plates.

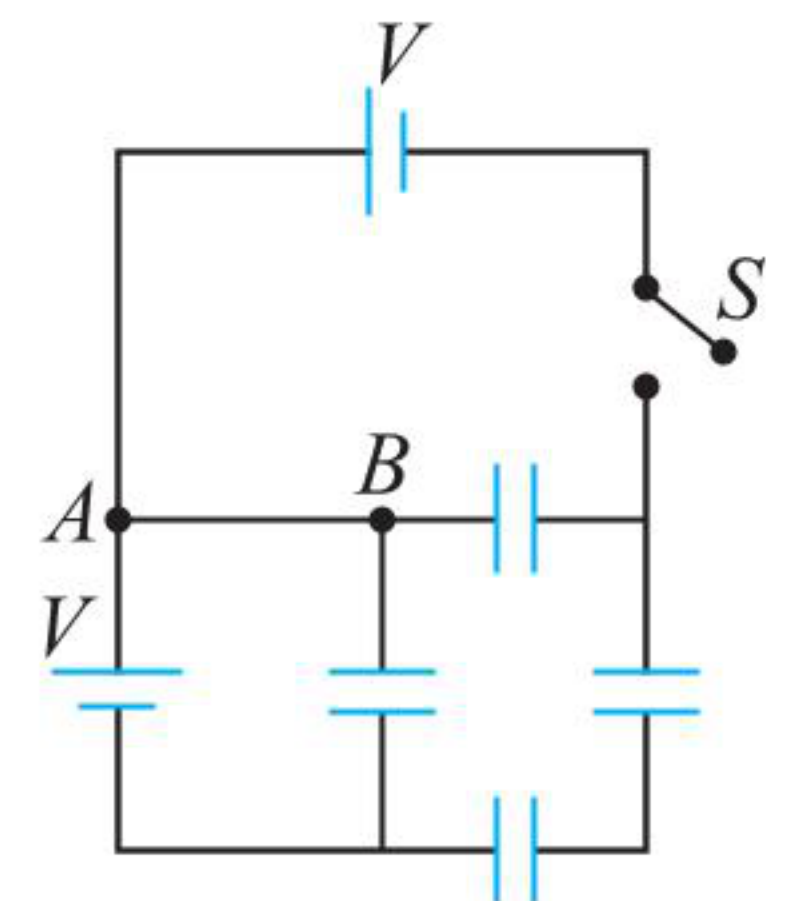
- (1) $\frac{m}{2} + \frac{E^2 \epsilon_0 A(k-1)}{2lgd}$ (2) $\frac{m}{2} - \frac{E^2 \epsilon_0 A(k-1)}{2lgd}$
 (3) $\frac{m}{2} + \frac{E^2 \epsilon_0 A(k-1)}{lgd}$ (4) $\frac{m}{2} - \frac{E^2 \epsilon_0 A(k-1)}{lgd}$

61. Two plates, each of area A , are placed parallel to each other at a distance d . One plate is connected to a battery of emf E and its negative is earthed. The other plate is also earthed. The charge drawn by plate is



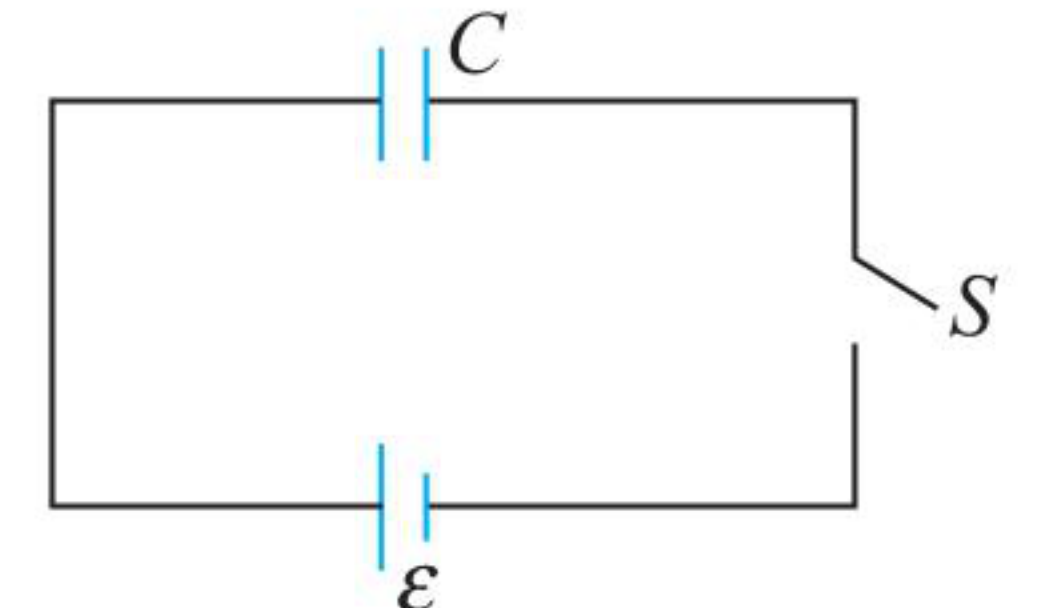
- (1) $\frac{2\epsilon_0 AE}{d}$ (2) $\frac{\epsilon_0 AE}{d}$
 (3) $\frac{\epsilon_0 AE}{2d}$ (4) $\frac{3\epsilon_0 AE}{d}$

62. Find the charge that will flow through the wire A to B if the switch S is closed. The capacitance of each capacitor shown in the figure is C .



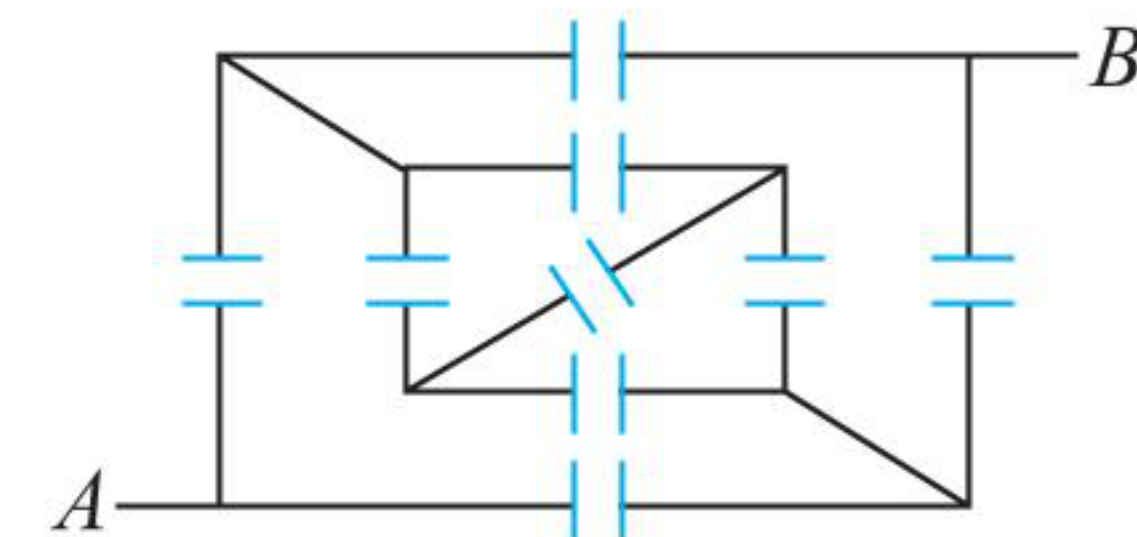
- (1) CV (2) $CV/3$
 (3) zero (4) $2CV/3$

63. The left plate of the capacitor shown in the figure carries a charge $+Q$ while the right plate is uncharged. The total final charge on the right plate after closing the switch S will be



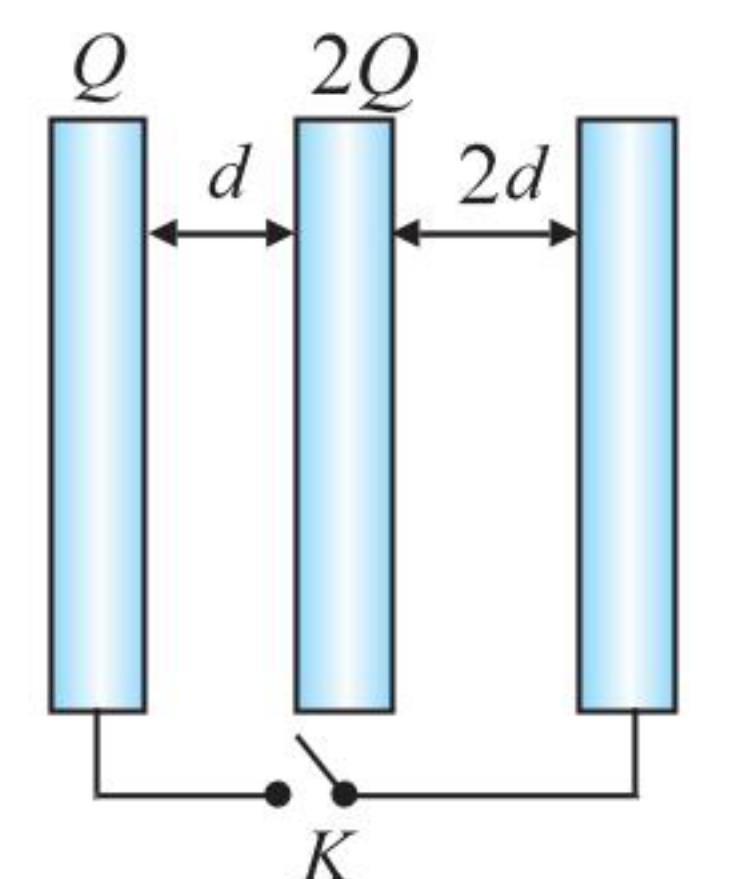
- (1) $\frac{Q}{2} + C\epsilon$ (2) $\frac{Q}{2} - C\epsilon$
 (3) $-\frac{Q}{2}$ (4) none of these

64. The effective capacitance between points A and B is (the capacitance of each of the capacitors is C)



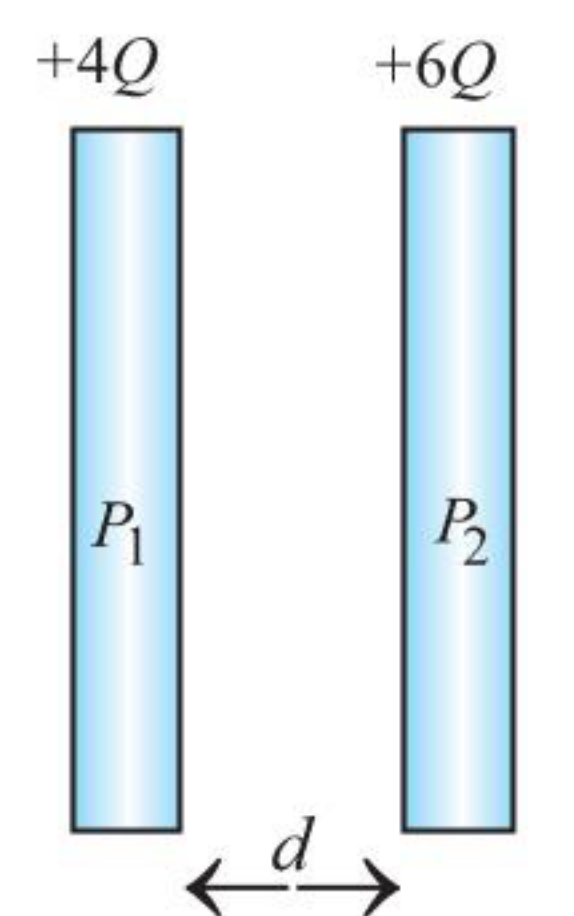
- (1) C (2) $C/2$
 (3) $36C/17$ (4) $42C/17$

65. Three large plates are arranged as shown. How much charge will flow through the key k if it is closed?



- (1) $5Q/6$ (2) $4Q/3$
 (3) $3Q/2$ (4) none

66. Two identical conducting very large plates P_1 and P_2 having charges $+4Q$ and $+6Q$ are placed very close to each other at separation d . The plate area of either face of the plate is A . The potential difference between plates P_1 and P_2 is



$$(1) V_{P_1} - V_{P_2} = \frac{Qd}{A\epsilon_0} \quad (2) V_{P_1} - V_{P_2} = \frac{-Qd}{A\epsilon_0}$$

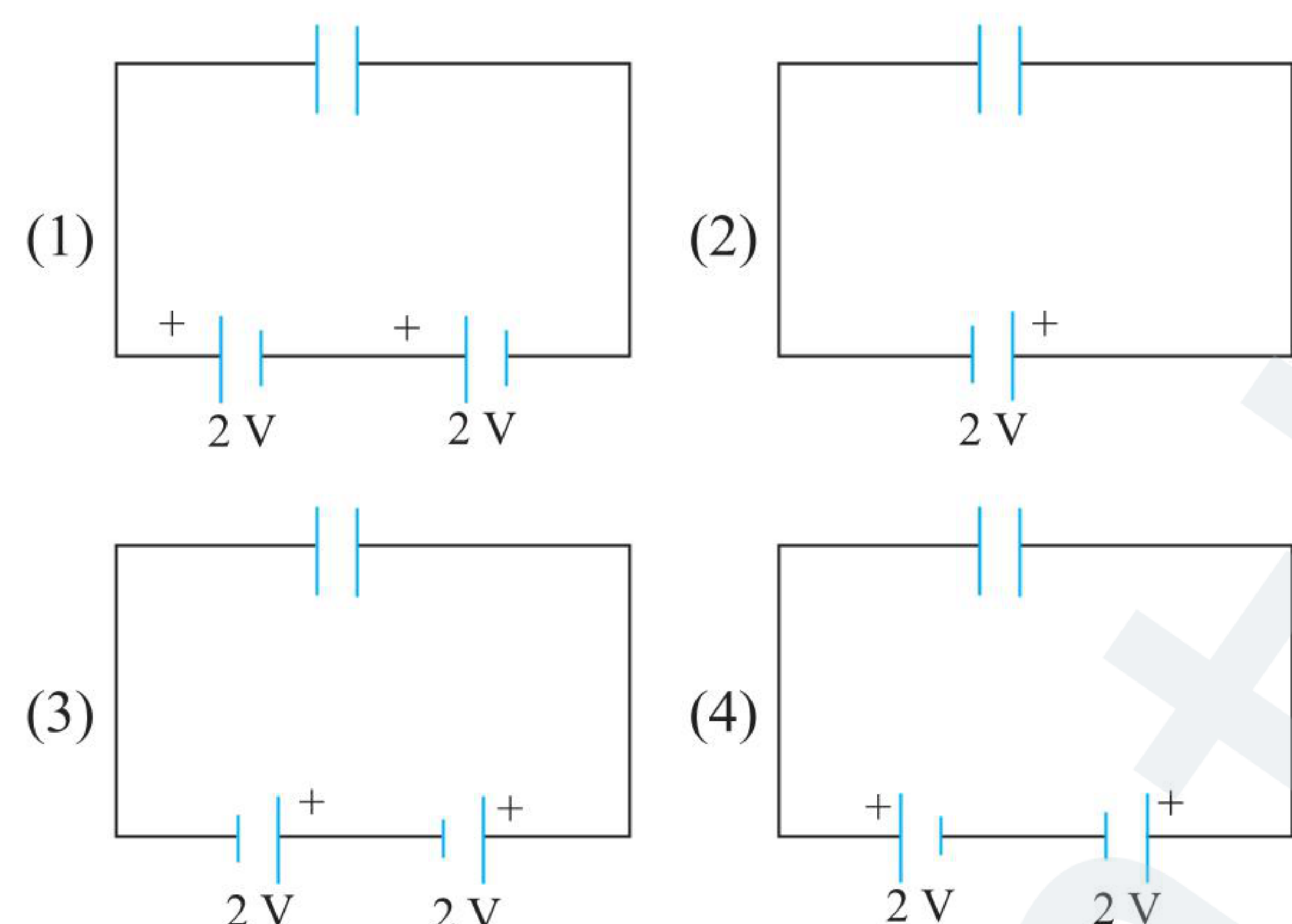
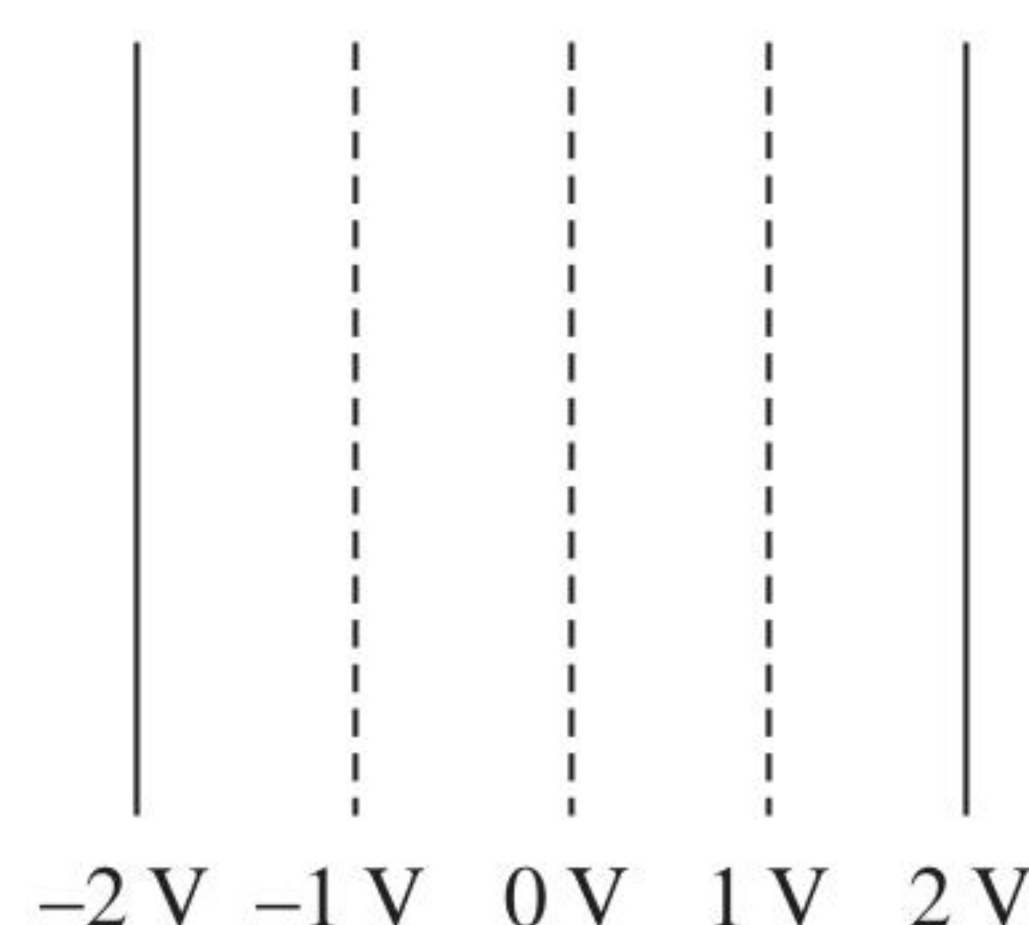
$$(3) V_{P_1} - V_{P_2} = \frac{5Qd}{A\epsilon_0} \quad (4) V_{P_1} - V_{P_2} = \frac{-5Qd}{A\epsilon_0}$$

67. In the above question, if plates P_1 and P_2 are connected by a thin conducting wire, then the amount of heat produced will be

$$(1) \frac{Q^2}{A\epsilon_0} d \quad (2) \frac{5Q^2}{A\epsilon_0} d$$

$$(3) \frac{2Q^2}{A\epsilon_0} d \quad (4) \text{none of these}$$

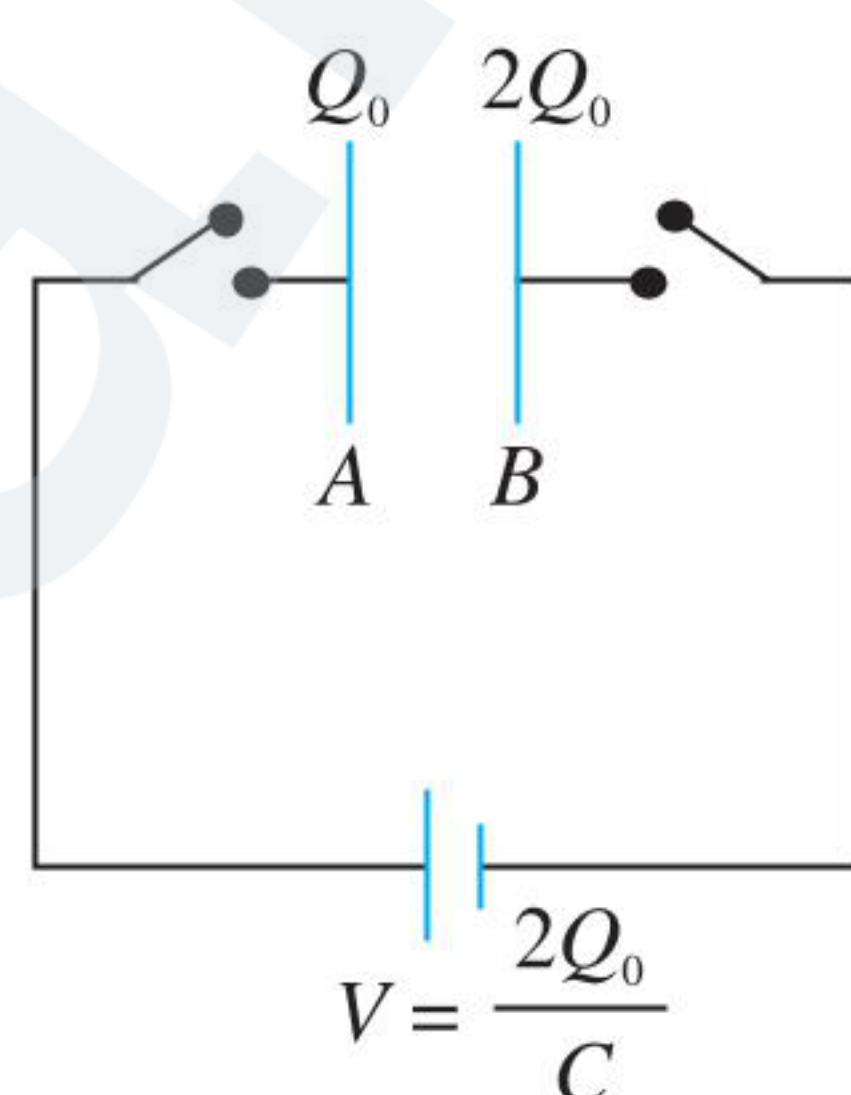
68. A battery (or batteries) connected to two parallel plates produces the equipotential lines between the plates as shown. Which of the following configurations is most likely to produce these equipotential lines?



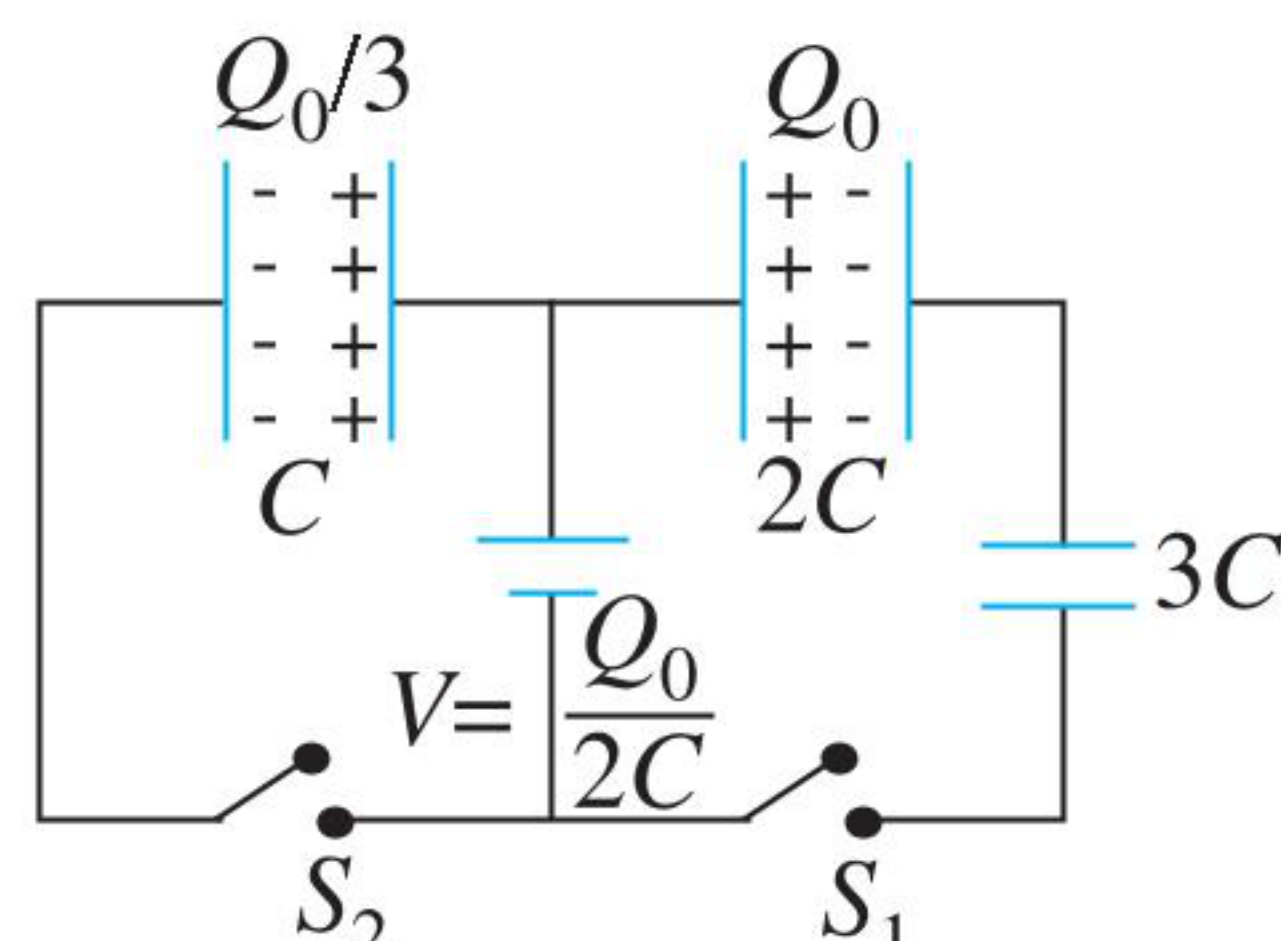
69. Charges Q_0 and $2Q_0$ are given to parallel plates A and B , respectively, and they are separated by a small distance. The capacitance of the given arrangement is C . Now, plates A and B are connected to positive and negative terminals of battery of potential difference $V = 2Q_0/C$ respectively, as shown, then the work done by the battery is

$$(1) \frac{2Q_0^2}{C} \quad (2) \frac{4Q_0^2}{C}$$

$$(3) \frac{5Q_0^2}{C} \quad (4) \frac{6Q_0^2}{C}$$



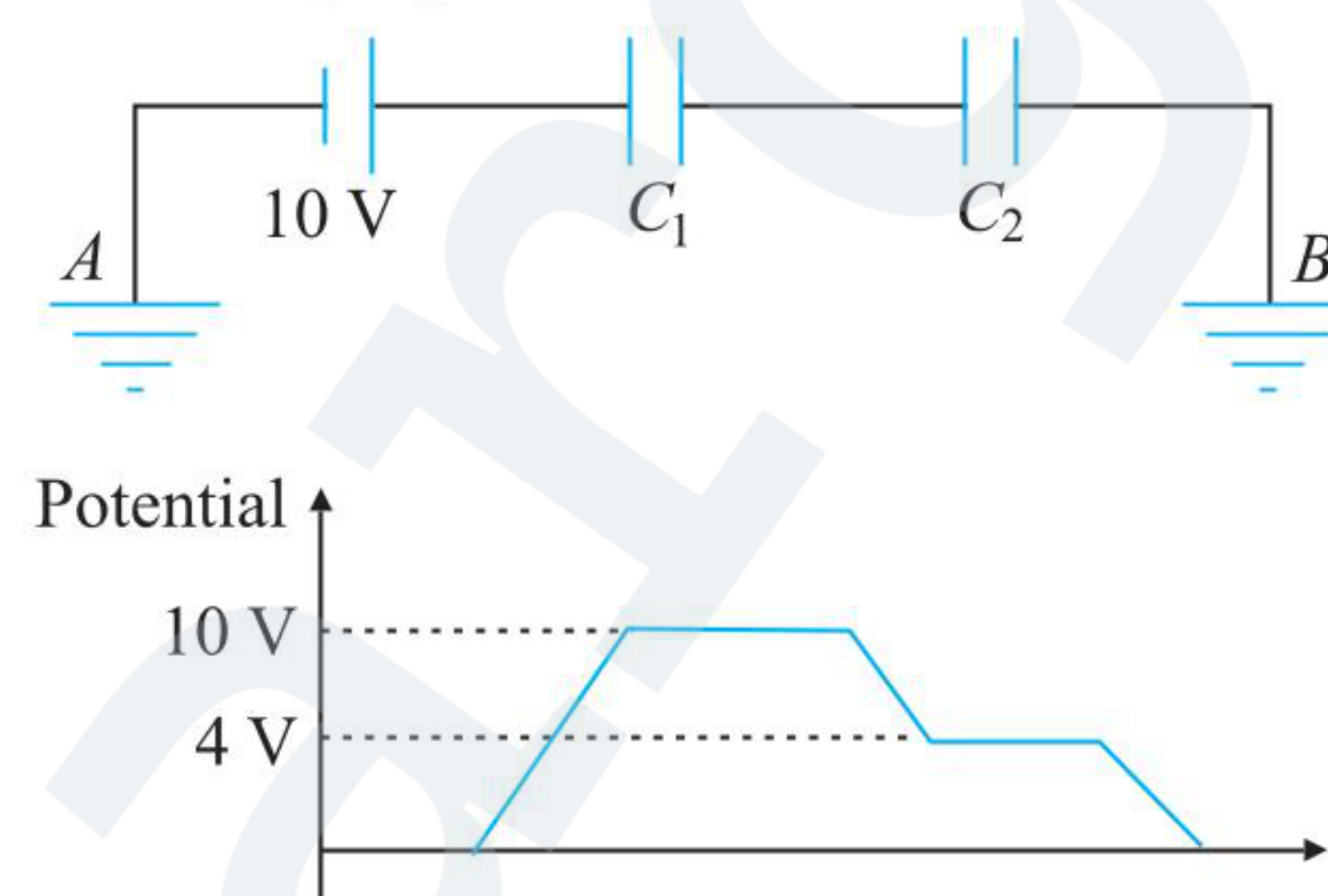
70. In the given circuit, the initial charges on the capacitors are shown in the figure. The charge flown through the switches S_1 and S_2 , respectively after closing the switches are



$$(1) \text{zero}, \frac{Q_0}{6} \quad (2) \frac{Q_0}{5}, \frac{Q_0}{2}$$

$$(3) \text{zero}, \frac{Q_0}{2} \quad (4) \frac{3}{5}Q_0, \frac{Q_0}{6}$$

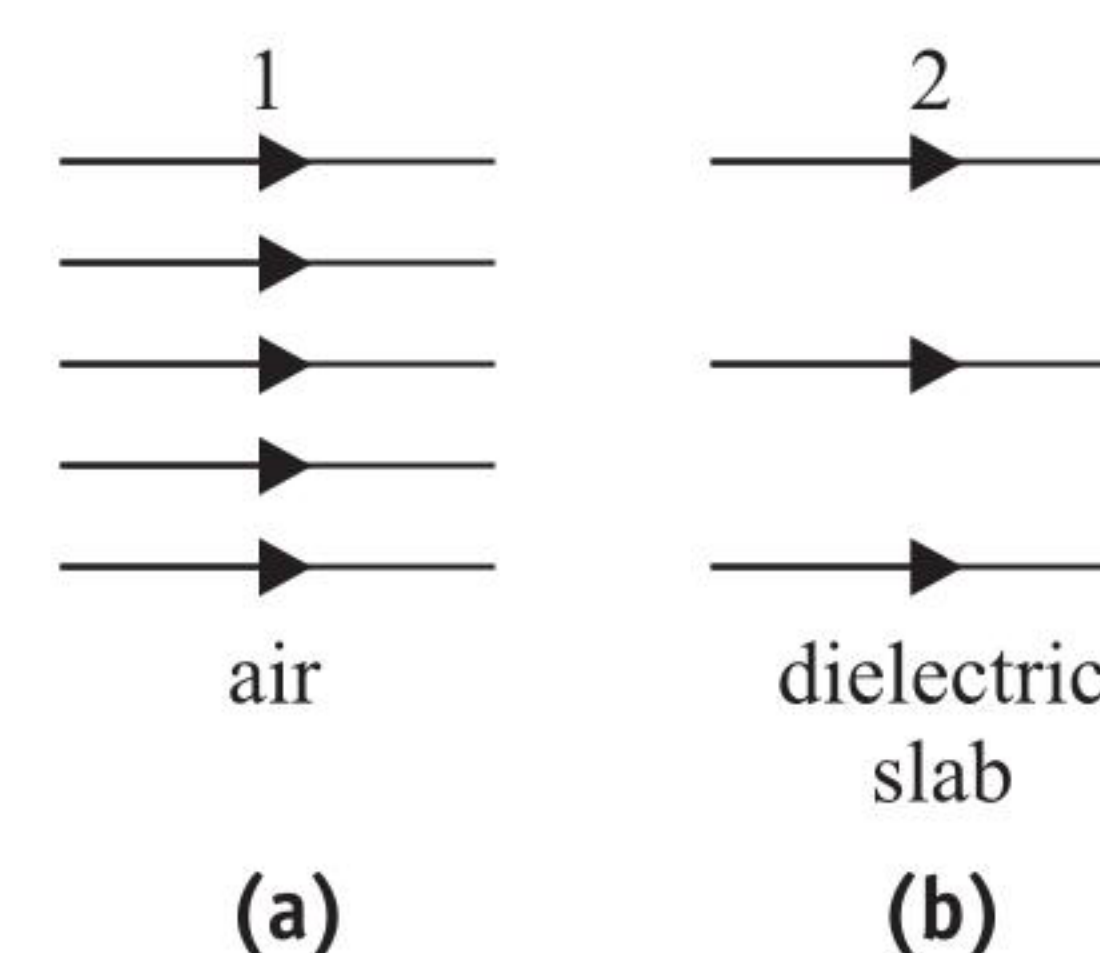
71. Figure shows two capacitors C_1 and C_2 connected with 10 V battery and terminal A and B are earthed. The graph shows the variation of potential as one moves from left to right. Then the ratio C_1/C_2 is



$$(1) 5/2 \quad (2) 2/3$$

$$(3) 2/5 \quad (4) 4/3$$

72. In figure, there are two patterns of electric field lines absent of dielectric [Fig. (a)] and in presence of dielectric slab [Fig. (b)]. The relative permittivity of the dielectric slab is



$$(1) \frac{5}{3} \quad (2) \frac{5}{3} \times \epsilon_0$$

$$(3) 4 \quad (4) \frac{5}{2}$$

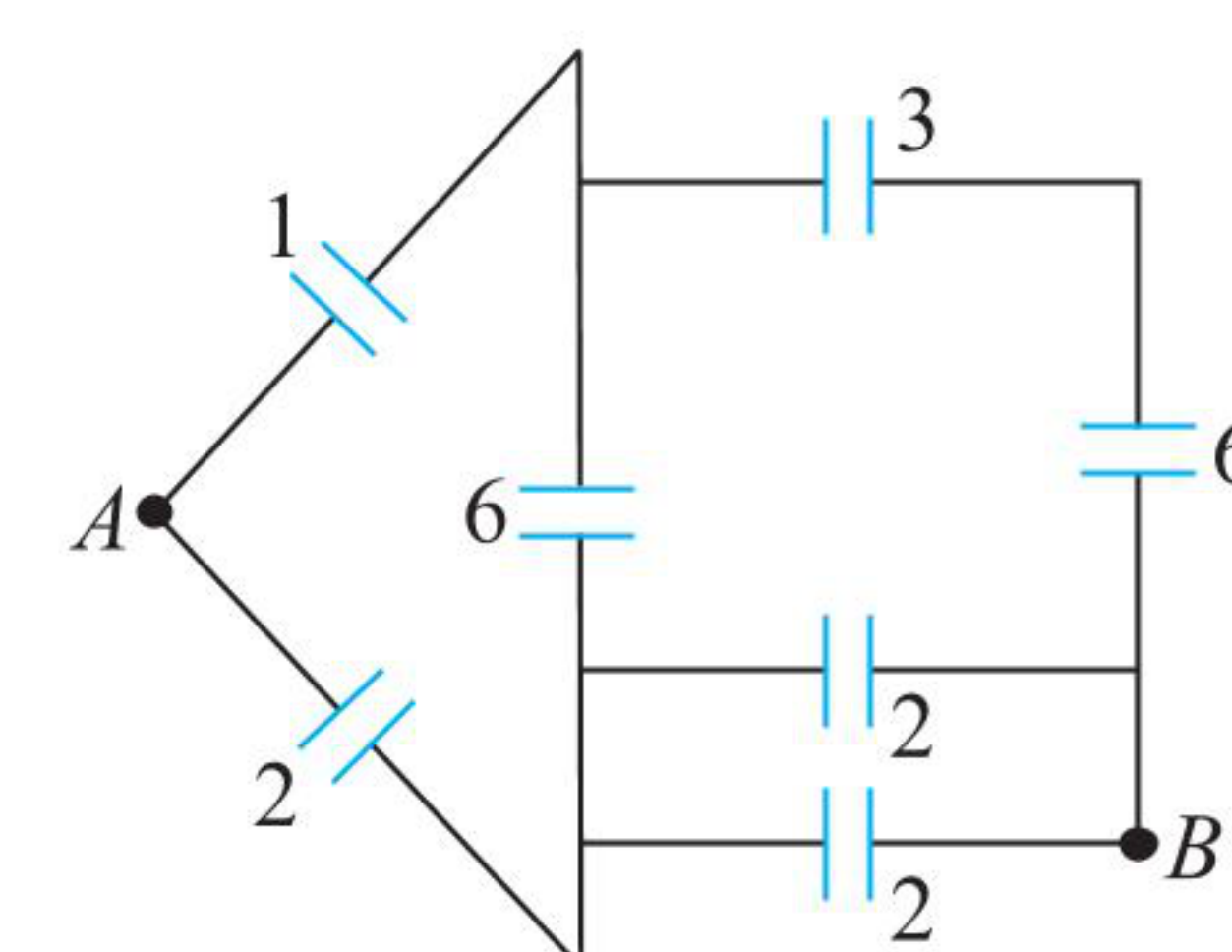
73. All the capacitances shown in figure are in μF . Find the equivalent capacitance between A and B .

$$(1) 2 \mu\text{F}$$

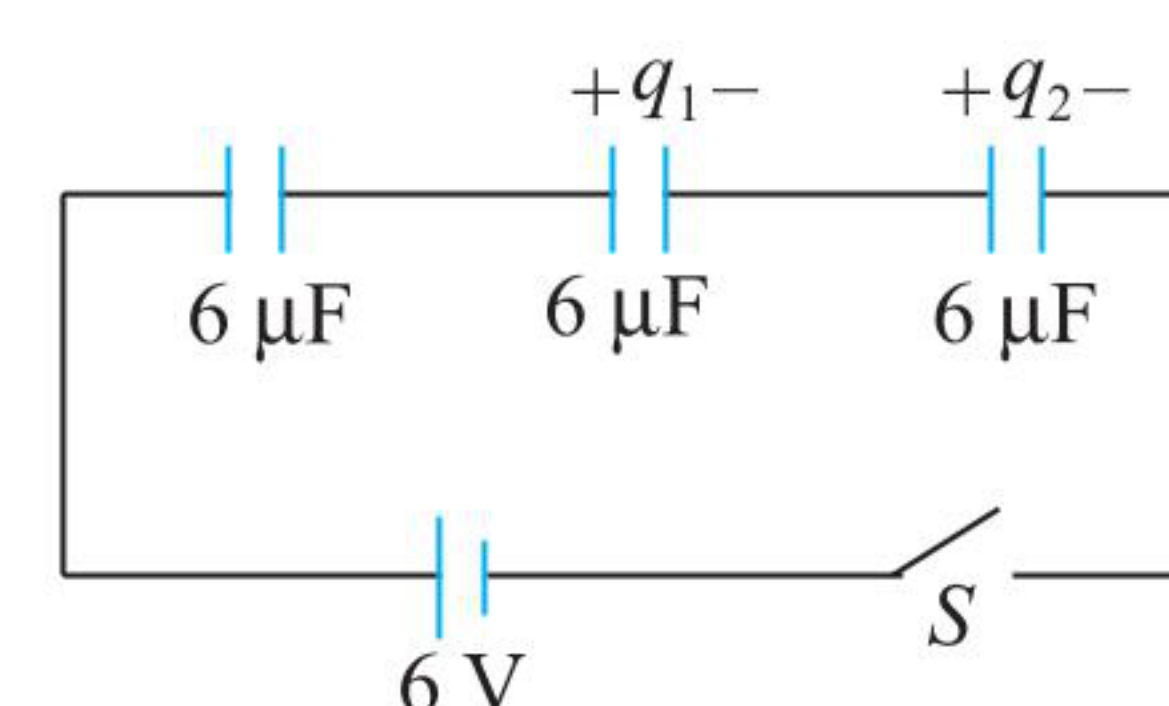
$$(2) 3 \mu\text{F}$$

$$(3) 6 \mu\text{F}$$

$$(4) 9 \mu\text{F}$$



74. The left most capacitor is uncharged. The other two capacitors carry charges $q_1 = 8 \mu\text{C}$ and $q_2 = 10 \mu\text{C}$. Now switch S is closed. Find the final charge on left most capacitor.



$$(1) 3 \mu\text{C} \quad (2) 6 \mu\text{C}$$

$$(3) 12 \mu\text{C} \quad (4) 18 \mu\text{C}$$

75. In the figure shown, all the capacitors are initially uncharged.

Case I: only switch S_1 is closed

Case II: only switch S_2 is closed

Case III: both switches are closed

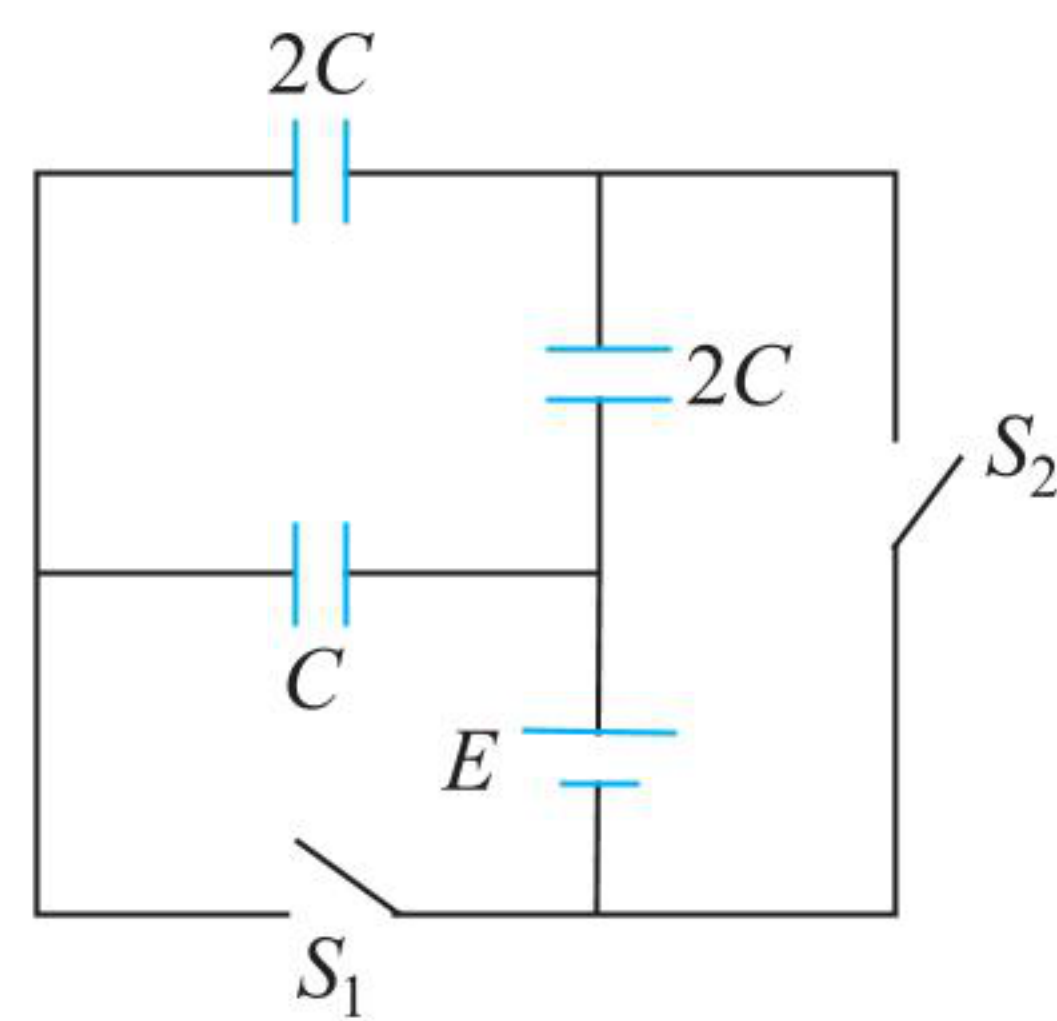
Find the ratio of the total energy stored in the system of capacitors for the cases I, II, and III, respectively.

(1) 1 : 2 : 3

(2) 5 : 7 : 9

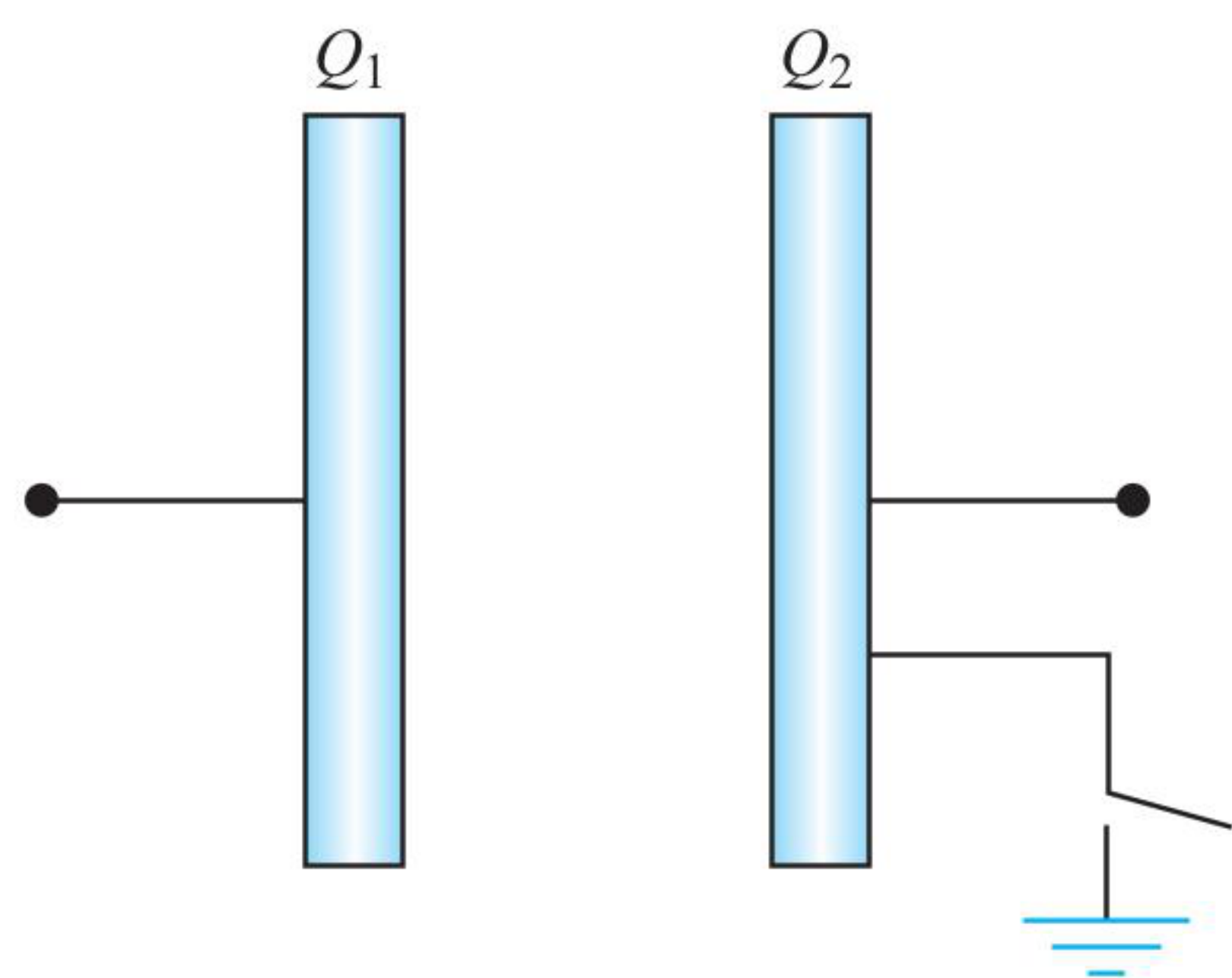
(3) 6 : 8 : 9

(4) 9 : 7 : 6



Multiple Correct Answers Type

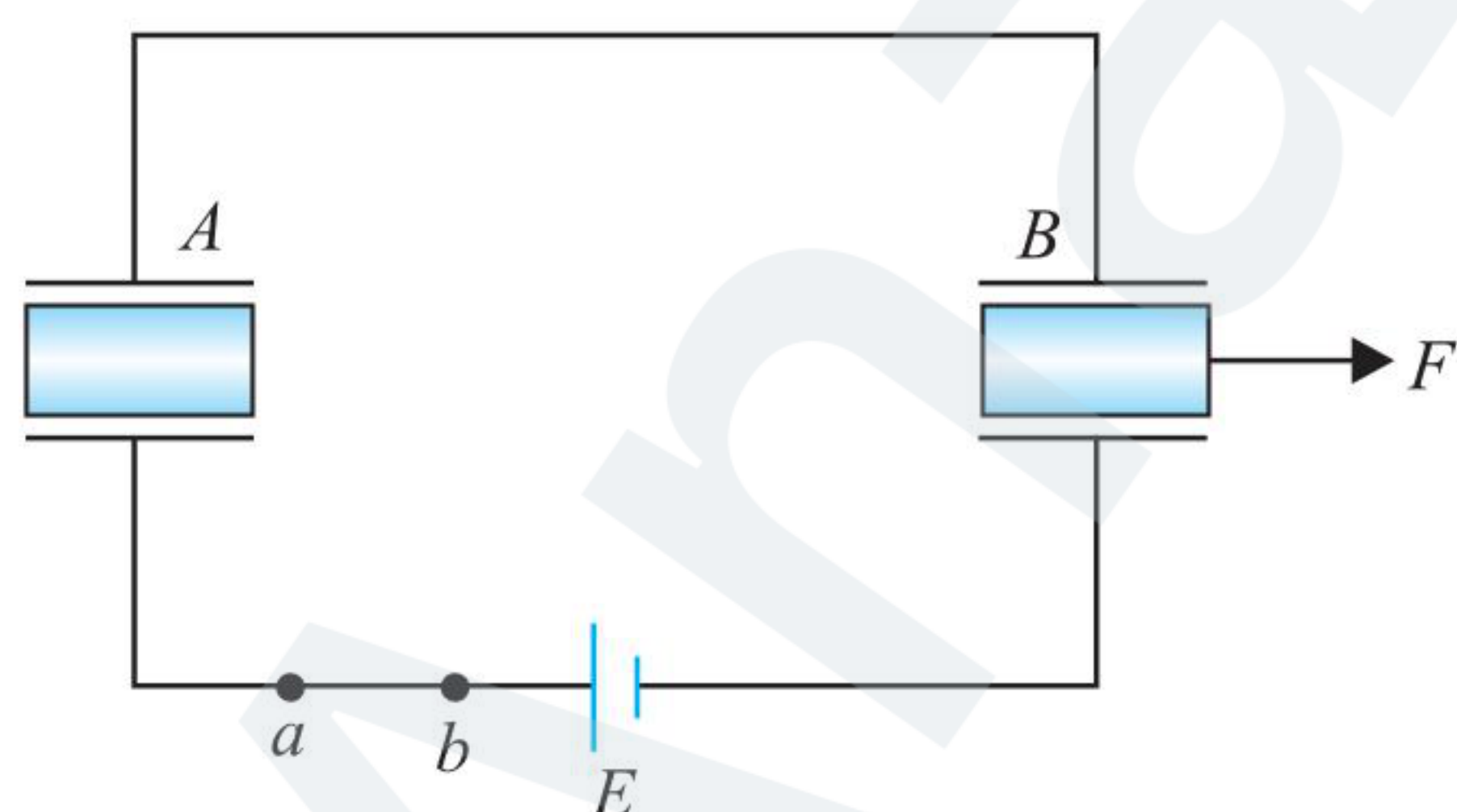
1. Charges Q_1 and Q_2 are given to two plates of a parallel plate capacitor. The capacity of the capacitor is C . When the switch is closed, mark the correct statement(s). (Assume both Q_1 and Q_2 to be positive.)



- (1) The charge flowing through the switch is zero.
- (2) The charge flowing through the switch is $Q_1 + Q_2$.
- (3) Potential difference across the capacitor plate is Q_1/C .
- (4) The charge of the capacitor is Q_1 .

2. Two identical capacitors with identical dielectric slabs in between them are connected in series as shown in figure. Now, the slab of one capacitor is pulled out slowly with the help of an external force F at steady state as shown.

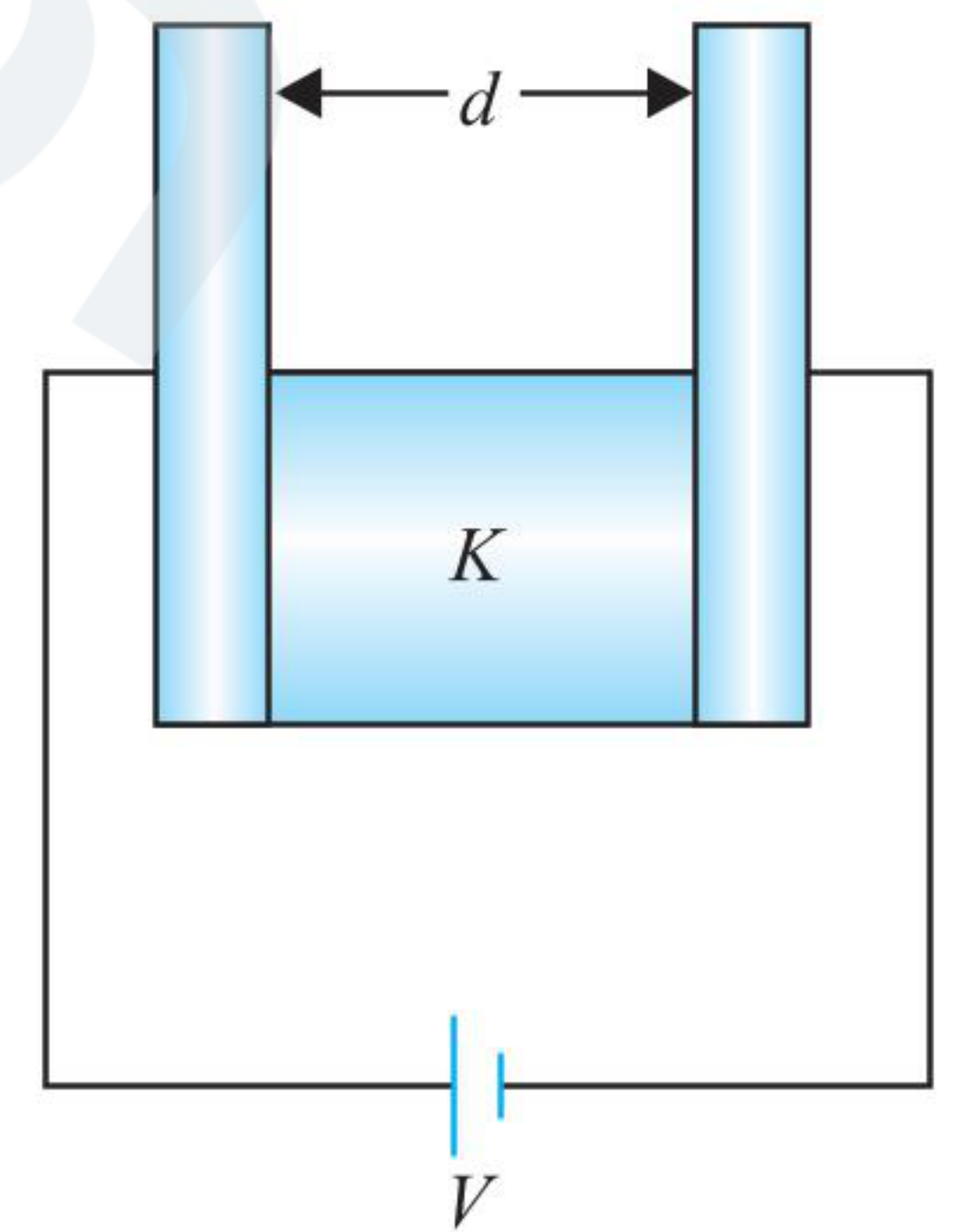
Mark the correct statement(s).



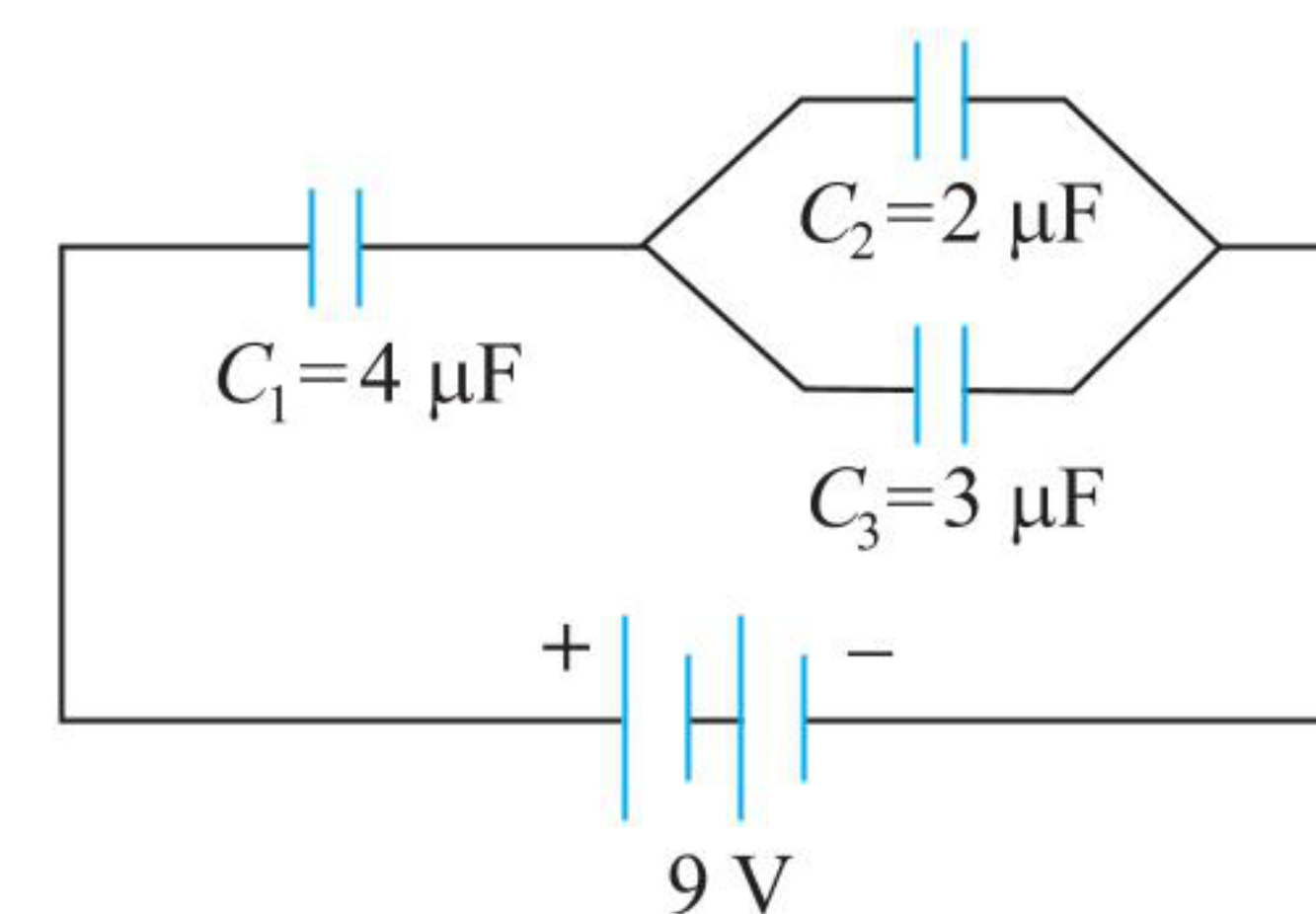
- (1) During the process, charge (positive) flows from b to a .
 - (2) During the process, the charge of capacitor B is equal to the charge on A at all instants.
 - (3) Work done by F is positive.
 - (4) During the process, the battery has been charged.
3. A parallel plate air capacitor has initial capacitance C . If plate separation is slowly increased from d_1 to d_2 , then mark the correct statement(s). (Take potential of the capacitor to be constant, i.e., throughout the process it remains connected to battery.)

- (1) Work done by electric force = negative of work done by external agent.
- (2) Work done by external force = $-\int \vec{F} \cdot d\vec{x}$, where $\vec{F}$ is the electric force of attraction between the plates at plate separation x .
- (3) Work done by electric force $\neq$ negative of work done by external agent.
- (4) Work done by battery = two times the change in electric potential energy stored in capacitor.

4. A dielectric slab fills the lower half of a parallel plate capacitor as shown in figure. (Take plate area as A)

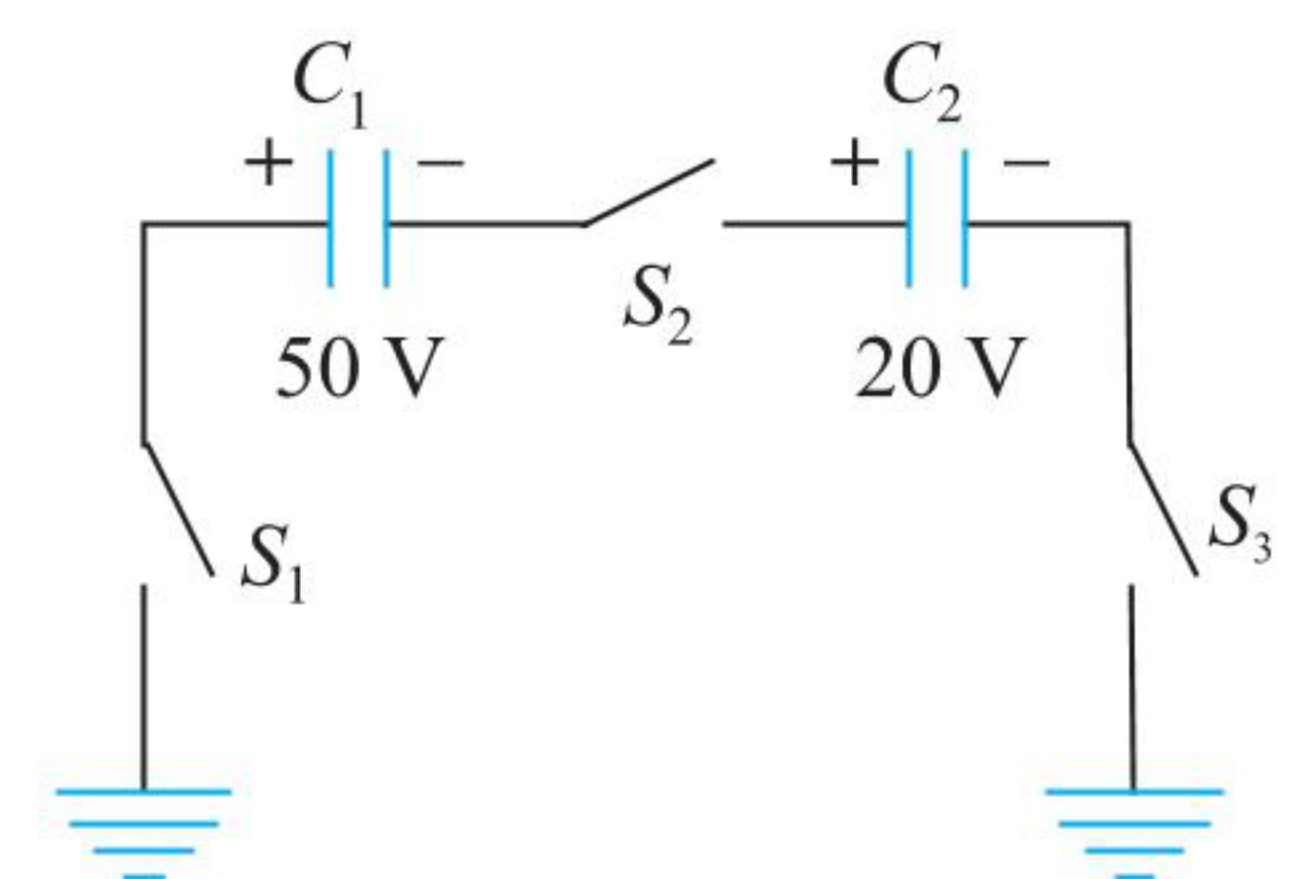


- (1) Equivalent capacity of the system is $(\epsilon_0 A/2d)(1 + K)$.
 - (2) The net charge of the lower half of the left hand plate is $1/K$ times the charge on the upper half of the plate.
 - (3) Net charges on the lower and upper halves of the left hand plate are different.
 - (4) Net charge on the lower half of the left hand plate is $\frac{K\epsilon_0 A}{2d} \times V$.
5. In figure, the charges on C_1 , C_2 , and C_3 are Q_1 , Q_2 , and Q_3 , respectively. Then,



- (1) $Q_2 = 8 \mu\text{C}$
- (2) $Q_3 = 12 \mu\text{C}$
- (3) $Q_1 = 20 \mu\text{C}$
- (4) $Q_2 = 12 \mu\text{C}$

6. In the circuit shown, $C_1 = C_2 = 2 \mu\text{F}$. Capacitor C_1 is charged to 50 V and C_2 is charged to 20 V. After charging, they are connected as shown. When S_1 , S_2 , and S_3 are closed,

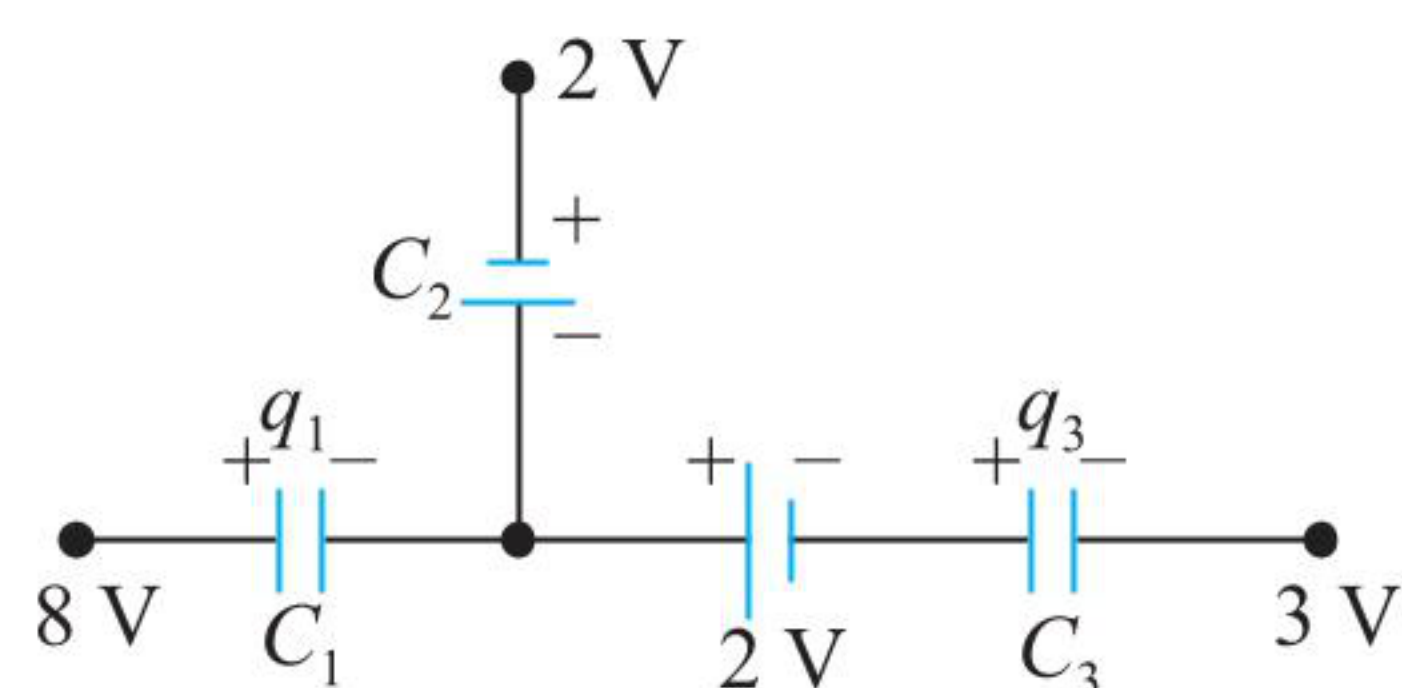


- (1) 70 μC of charge will pass through S_1
- (2) 100 μC of charge will pass through S_1
- (3) 70 μC of charge will pass through S_3
- (4) 40 μC of charge will pass through S_3

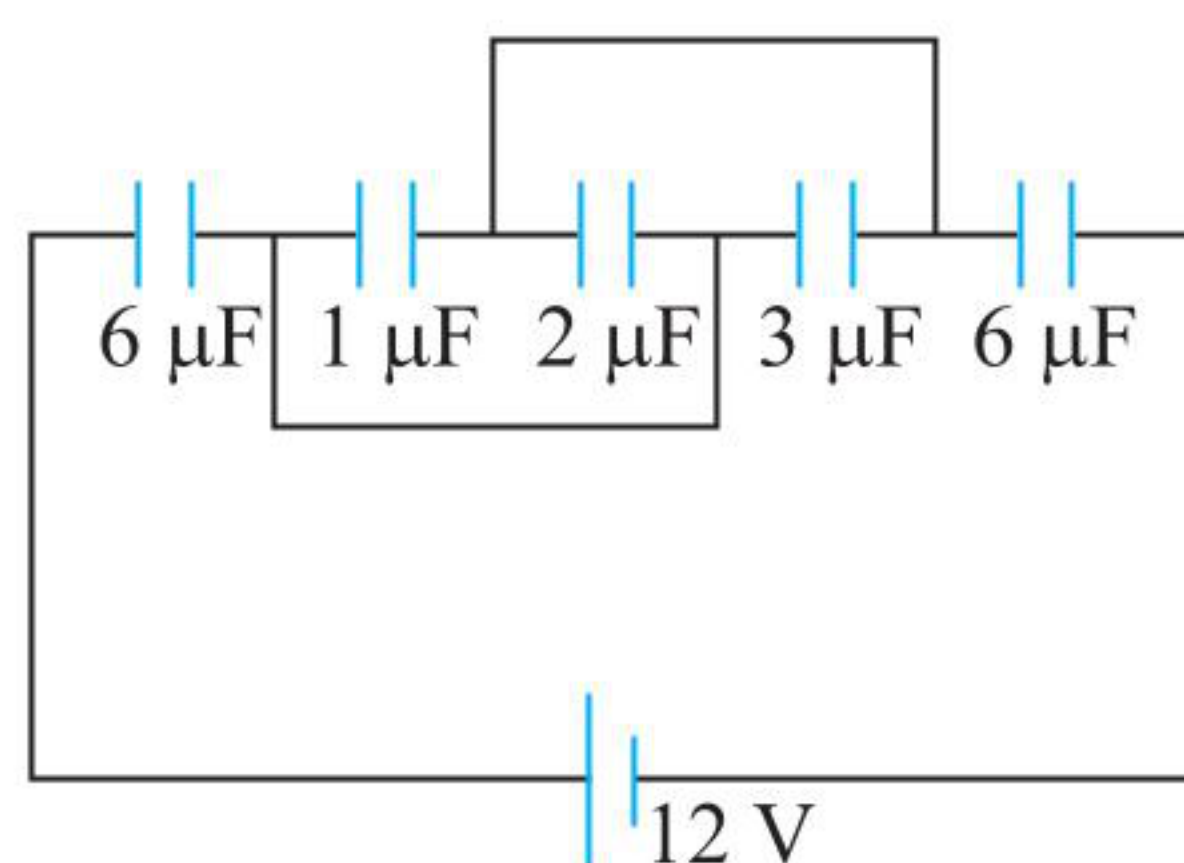
7. A capacitor of 5 μF is charged to a potential of 100 V. Now, this charged capacitor is connected to a battery of 100 V with the positive terminal of the battery connected to the negative plate of the capacitor. For the given situation, mark the correct statement(s).

- (1) The charge flowing through the 100 V battery is 500 μC .
- (2) The charge flowing through the 100 V battery is 1000 μC .
- (3) Heat dissipated in the circuit is 0.1 J.
- (4) Work done on the battery is 0.1 J.

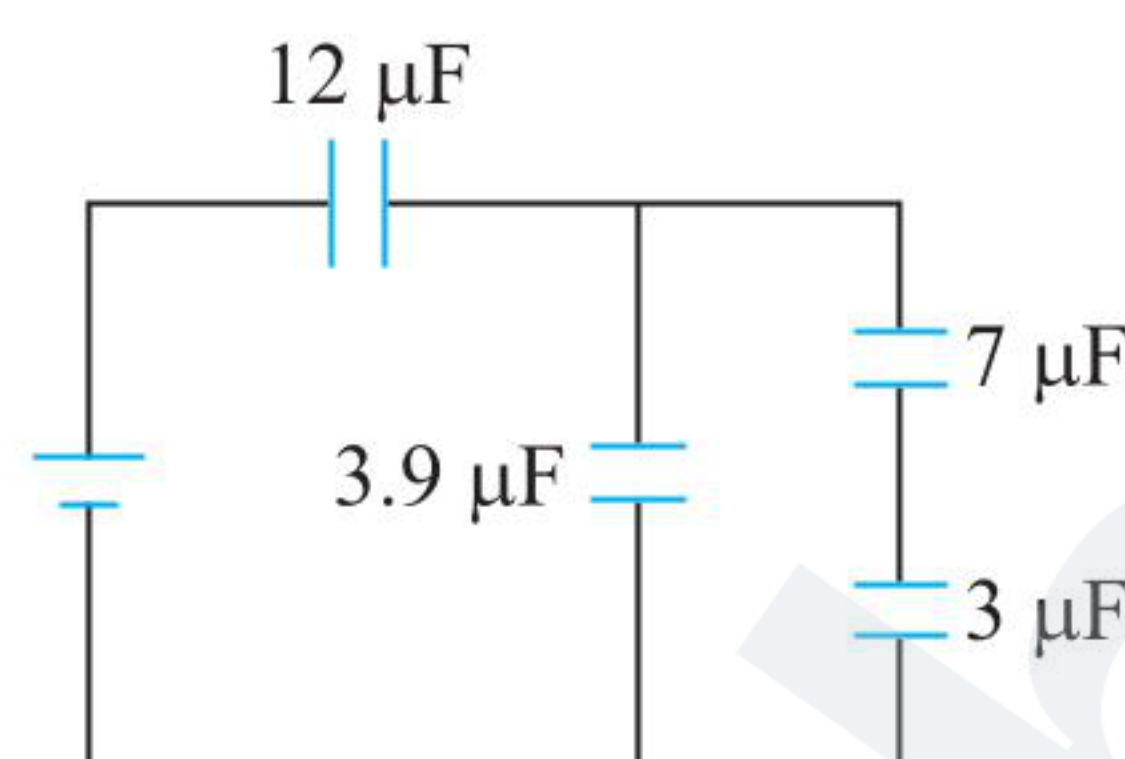
8. Figure shows a part of a circuit. If all the capacitors have a capacitance of $2 \mu\text{F}$, then the



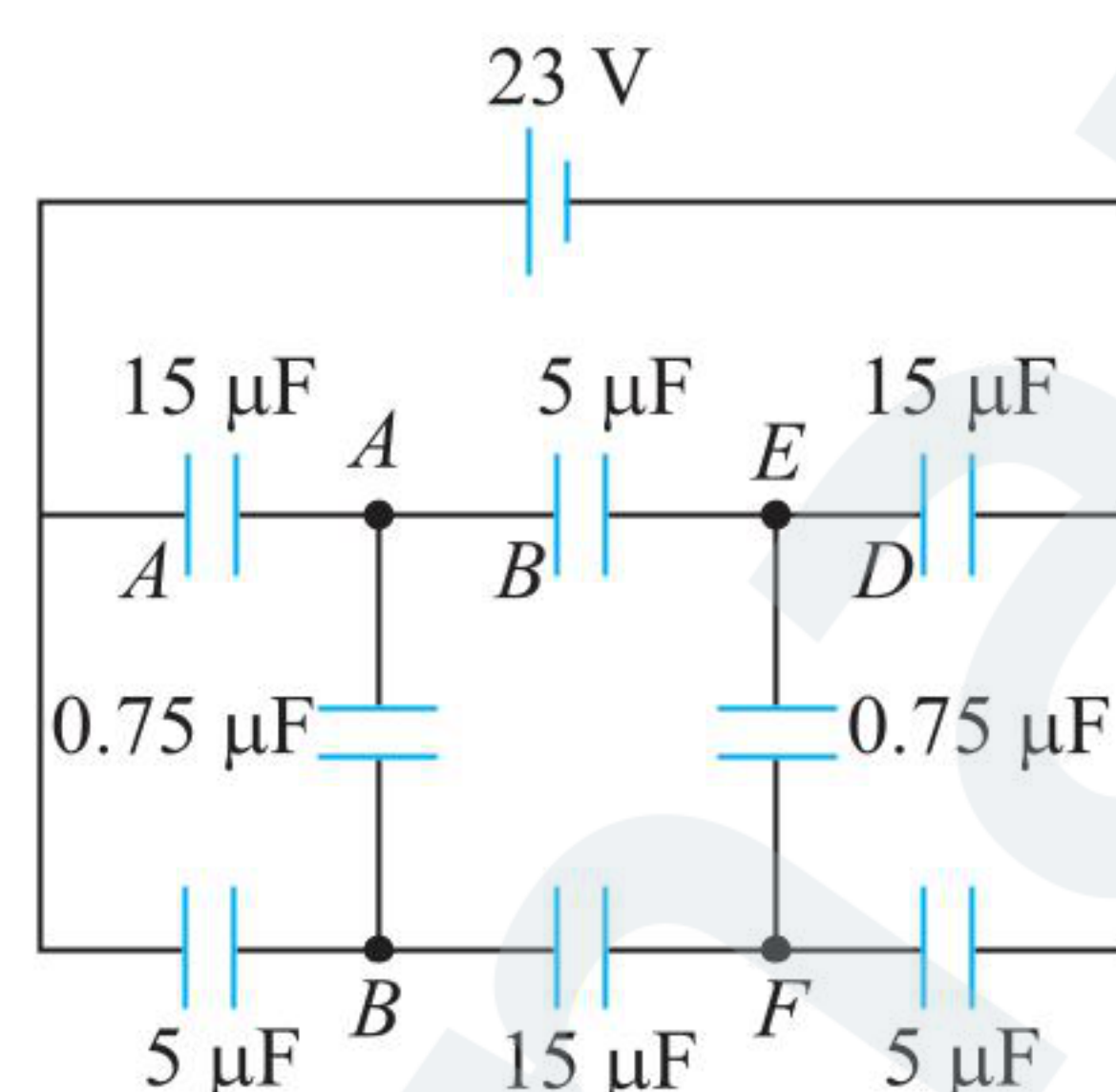
- (1) charge on C_3 is zero (2) charge on C_3 is $12 \mu\text{C}$
 (3) charge on C_1 is $6 \mu\text{C}$ (4) charge on C_2 is $6 \mu\text{C}$
9. In the circuit diagram shown in figure,



- (1) the charge on $2 \mu\text{F}$ capacitor is $8 \mu\text{C}$
 (2) the charge on each $6 \mu\text{F}$ capacitor is $72 \mu\text{C}$
 (3) the potential drop across $1 \mu\text{F}$ capacitor is 4 V
 (4) the potential drop across $3 \mu\text{F}$ is 4 V
10. Four capacitors and a battery are connected as shown in figure. If the potential difference across the $7 \mu\text{F}$ capacitor is 6 V , then which of the following statement(s) is/are correct?
- (1) The potential drop across the $12 \mu\text{F}$ capacitor is 10 V .
 (2) The charge in the $3 \mu\text{F}$ capacitor is $42 \mu\text{C}$.
 (3) The potential drop across the $3 \mu\text{F}$ capacitor is 10 V .
 (4) The emf of the battery is 30 V .

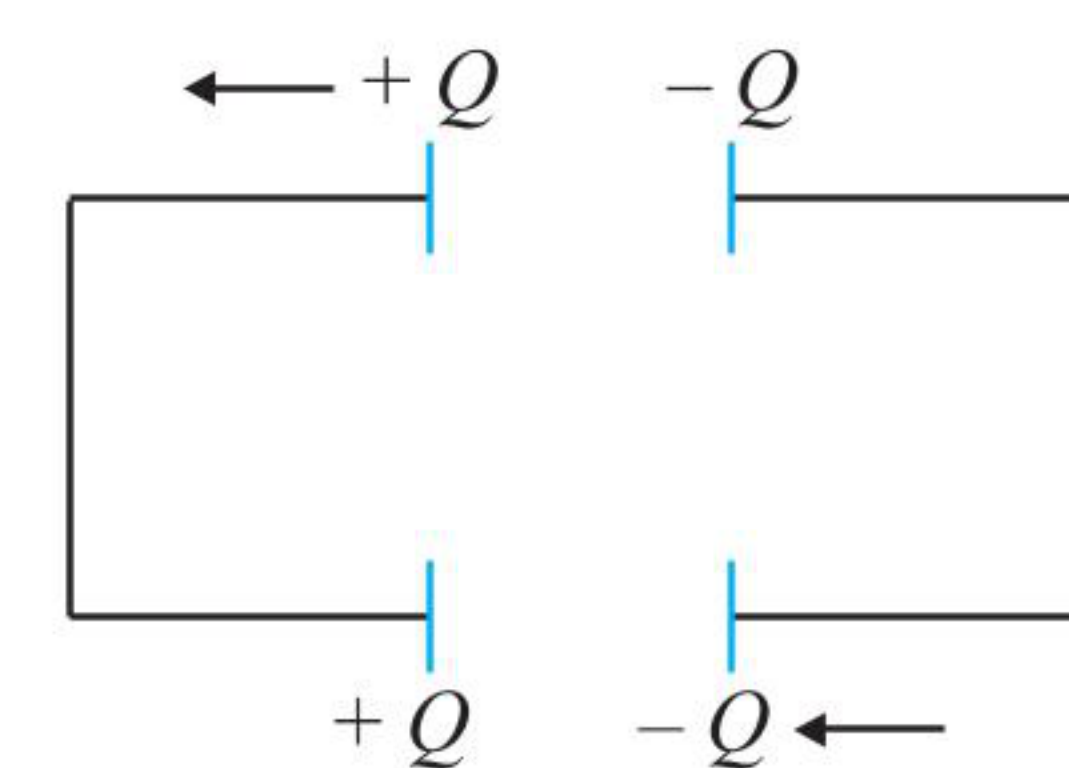


11. Find the potential difference between the points A and B and that between E and F of the circuit shown in figure.



- (1) $V_{AB} = 5 \text{ V}$ (2) $V_{EF} = 5 \text{ V}$
 (3) $V_{AB} = 0$ (4) $V_{EF} = 0$
12. Two identical parallel plate capacitors are connected in one case in parallel and in the other in series. In each case the plates of one capacitor are brought closer by a distance a and the plates of the other are moved apart by the same distance a . Then
- (1) total capacitance of first system increases
 (2) total capacitance of first system decreases
 (3) total capacitance of second system decreases
 (4) total capacitance of second system remains constant

13. A charge Q is imparted to two identical capacitors in parallel. Separation of the plates in each capacitor is d_0 . Suddenly, the first plate of the first capacitor and the second plate of the second capacitor start moving to the left with speed v , then



- (1) charges on the two capacitors as a function of time are

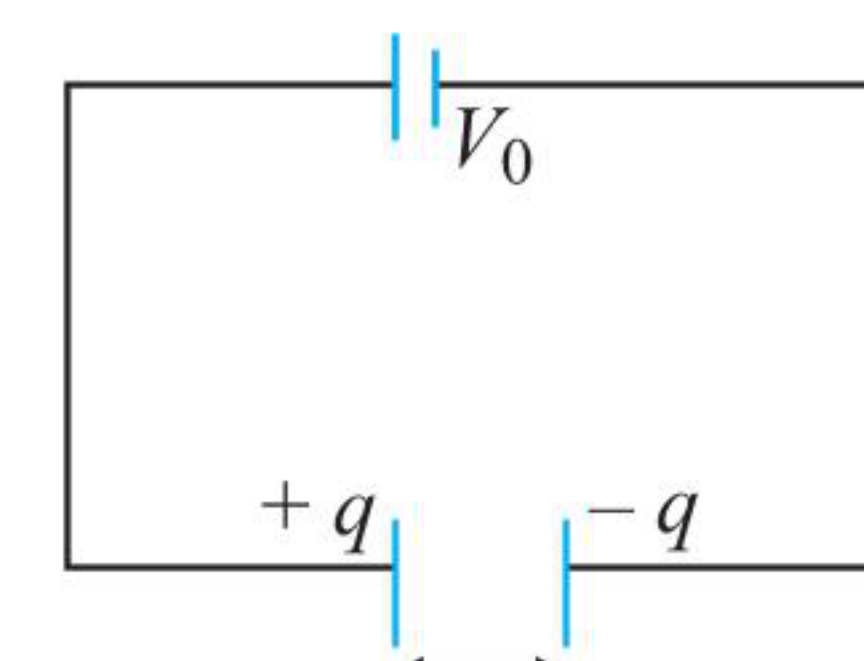
$$\frac{Q(d_0 - vt)}{2d_0}, \frac{Q(d_0 + vt)}{2d_0}$$

- (2) charges on the two capacitors as a function of time are

$$\frac{Qd_0}{2(d_0 - vt)}, \frac{Qd_0}{2(d_0 + vt)}$$

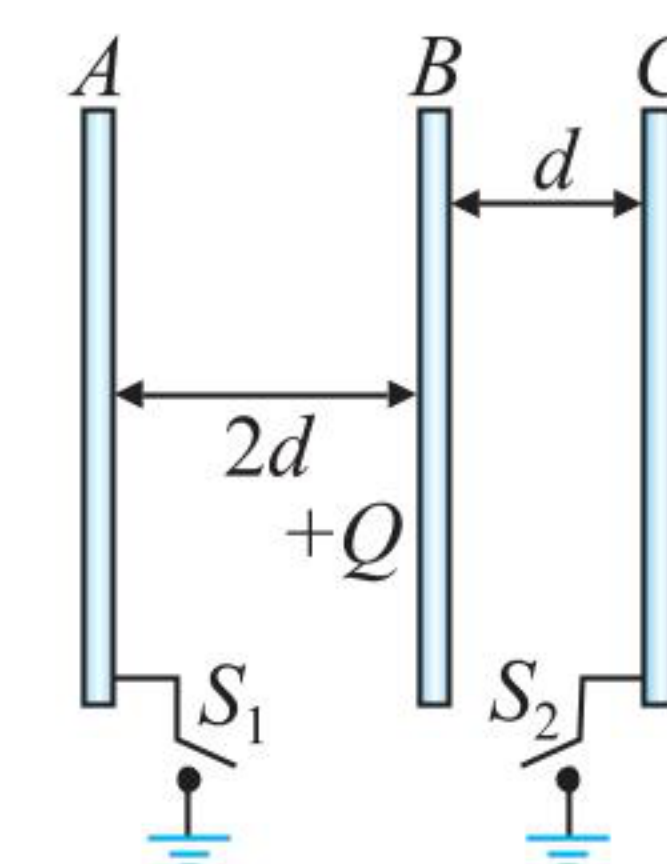
- (3) current in the circuit will increase as time passes on
 (4) current in the circuit will be constant

14. Two plates of a parallel plate capacitor carry charges q and $-q$ and are separated by a distance a from each other. The capacitor is connected to a constant voltage source V_0 . The distance between the plates is changed to $x + dx$. Then in steady state



- (1) change in electrostatic energy stored in the capacitor is $-Udx/x$
 (2) change in electrostatic energy in the capacitor is $-Udx/dx$
 (3) attraction force between the plates is $1/2qE$
 (4) attraction force between the plates is qE (where E is electric field between the plates)

15. Three identical parallel conducting plates A , B , and C are placed as shown. Switches S_1 and S_2 are open, and can connect A and C to earth when closed. $+Q$ charge is given to B .

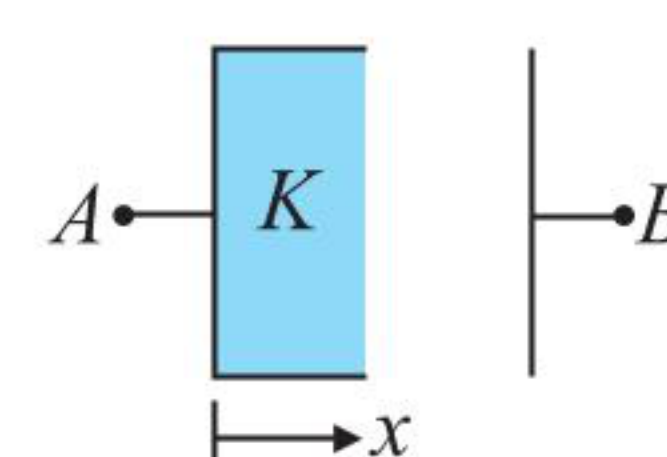


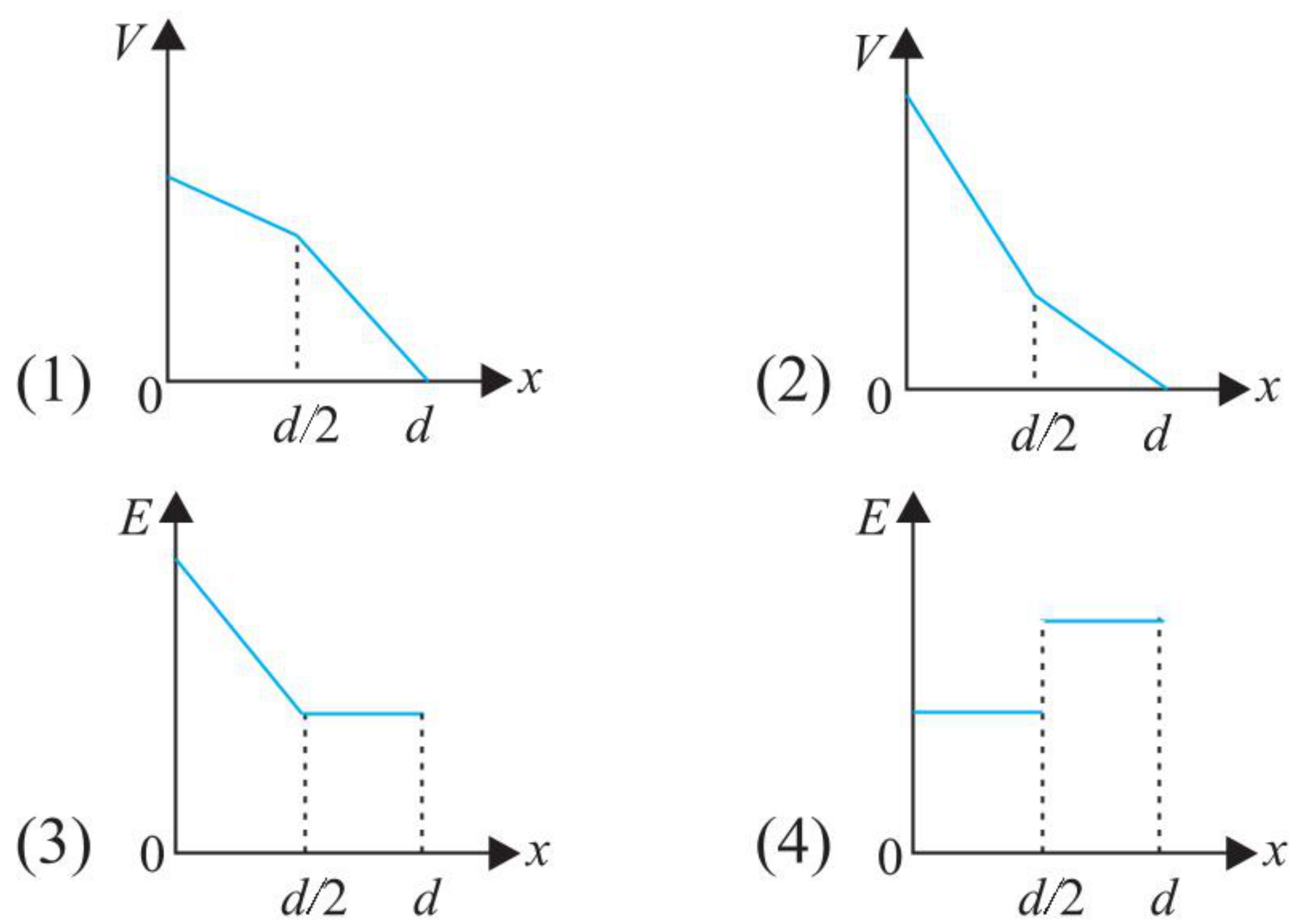
- (1) If S_1 is closed with S_2 open, a charge of amount Q will pass through S_1 .
 (2) If instead S_2 were closed with S_1 open, a charge of amount Q will pass through S_2 .
 (3) If S_1 and S_2 are closed together, a charge of amount $Q/3$ will pass through S_1 , and a charge of amount $2Q/3$ will pass through S_2 .
 (4) If S_1 and S_2 are closed together, a charge of amount $2Q/3$ will pass through S_1 , and a charge of amount $Q/3$ will pass through S_2 .

16. A parallel plate capacitor is charged from a cell and then isolated from it. The separation between the plates is now increased.

- (1) The force of attraction between the plates will decrease.
 (2) The field in the region between the plates will not change.
 (3) The energy stored in the capacitor will increase.
 (4) The potential difference between the plates will decrease.

17. In figure, half of the space between the plates of a parallel plate capacitor is filled with dielectric material of constant K . Then which of the plots are possible?

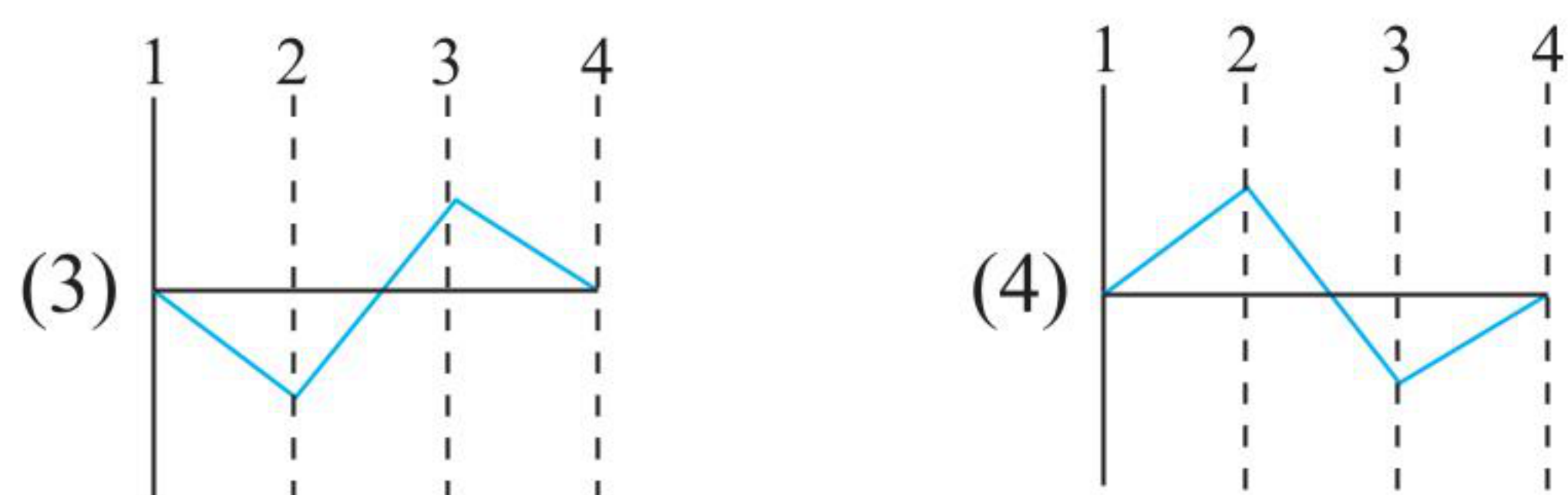




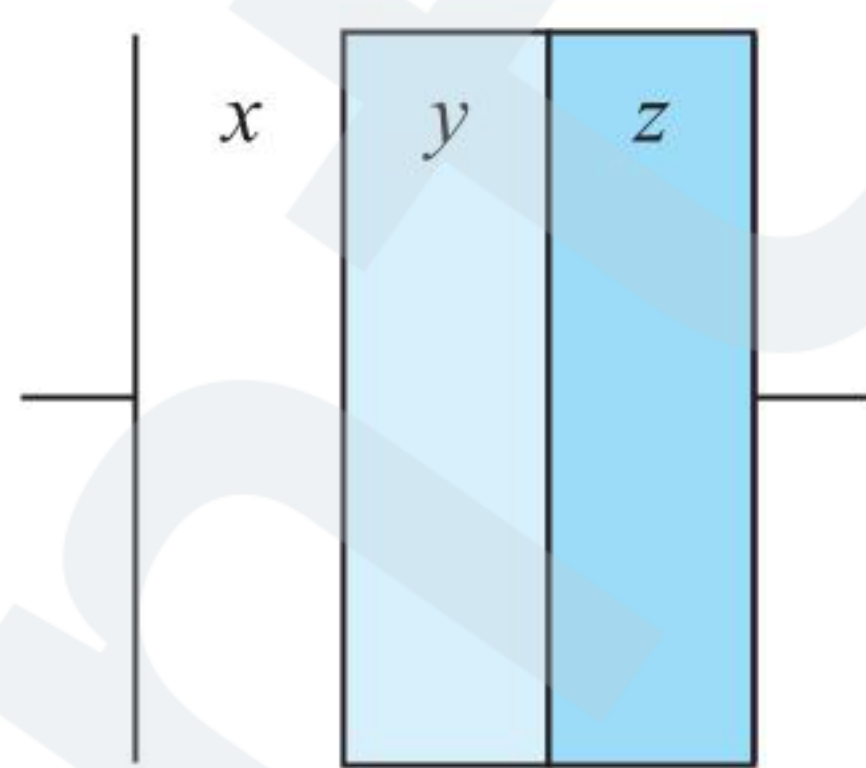
18. Four identical metallic plates (1, 2, 3 and 4) are arranged in air at same distance d from each other with their outer plates being connected together and earthed. If the plates 2 and 3 are connected with a cell of constant emf E , then ratio of electric fields between the plate is

- (1) $E_1 : E_2 : E_3 = \frac{2}{3} : 1 : \frac{2}{3}$
- (2) $E_1 : E_2 : E_3 = \frac{1}{2} : 1 : \frac{1}{2}$

and variation of electric potential will be

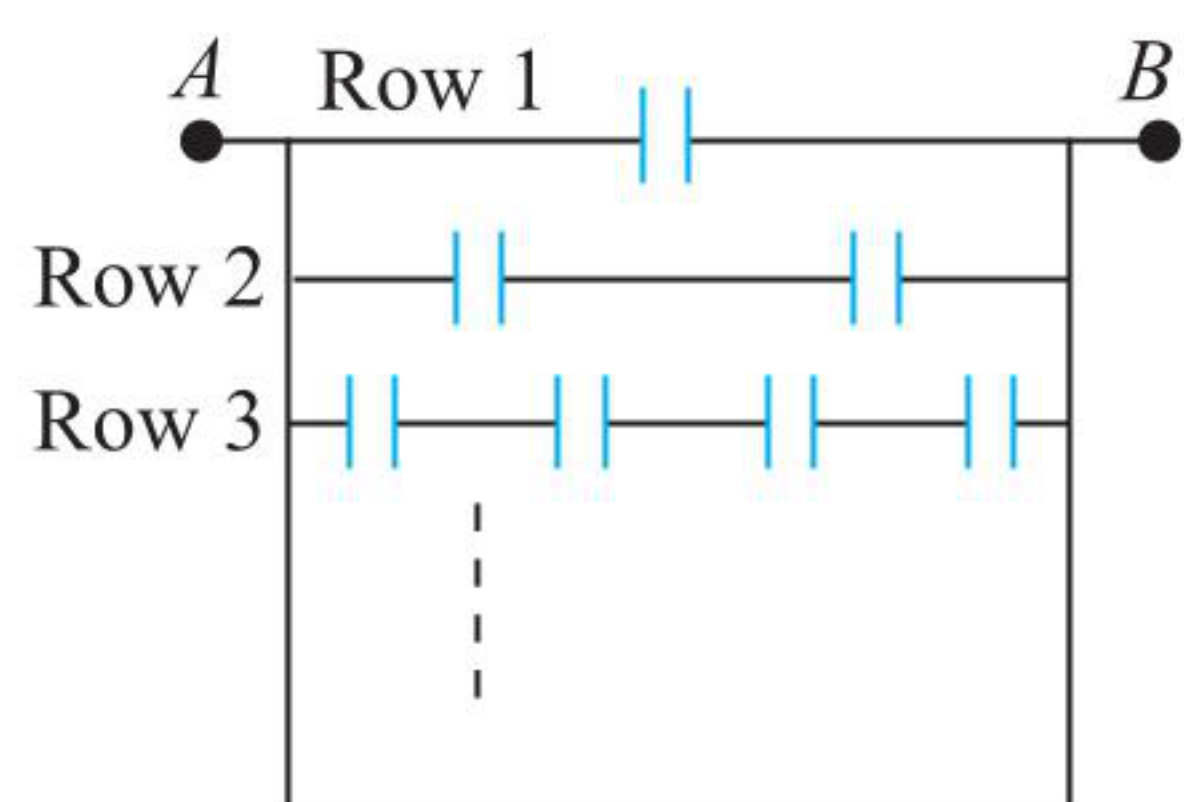


19. Figure shows a capacitor having three layers between its plates. Layer x is vacuum, y is conductor, and z is a dielectric. Which of the following change (s) will result in increase in capacitance?



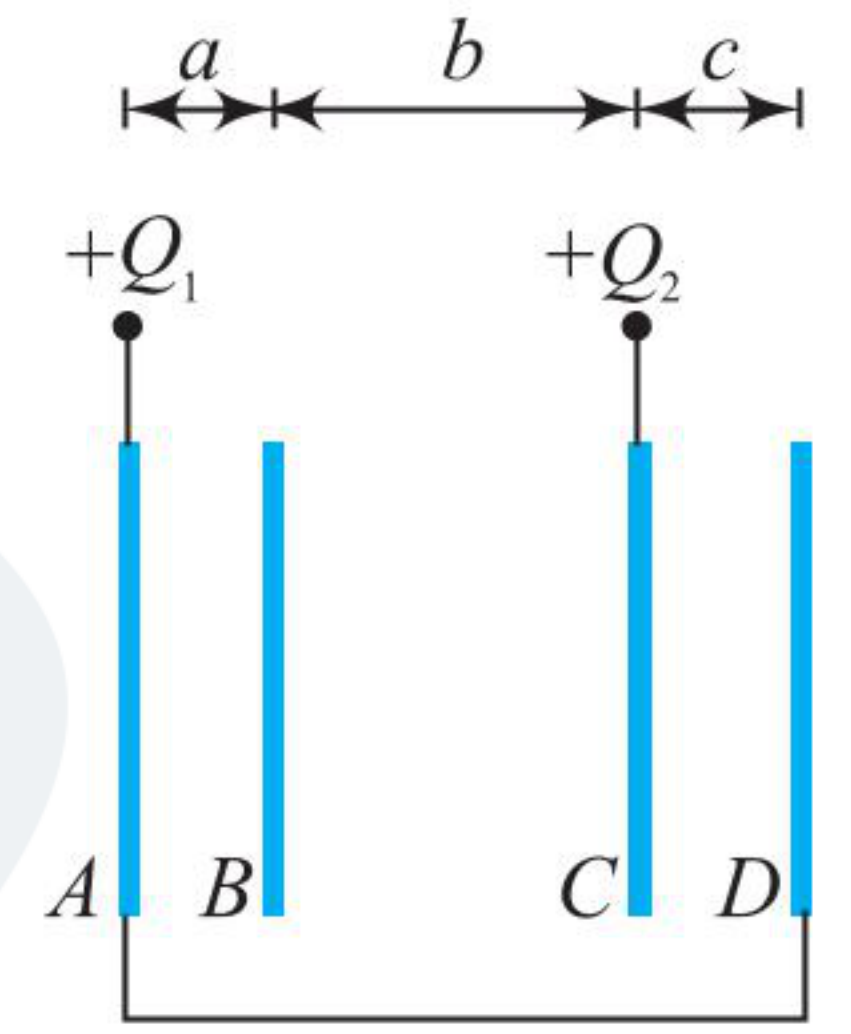
- (1) replace x by conductor (2) replace y by dielectric
 (3) replace z by conductor (4) replace x by dielectric
20. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles.
- (1) The charge on the capacitor increases.
 (2) The voltage across the plates increases.
 (3) The capacitance increases.
 (4) The electrostatic energy stored in the capacitor increases.
21. The plates of a parallel plate capacitor are not exactly parallel. The surface charge density is, therefore,
- (1) higher at the closer end
 (2) the surface charge density will not be uniform
 (3) each plate will have the same potential at each point
 (4) the electric field is smallest where the plates are closest

22. Rows of capacitors containing 1, 2, 4, 8, ..., ∞ capacitors, each of capacitance $2F$, are connected in parallel as shown in figure. The potential difference across AB is 10 V, then



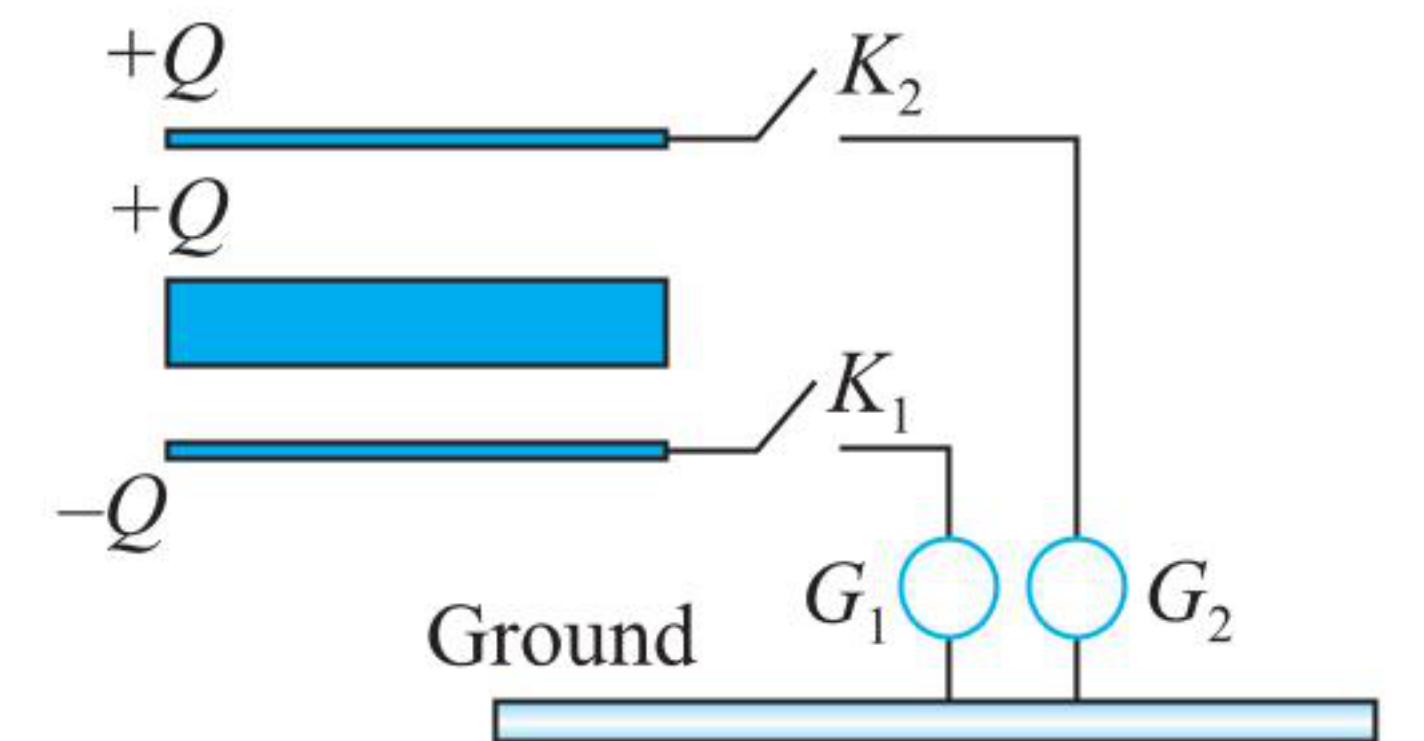
- (1) total capacitance across AB is $4F$
 (2) charge on each capacitor will be same
 (3) charge on the capacitor in the first row is more than on any other capacitor
 (4) energy of all the capacitors is 50 J

23. Figure shows an arrangement of four identical rectangular plates A , B , C , and D each of area S . Find the charges appearing on each face (from left to right) of the plates. Ignore the separation between the plates in comparison to the plate dimensions.

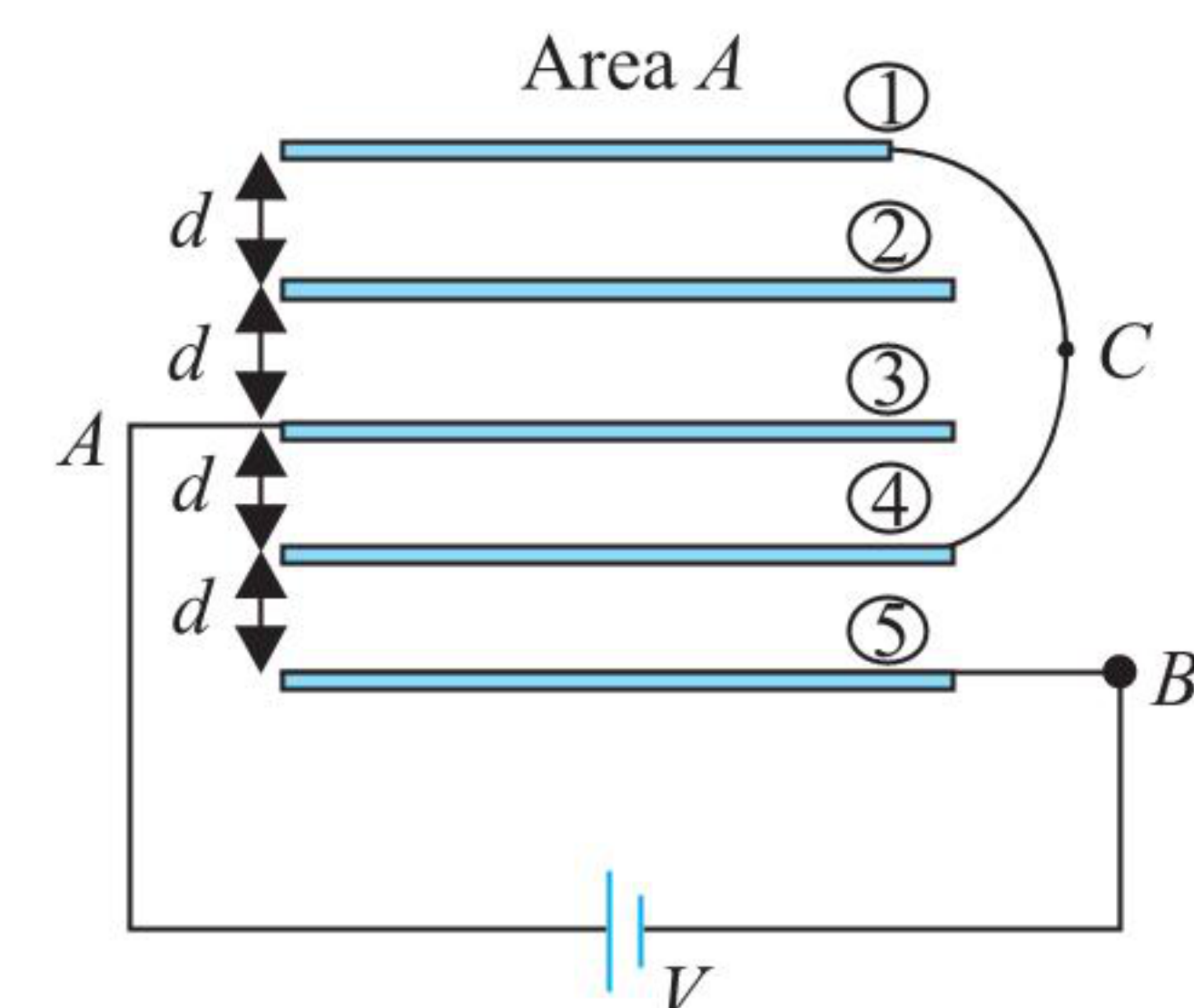


- (1) Potential difference between plates A and B is independent of Q_1 .
 (2) Potential difference between plates C and D is independent of Q_1 .
 (3) Potential difference between plates A and B is independent of Q_2 .
 (4) Potential difference between plates C and D is independent of Q_2 .

24. A parallel plate capacitor is charged as shown (Q is given). A metal slab with the total charge $+Q$ is placed inside the capacitor as shown. The thickness of the slab is d . The distance between the top plate and the top of the slab is $2d$, and the distance between the bottom plate and the bottom of the slab is d . Each plate is grounded through a galvanometer as shown. Find the charge that passes through each galvanometer after both switches are closed simultaneously.



- (1) A total charge of $-Q/3$ flows (through G_1) from the bottom plate to ground.
 (2) A total charge of $+4Q/3$ flows (through G_2) from the top plate to ground.
 (3) A total charge of $+Q/3$ flows (through G_1) from the bottom plate to ground.
 (4) A total charge of $-4Q/3$ flows (through G_2) from the top plate to ground.
25. Five plates are arranged as shown in the figure and connected across a battery of emf V . The separation between each plate is d and surface area of each plate is A .



- (1) Equivalent capacity between A and B is $\epsilon_0 A/5d$
 (2) Equivalent capacity between A and B is $3\epsilon_0 A/5d$
 (3) Charge on plate 1 is $\epsilon_0 AV/5d$
 (4) Charge on plate 3 is $3\epsilon_0 AV/5d$

Linked Comprehension Type

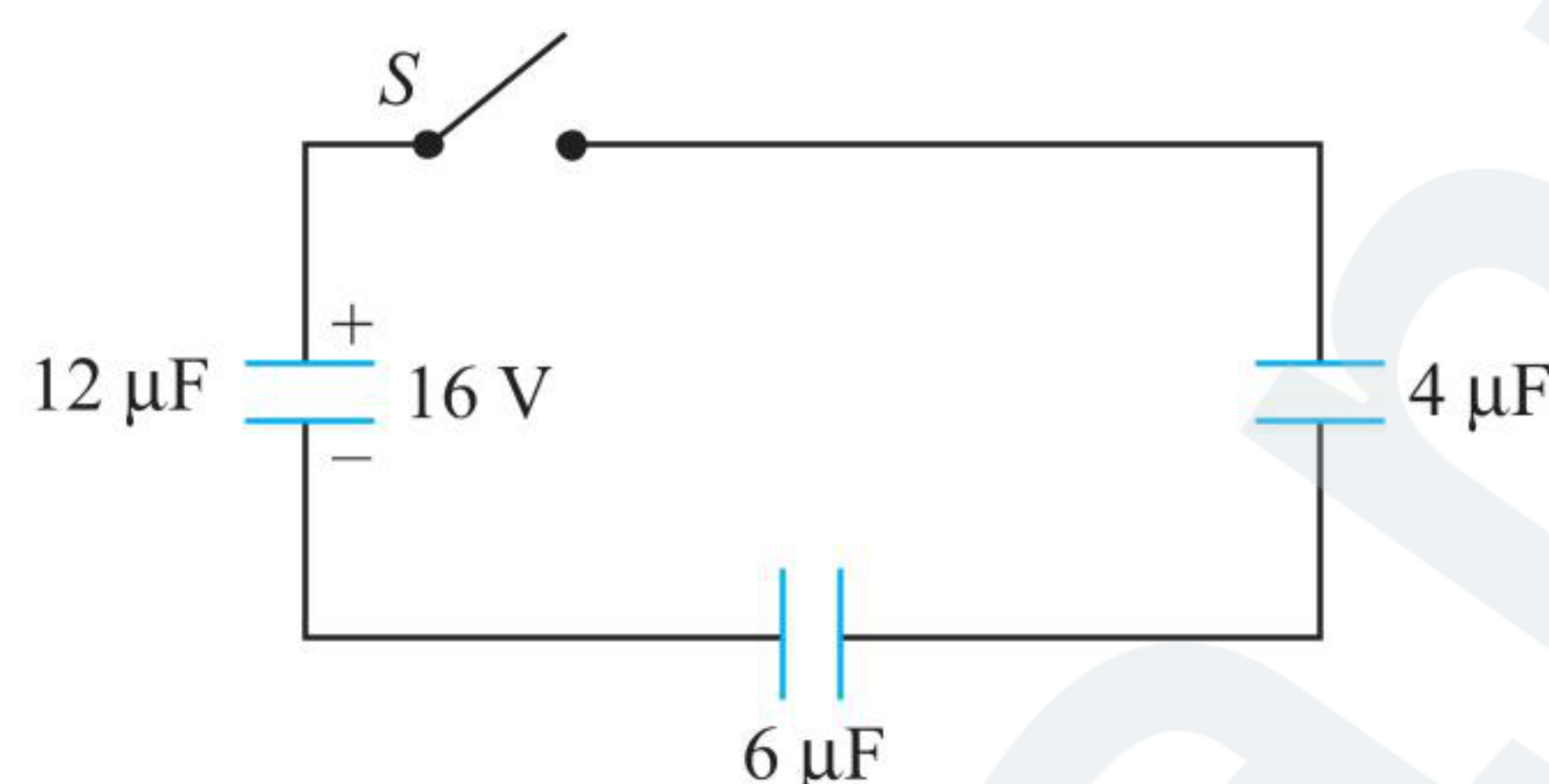
For Problems 1–2

A $1\ \mu\text{F}$ and a $2\ \mu\text{F}$ capacitor are connected in series across a $1200\ \text{V}$ supply.

- The charged capacitors are disconnected from the line and from each other and are now reconnected with the terminals of like sign together. Find the final charge on each capacitor and the voltage across each capacitor.
 - Charges on capacitors are $1400/3\ \mu\text{C}$ and $3200/3\ \mu\text{C}$, and potential difference across each capacitor is $1600/3\ \text{V}$.
 - Charges on capacitors are $1600/3\ \mu\text{C}$ and $3200/3\ \mu\text{C}$, and potential difference across each capacitor is $1600/3\ \text{V}$.
 - Charge on each capacitor is $1600\ \mu\text{C}$, and potential difference across each capacitor is $800\ \text{V}$.
 - Charge and potential difference across each capacitor are zero.
- If the charged capacitors are reconnected with the terminals of opposite signs together, find the final charge and voltage across each capacitor.
 - Charges on capacitors are $1400/3\ \mu\text{C}$ and $3200/3\ \mu\text{C}$, and potential difference across each capacitor is $1600/3\ \text{V}$.
 - Charges on capacitors are $1600/3\ \mu\text{C}$ and $3200/3\ \mu\text{C}$, and potential difference across each capacitor is $1600/3\ \text{V}$.
 - Charge on each capacitor is $1600\ \mu\text{C}$, and potential difference across each capacitor is $800\ \text{V}$.
 - Charge and potential difference across each capacitor are zero.

For Problems 3–5

In the arrangement shown in figure, when the switch S is closed, find



- the final charge on the $6\ \mu\text{F}$ capacitor
 - $12\ \mu\text{C}$
 - $24\ \mu\text{C}$
 - $32\ \mu\text{C}$
 - $48\ \mu\text{C}$
- the final potential difference across the $4\ \mu\text{F}$ capacitor
 - $12\ \text{V}$
 - $8\ \text{V}$
 - $20\ \text{V}$
 - $32\ \text{V}$
- the final potential difference across the $12\ \mu\text{F}$ capacitor
 - $\frac{40}{3}\ \text{V}$
 - $\frac{20}{3}\ \text{V}$
 - $12\ \text{V}$
 - $24\ \text{V}$

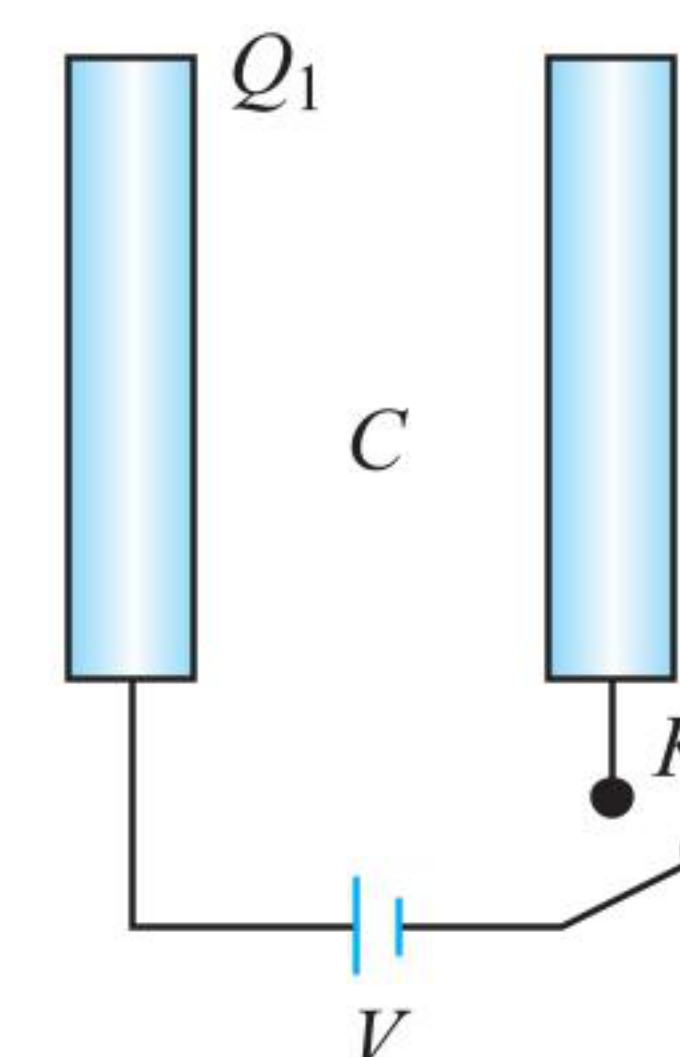
For Problems 6–7

Each plate of a parallel plate air capacitor has area $S = 5 \times 10^{-3}\ \text{m}^2$ and the distance between the plates is $d = 8.80\ \text{mm}$. Plate A has positive charge $q_1 = +10^{-10}\ \text{C}$, and plate B has charge $q_2 = +2 \times 10^{-10}\ \text{C}$. A battery of emf $E = 10\ \text{V}$ has its positive terminal connected to plate A and the negative terminal to plate B . (Given $\epsilon_0 = 8.8 \times 10^{-12}\ \text{Nm}^2\ \text{C}^{-2}$)

- Charge supplied by the battery is
 - $120\ \text{pC}$
 - $100\ \text{pC}$
 - $60\ \text{pC}$
 - $50\ \text{pC}$
- Energy supplied by the battery is
 - $10^{-9}\ \text{J}$
 - $5 \times 10^{-9}\ \text{J}$
 - $50 \times 10^{-9}\ \text{J}$
 - $25 \times 10^{-9}\ \text{J}$

For Problems 8–9

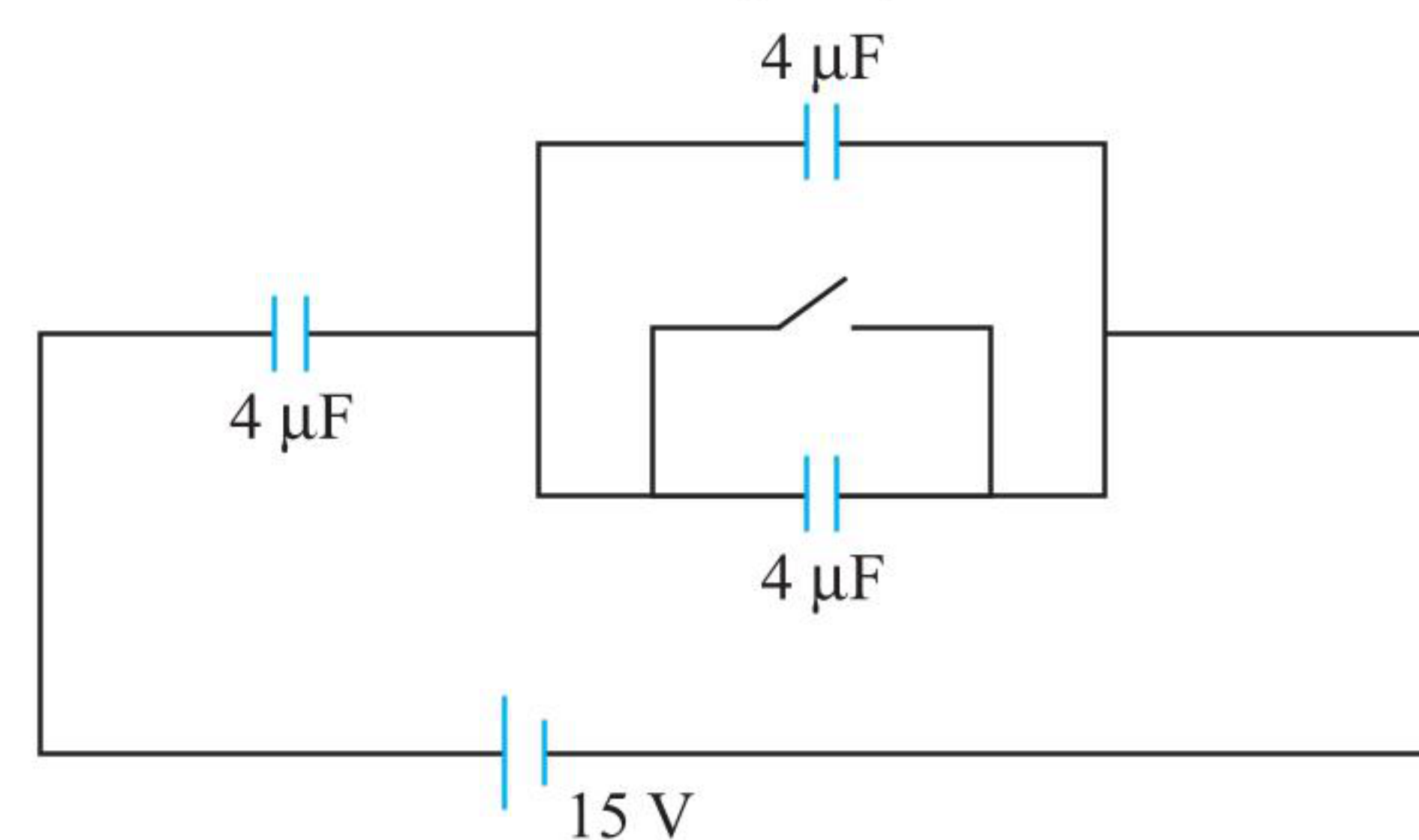
For the system shown in figure, capacitance is C . The left plate is given a charge Q_1 , and the right plate is uncharged. Now, the switch is closed.



- Find the amount of charge that will flow through the battery before the steady state is achieved.
 - CV
 - $CV - Q_1$
 - $CV + \frac{Q_1}{2}$
 - $CV - \frac{Q_1}{2}$
- Find the charge appearing on the inner face of the left plate.
 - $CV - \frac{Q_1}{2}$
 - $CV + Q_1$
 - $CV + \frac{Q_1}{2}$
 - CV

For Problems 10–11

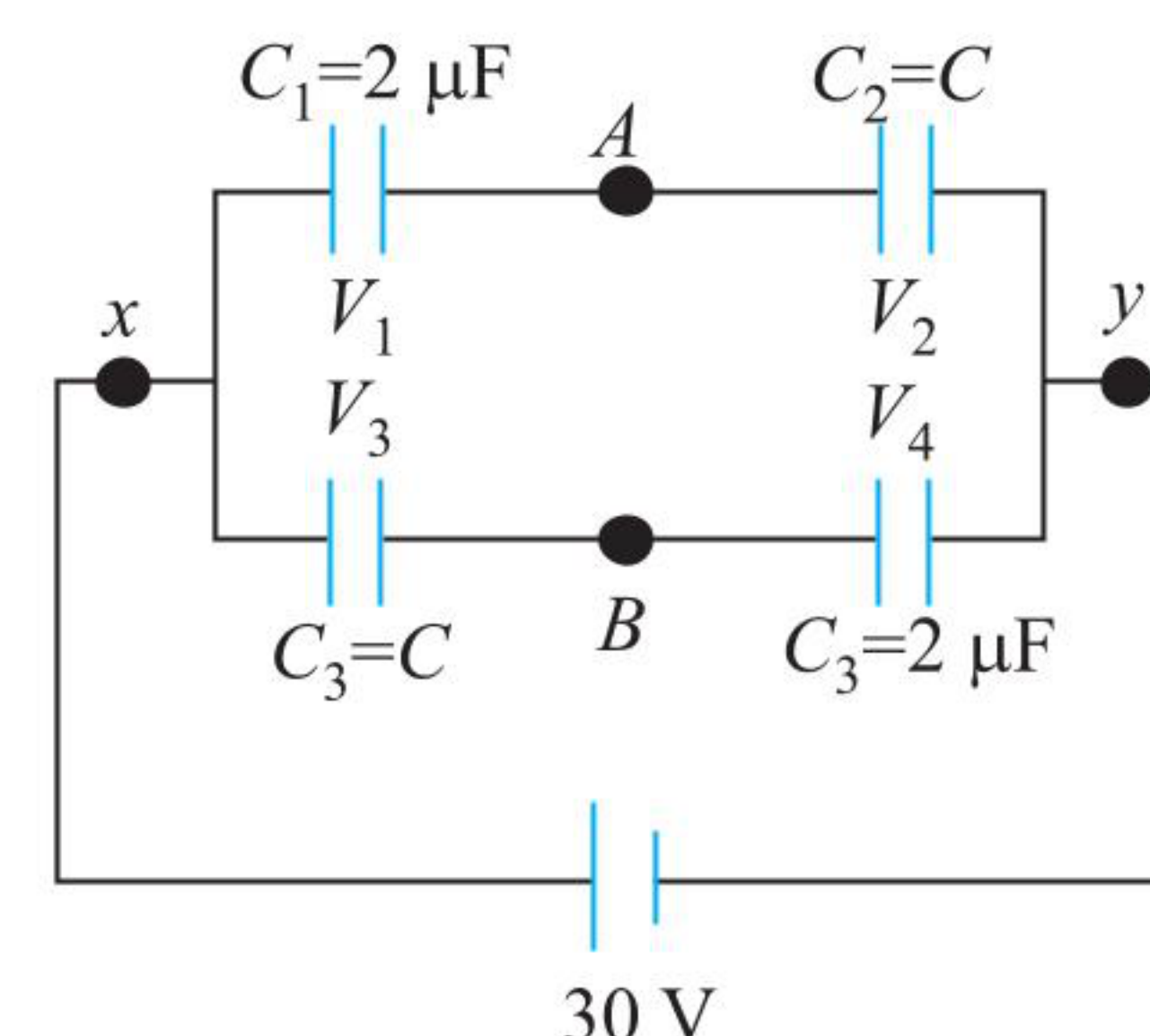
Consider the circuit shown in figure, after switch S is closed.



- What amount of charge will flow through the battery?
 - $20\ \mu\text{C}$
 - $60\ \mu\text{C}$
 - $40\ \mu\text{C}$
 - no charge will flow
- What amount of charge will flow through the switch?
 - $20\ \mu\text{C}$
 - $60\ \mu\text{C}$
 - $40\ \mu\text{C}$
 - no charge will flow

For Problems 12–16

The given circuit shows an arrangement of four capacitors. A potential difference $30\ \text{V}$ is applied across the combination. It is observed that potentials at points A and B differ by $5\ \text{V}$. Also if a conducting wire is connected between A and B , electrons will flow from A to B . Of course, we have not actually connected any wire between A and B , we have described only an 'if' situation. Answer the following questions.



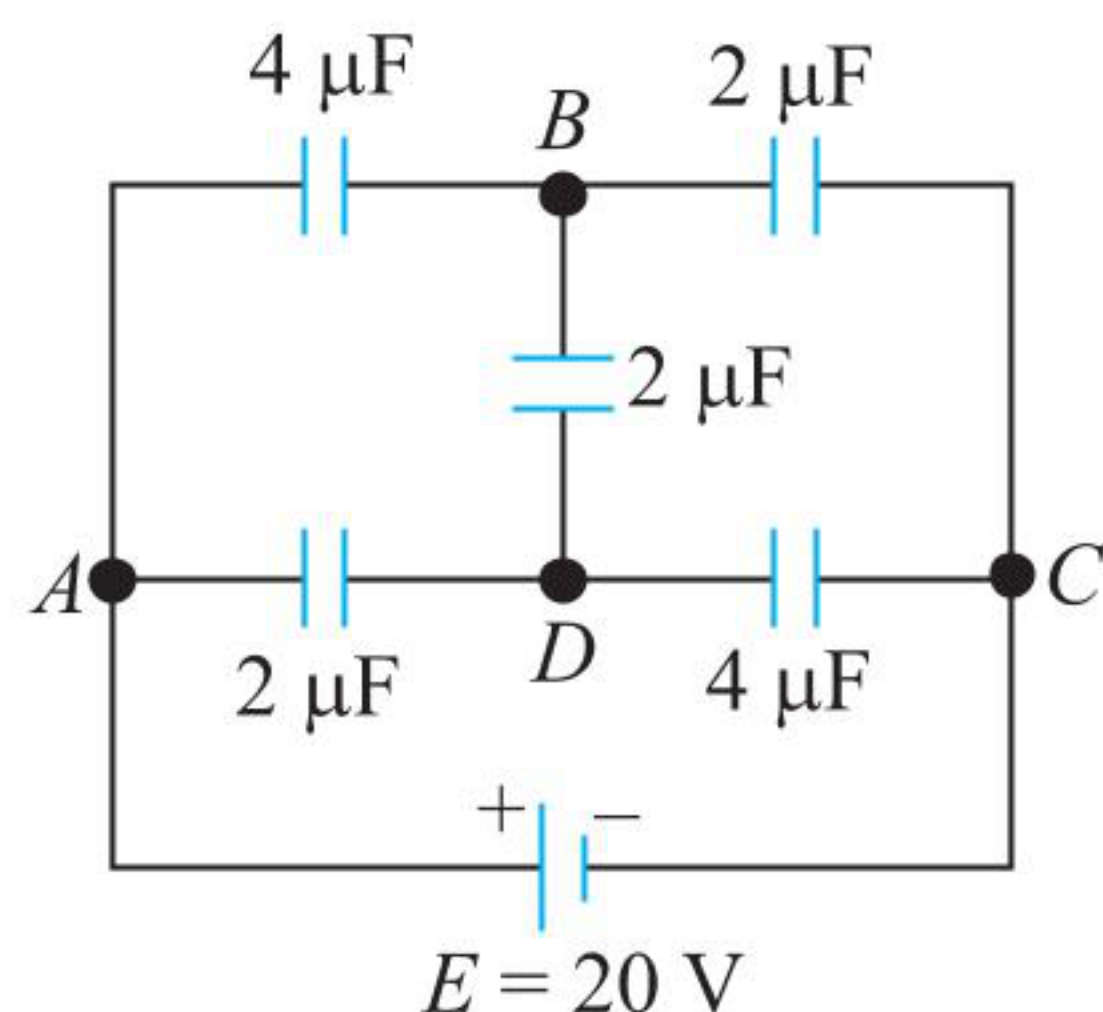
12. Potential difference across C_4 is
 (1) 12.5 V (2) 15.5 V
 (3) 17.5 V (4) 22.5 V
13. Equivalent capacitor between X and Y is
 (1) $2.34 \mu\text{F}$ (2) $1.54 \mu\text{F}$
 (3) $1.22 \mu\text{F}$ (4) $0.77 \mu\text{F}$
14. Charge on capacitor C_2 is
 (1) $60 \mu\text{C}$ (2) $52 \mu\text{C}$
 (3) $42 \mu\text{C}$ (4) $35 \mu\text{C}$

Let us now connect two more capacitors in the circuit. One of them, C_5 , is connected in the part of the circuit between X and A . It could be either in series or in parallel with C_1 . The other, C_6 , is connected between A and B . It is observed whether we increase C_6 or reduce it, equivalent capacitance between X and Y has the same value.

15. Capacitance C_5 is
 (1) $1.24 \mu\text{F}$ in series with C_1
 (2) $1.92 \mu\text{F}$ in parallel with C_1
 (3) $2.28 \mu\text{F}$ in series with C_1
 (4) $2.56 \mu\text{F}$ in parallel with C_1
16. Charge on C_5 will be
 (1) $32 \mu\text{C}$ (2) $28 \mu\text{C}$
 (3) $24 \mu\text{C}$ (4) $16 \mu\text{C}$

For Problems 17–18

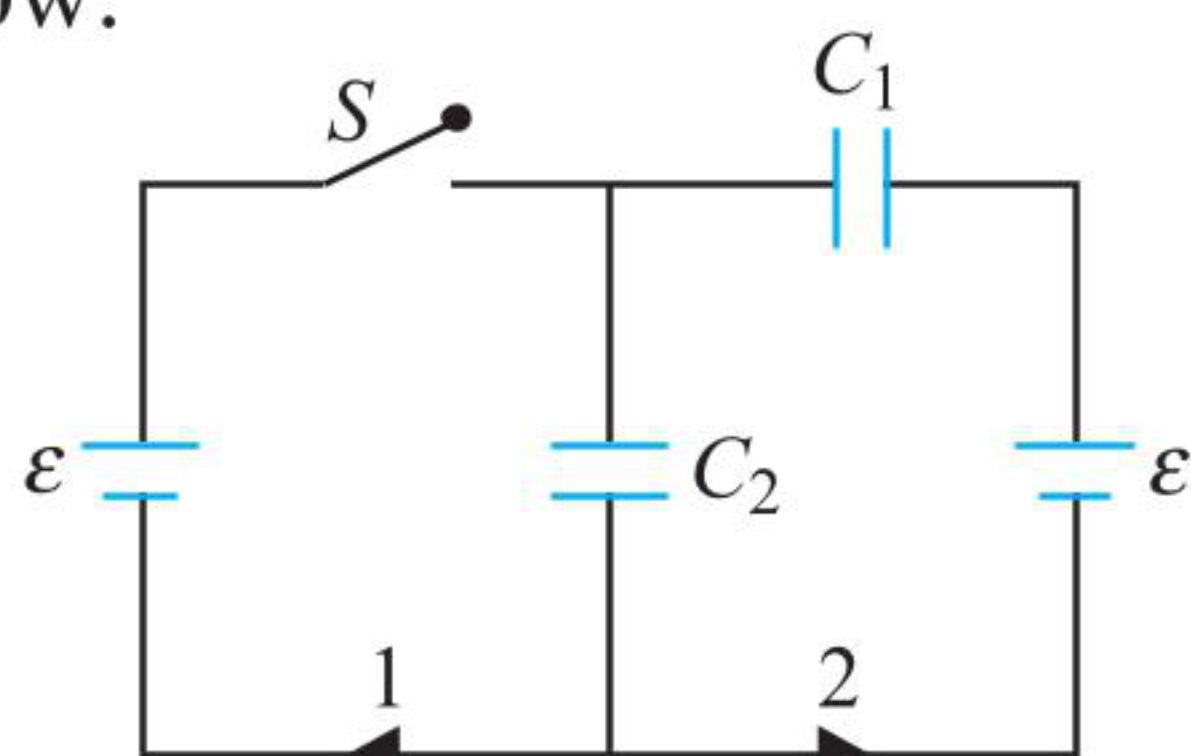
Figure shows a diagonal symmetric arrangement of capacitors and a battery.



17. Identify the correct statements.
 (1) Both the $4 \mu\text{F}$ capacitors carry equal charges in opposite sense.
 (2) Both the $4 \mu\text{F}$ capacitors carry equal charges in the same sense.
 (3) $V_B - V_D = 0$
 (4) $V_D - V_B > 0$
18. If the potential of C is zero, then identify the incorrect statement.
 (1) $V_A = +15 \text{ V}$
 (2) $4(V_A - V_B) + 2(V_D - V_B) = 2V_B$
 (3) $2(V_A - V_D) + 2(V_B - V_D) = 4V_D$
 (4) $V_A = V_B + V_D$

For Problems 19–20

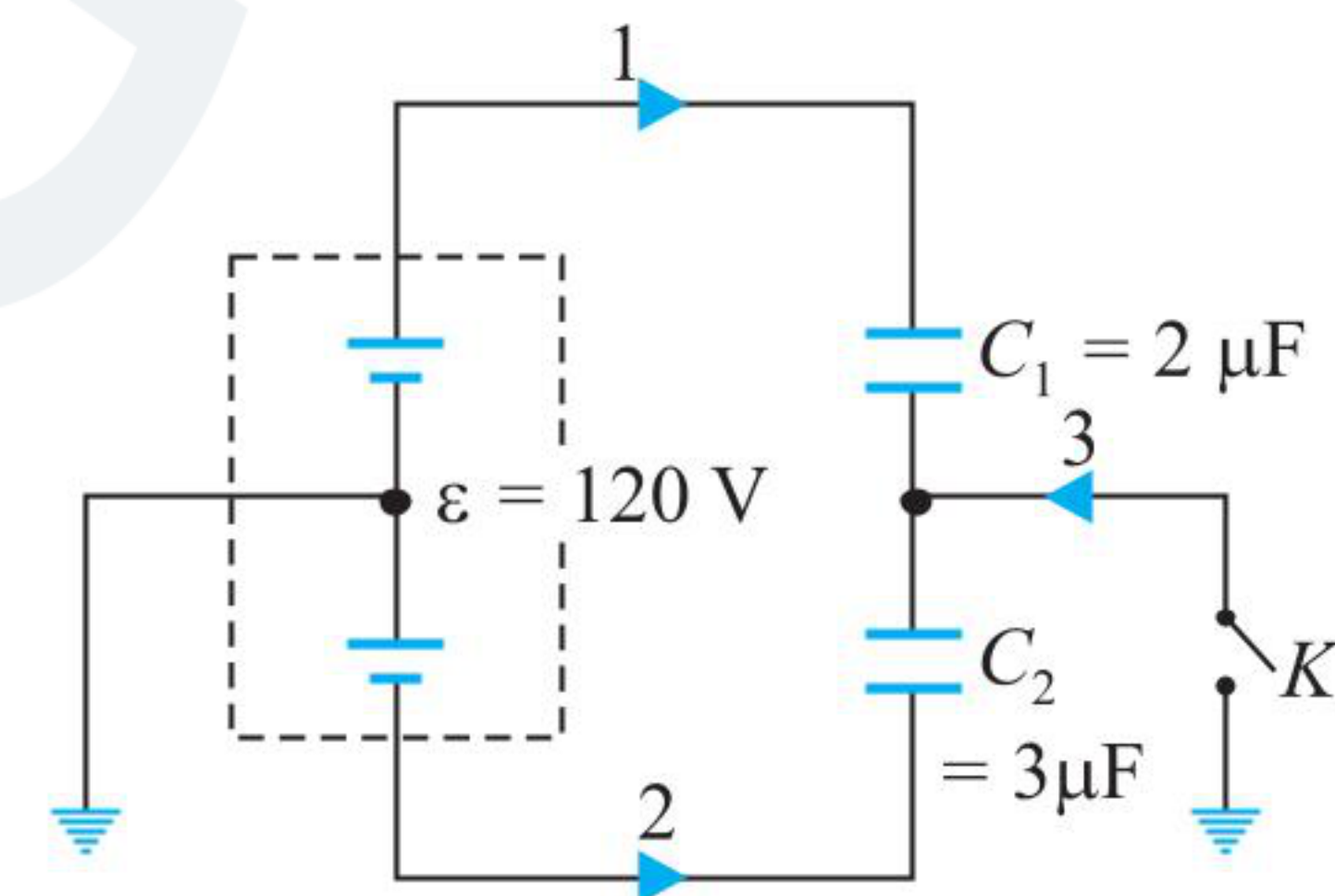
In the circuit shown in figure, initially the switch is opened. The switch is closed now.



19. The charge that will flow in direction 1 is
 (1) $-\frac{C_2^2 \epsilon}{C_1 + C_2}$ (2) $-\left(\frac{C_1 C_2}{C_1 + C_2}\right) \epsilon$
 (3) $\frac{C_1^2 \epsilon}{C_1 + C_2}$ (4) $C_2 \epsilon$
20. The charge that will flow in direction 2 is
 (1) $-\frac{C_2^2 \epsilon}{C_1 + C_2}$ (2) $\left(\frac{C_1 C_2}{C_1 + C_2}\right) \epsilon$
 (3) $\frac{C_1^2 \epsilon}{C_1 + C_2}$ (4) $C_2 \epsilon$

For Problems 21–23

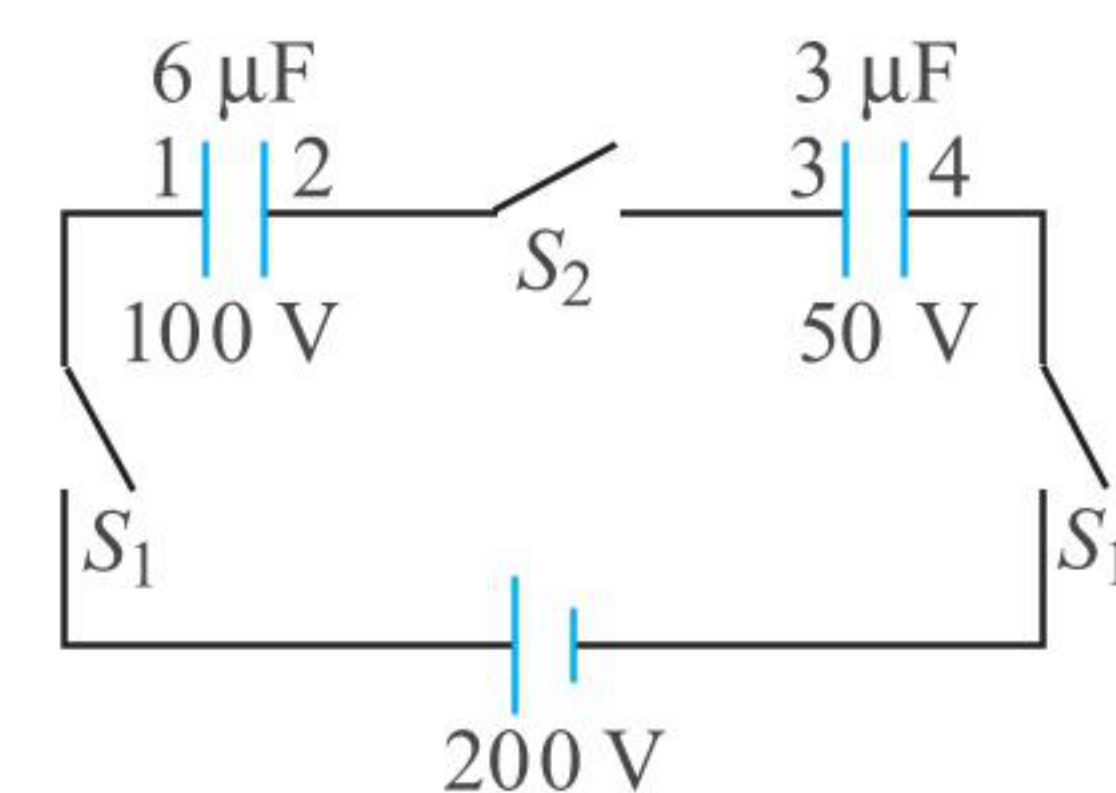
Capacitors $C_1 = 2 \mu\text{F}$ and $C_2 = 3 \mu\text{F}$ are connected in series to a battery of emf $\epsilon = 120 \text{ V}$ whose midpoint is earthed. The wire connecting the capacitors can be earthed through a key K . Now, key K is closed. Determine the charge flowing through the sections 1, 2, and 3 in the directions indicated in figure.



21. In section 1,
 (1) $-24 \mu\text{C}$ (2) $-36 \mu\text{C}$
 (3) $-60 \mu\text{C}$ (4) $60 \mu\text{C}$
22. In section 2,
 (1) $-24 \mu\text{C}$ (2) $-36 \mu\text{C}$
 (3) $-60 \mu\text{C}$ (4) $60 \mu\text{C}$
23. In section 3,
 (1) $-24 \mu\text{C}$ (2) $-36 \mu\text{C}$
 (3) $-60 \mu\text{C}$ (4) $60 \mu\text{C}$

For Problems 24–26

Two capacitors of capacity $6 \mu\text{F}$ and $3 \mu\text{F}$ are charged to 100 V and 50 V separately and connected as shown in figure. Now all the three switches S_1 , S_2 , and S_3 are closed.



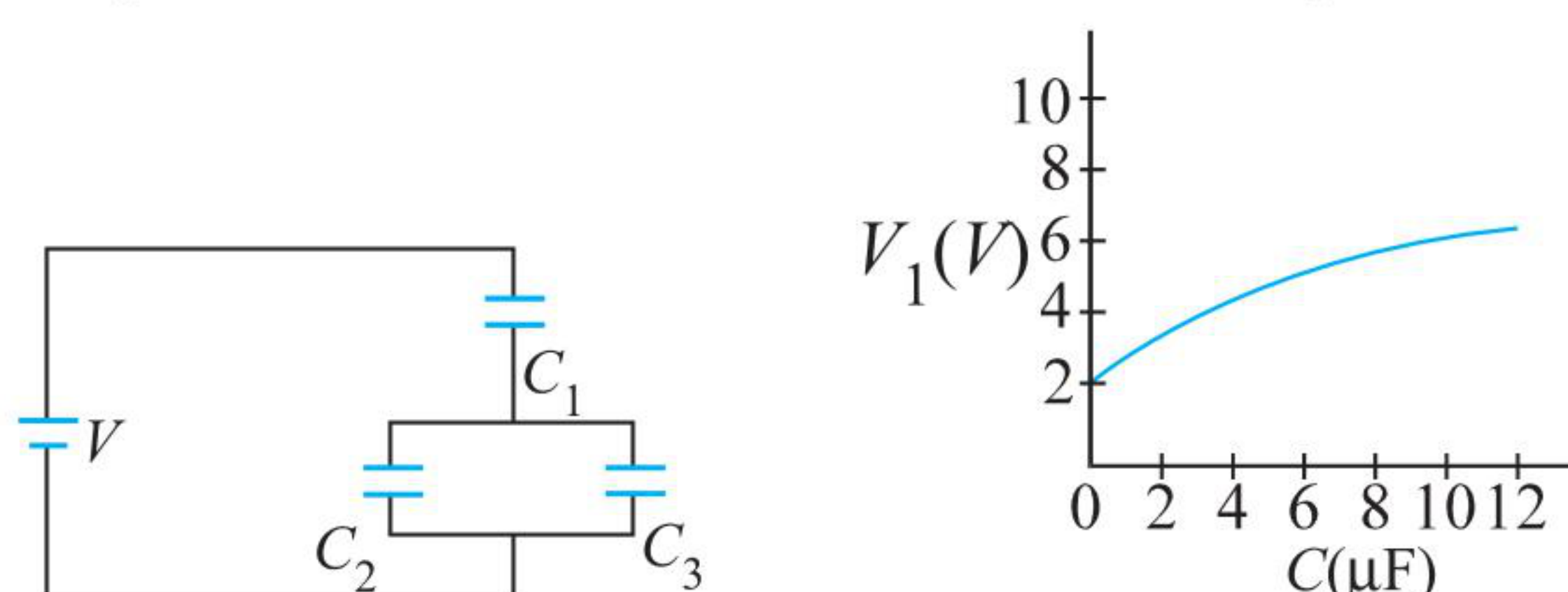
24. Which plate(s) form an isolated system?
 (1) Plate 1 and plate 4 separately
 (2) Plate 2 and plate 3 separately
 (3) Plate 1 and plate 4 jointly
 (4) Plate 2 and plate 3 jointly
25. Charges on $6 \mu\text{F}$ and $3 \mu\text{F}$ capacitors in steady state will be
 (1) $400 \mu\text{C}$, $400 \mu\text{C}$ (2) $700 \mu\text{C}$, $250 \mu\text{C}$
 (3) $800 \mu\text{C}$, $350 \mu\text{C}$ (4) $300 \mu\text{C}$, $450 \mu\text{C}$

26. Let q_1 , q_2 , and q_3 be the magnitudes of charges flown from switches S_1 , S_2 , and S_3 after they are closed. Then

- (1) $q_1 = q_3$ and $q_2 = 0$ (2) $q_1 = q_3 = q_2/2$
 (3) $q_1 = q_3 = 3q_2$ (4) $q_1 = q_2 = q_3$

For Problems 27–29

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). C_1 and C_2 are of fixed values. Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3 . Electric potential V_1 approaches on asymptote of 10 V as $C_3 \rightarrow \infty$.



27. The electric potential V across the battery is equal to

- (1) 10 V (2) 12 V
 (3) 16 V (4) 20 V

28. Relation between C_1 and C_2 is

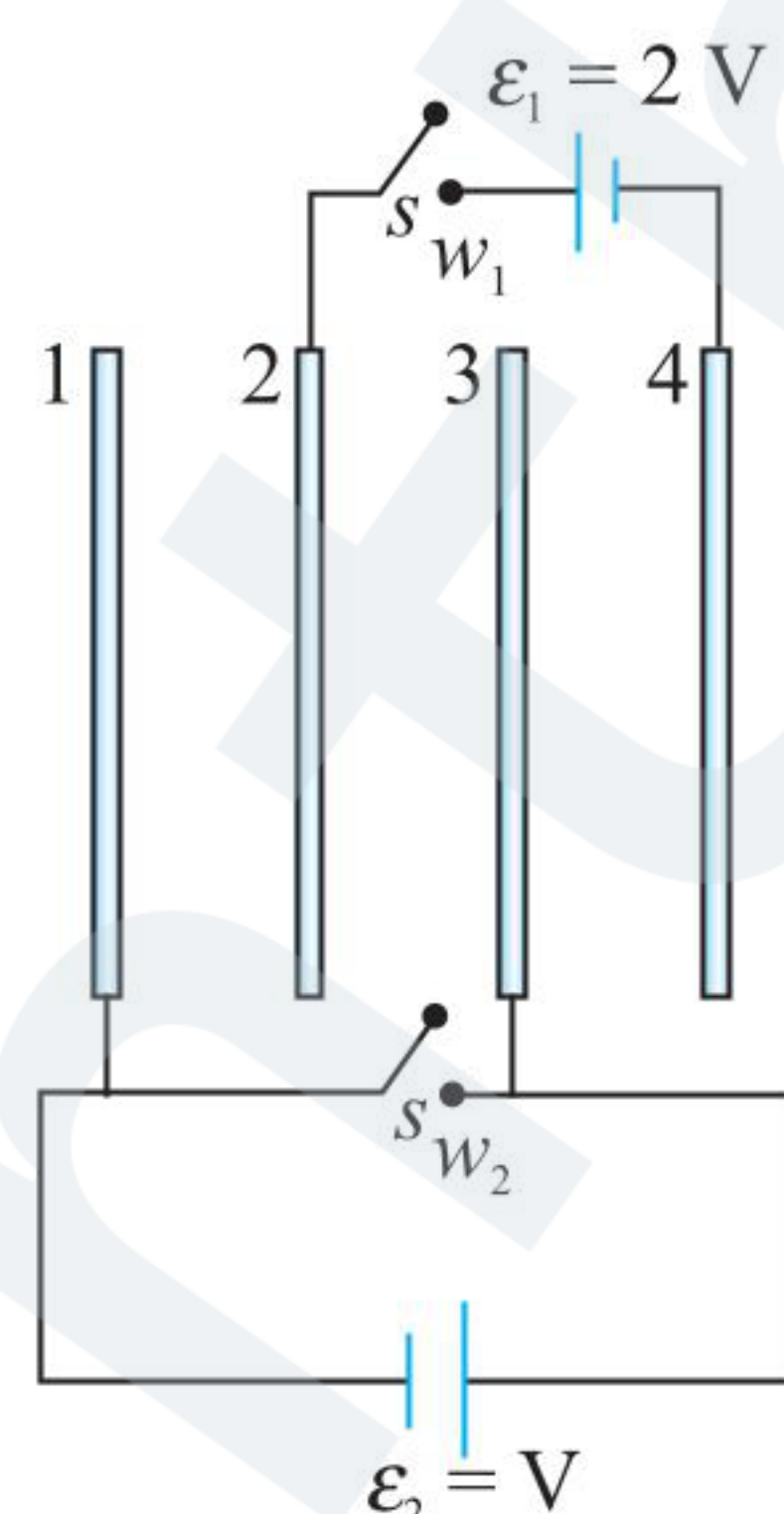
- (1) $C_1 = C_2$ (2) $C_1 = 4C_2$
 (3) $4C_1 = C_2$ (4) any relation

29. When $V_1 = 4$ V, then C_3 is equal to

- (1) $5C_2/2$ (2) $5C_1/3$
 (3) $5C_2/3$ (4) none of these

For Problems 30–32

Figure shows four plates each of plate area A and separated between plates is d . Two switches S_{w_1} and S_{w_2} can activate two batteries in the circuit.



30. Switch S_{w_1} and S_{w_2} are closed. What is the charge on plate 2?

- (1) $\frac{3 \epsilon_0 A V}{2 D}$ (2) $\frac{3 \epsilon_0 A V}{2 d}$
 (3) $\frac{1 \epsilon_0 A V}{3 d}$ (4) $\frac{4 \epsilon_0 A V}{3 d}$

31. What is the charge passing through battery 1?

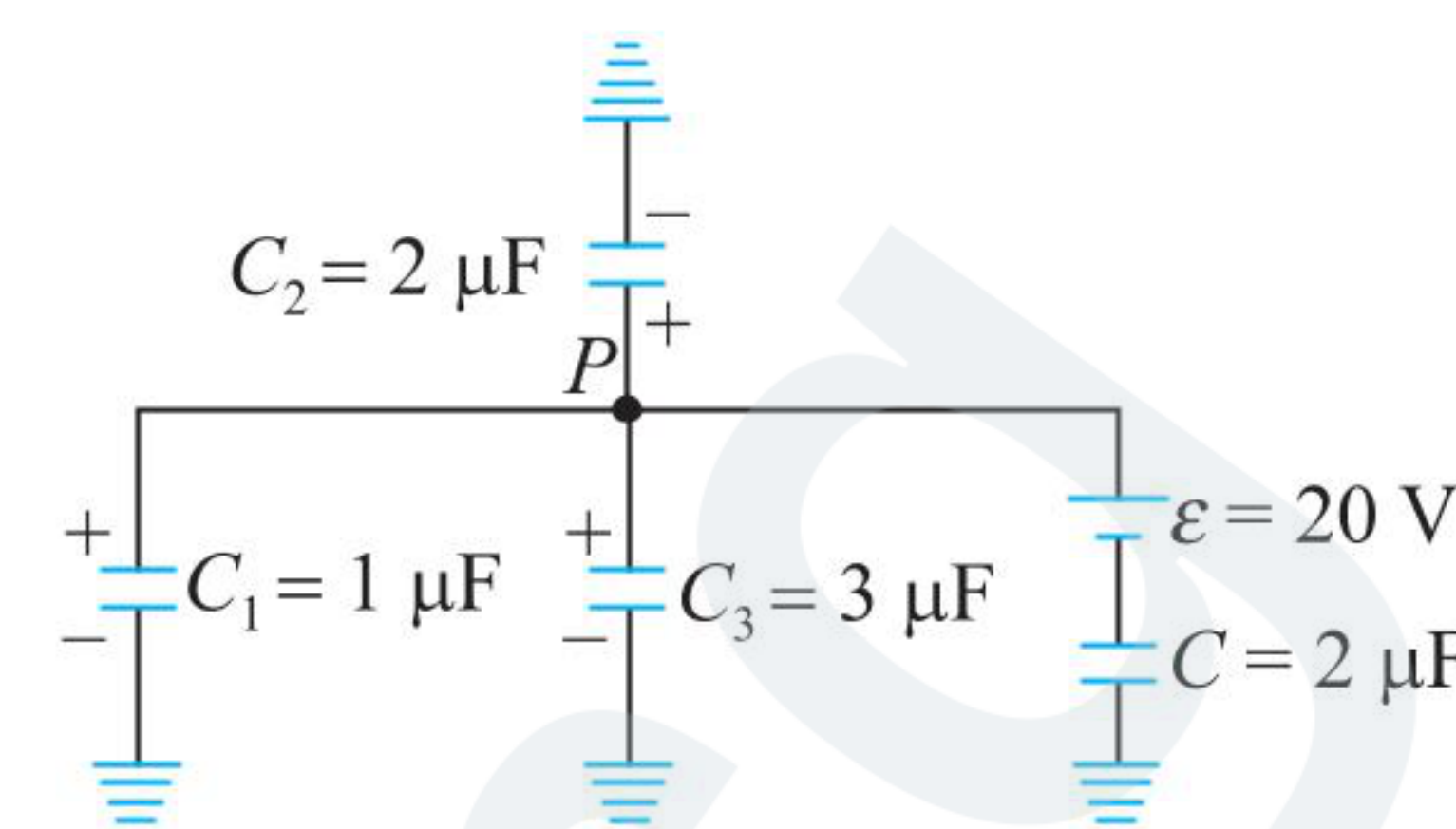
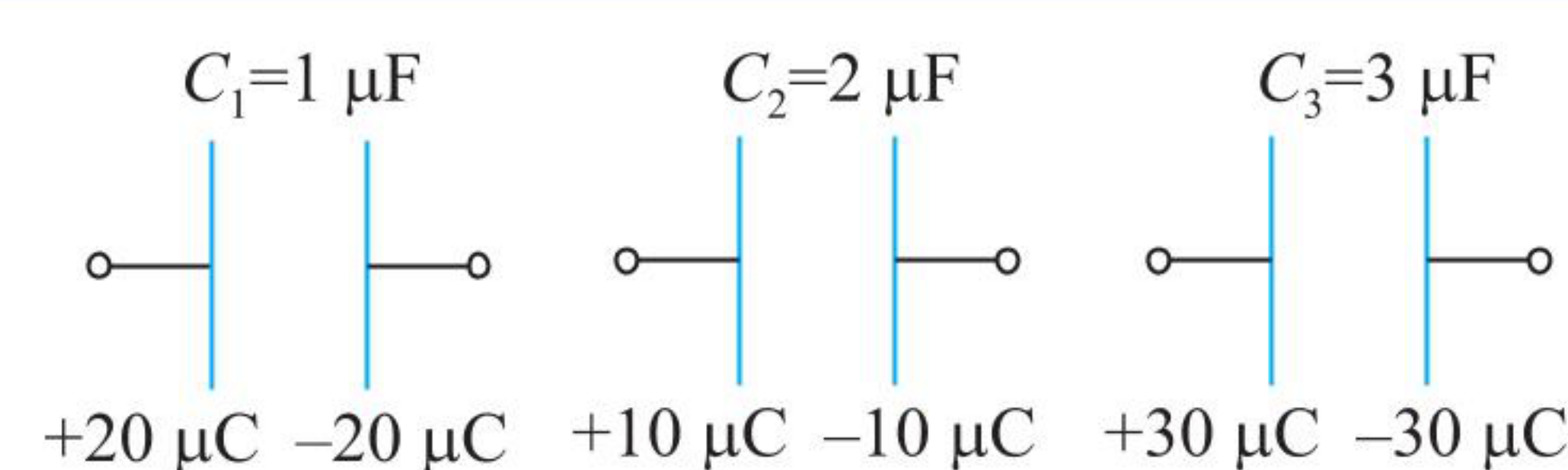
- (1) $\frac{3 \epsilon_0 A V}{2 D}$ (2) $\frac{3 \epsilon_0 A V}{2 d}$
 (3) $\frac{1 \epsilon_0 A V}{3 d}$ (4) $\frac{4 \epsilon_0 A V}{3 d}$

32. Now switch S_{w_2} is opened, what is the charge on plate 4?

- (1) $-\frac{5 \epsilon_0 A V}{3 d}$ (2) $+\frac{5 \epsilon_0 A V}{3 d}$
 (3) $-\frac{1 \epsilon_0 A V}{3 d}$ (4) $+\frac{1 \epsilon_0 A V}{3 d}$

For Problems 33–34

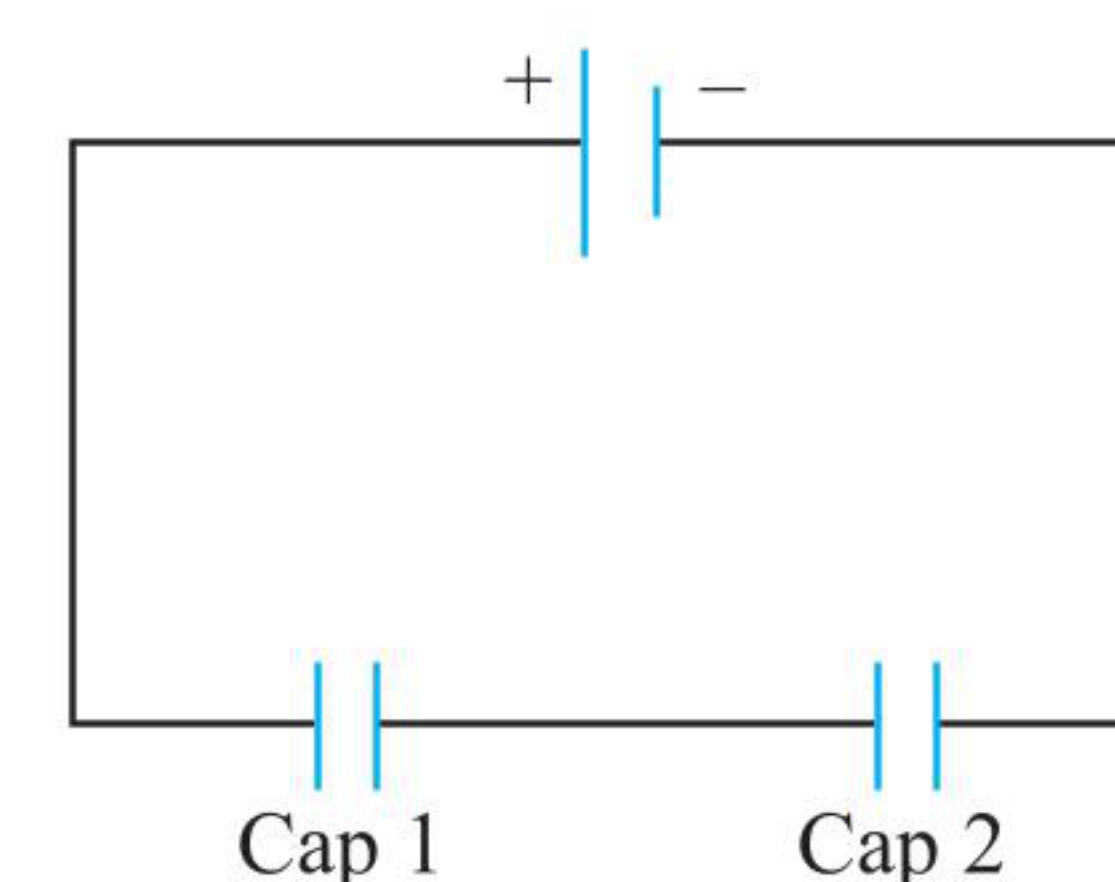
Three capacitors C_1 , C_2 , and C_3 of capacitance $1 \mu\text{F}$, $2 \mu\text{F}$, and $3 \mu\text{F}$, respectively, are charged separately as shown in the figure. Now these charged capacitors are connected to a battery of $\epsilon = 20$ V and an uncharged capacitor of $C = 2 \mu\text{F}$ as shown in figure.



33. The charge on capacitor marked C is
 (1) $3.75 \mu\text{C}$ (2) $7.5 \mu\text{C}$
 (3) $15 \mu\text{C}$ (4) none of these
34. The potential of point P is
 (1) 12.5 V (2) 25 V
 (3) 6.25 V (4) none of these

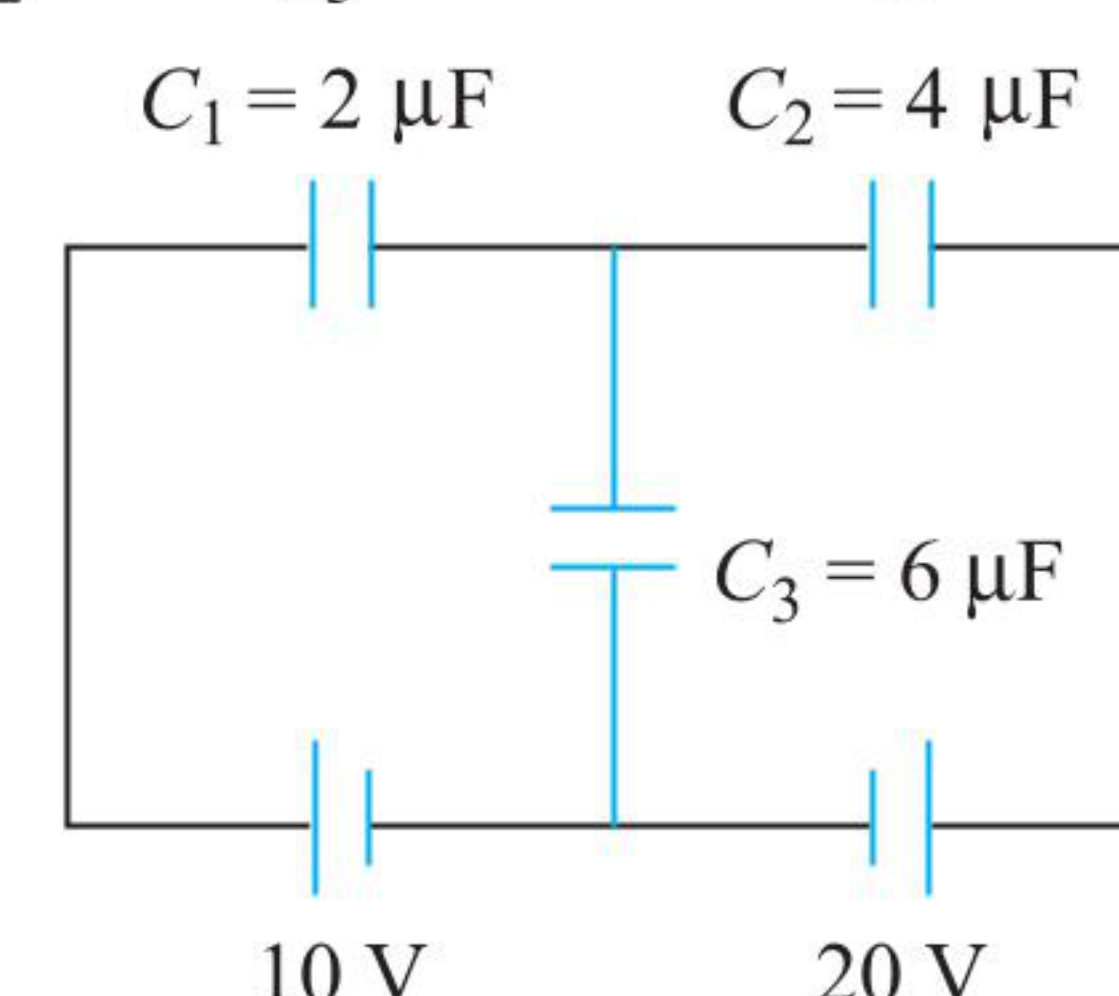
Matrix Match Type

1. Two identical capacitors are connected in series, and the combination is connected with a battery, as shown. Some changes in capacitor 1 are now made independently after the steady state is achieved as listed in column I. Some effects that may occur in the new steady state due to these changes on capacitor 2 are listed in column II. Match the changes on capacitor 1 in column I with the corresponding effect on capacitor 2 in column II.



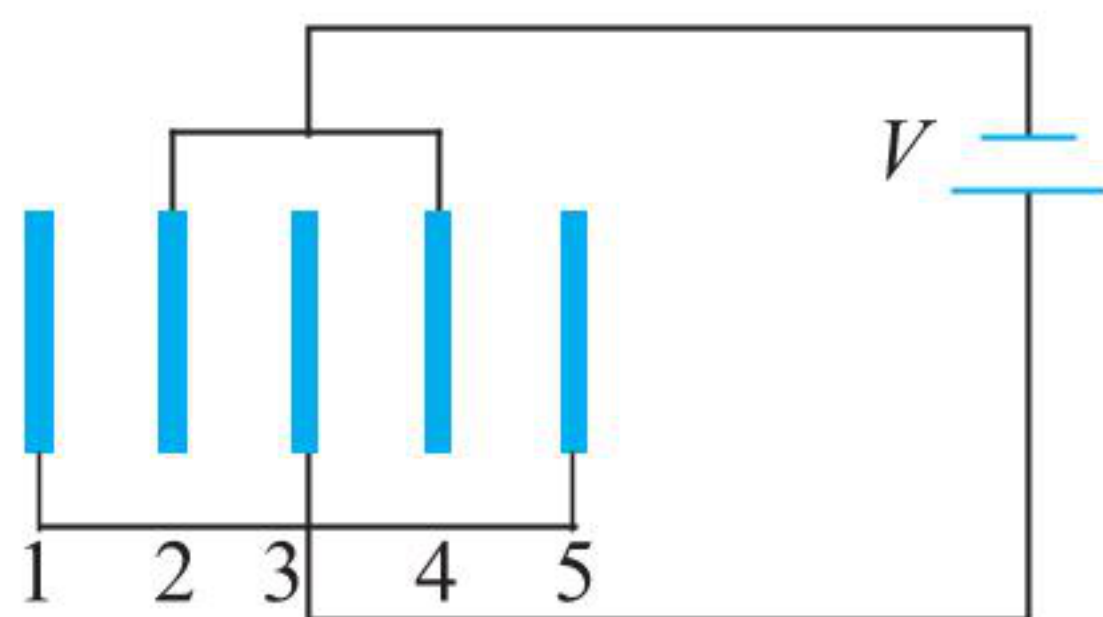
Column I	Column II
i. A dielectric slab is inserted	a. Charge on the capacitor increases
ii. Separation between plates is increased	b. Charge on the capacitor decreases
iii. A metal plate is inserted connecting both plates	c. Energy stored in the capacitor increases
iv. The left plate is grounded	d. No change occurs

2. Observe the circuit in figure and match the following (assume q_1 , q_2 , and q_3 be the charges on three capacitors).



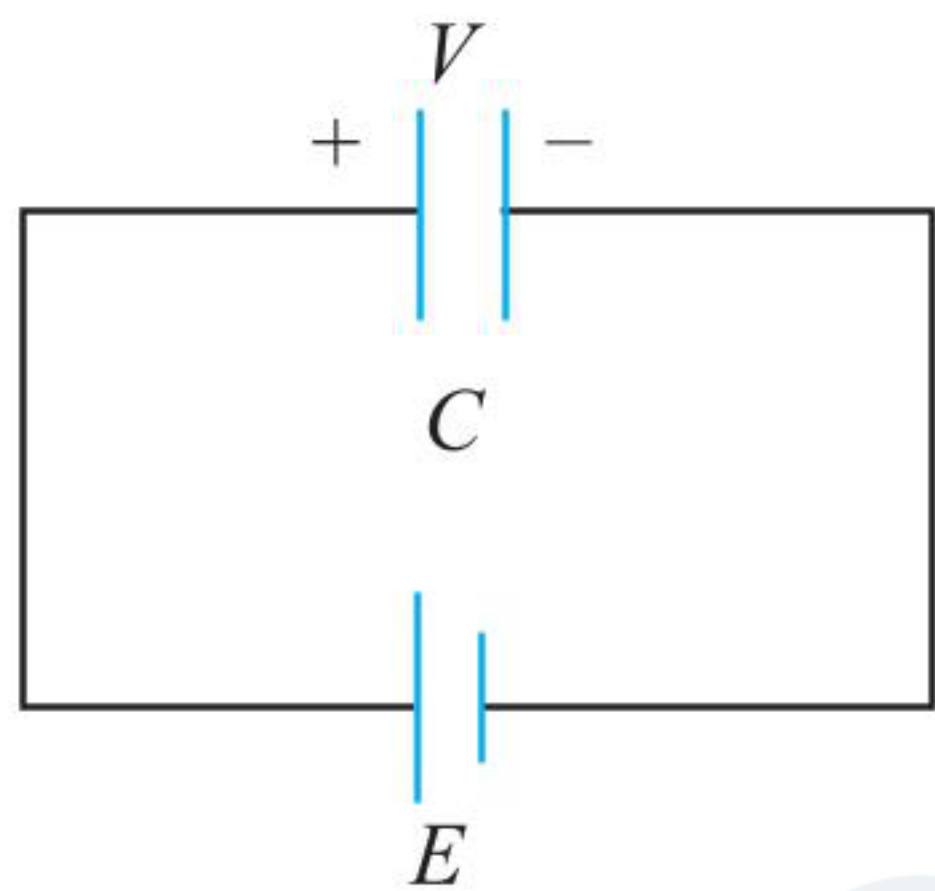
Column I	Column II
i. q_1 (in μC)	a. 50
ii. q_2 (in μC)	b. $\frac{10}{3}$
iii. q_3 (in μC)	c. $\frac{140}{3}$
iv. Potential difference across the $6\ \mu\text{F}$ capacitor is (in volt)	d. $\frac{25}{3}$

3. Five identical capacitor plates, each of area A , are arranged such that the adjacent plates are at a distance d apart. The plates are connected to a source of emf V as shown in figure. Match the following.



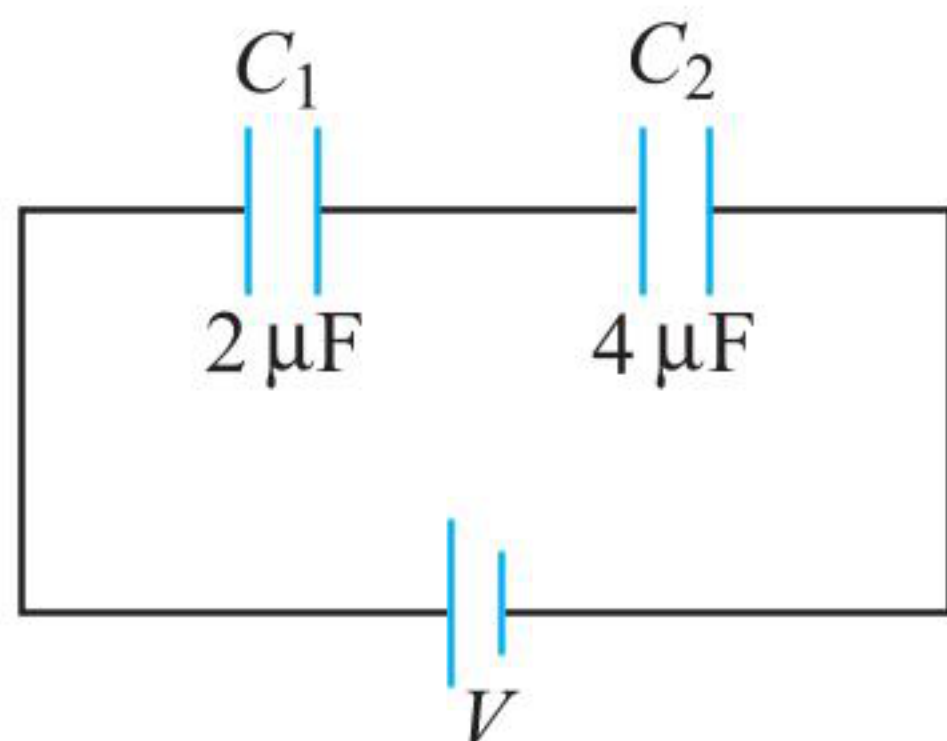
Column I	Column II
i. Charge on plate 1	a. $-2\epsilon_0 AV/d$
ii. Charge on plate 4	b. $+\epsilon_0 AV/d$
iii. Potential difference between plates 2 and 3	c. zero
iv. Potential difference between plates 1 and 5	d. V

4. A capacitor of capacitance C is charged to a potential V . Now it is connected to a battery of emf E as shown in figure. Based on this information, match the entries of Column I with the entries of Column II in the following table.



Column I	Column II
i. If $V = E$, then	a. charge flows in the circuit
ii. If $V > E$, then	b. no charge flows in the circuit
iii. If $V < E$, then	c. nonzero thermal energy will be dissipated in the circuit
iv. If the plates of capacitor are shorted, then	d. outer surfaces of the plates of capacitor have zero charge

5. In figure, the separation between the plates of C_1 is slowly increased to double of its initial value. Now match the entries in columns I and II.

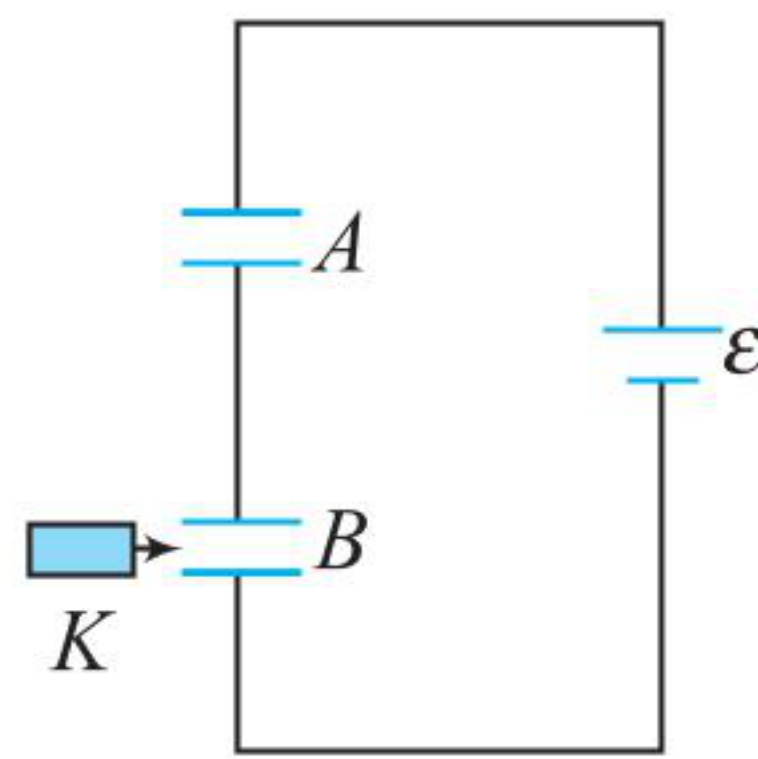


Column I	Column II
i. The potential difference across C_1	a. increases
ii. The potential difference across C_2	b. decreases
iii. The energy stored in C_1	c. increases by a factor of 6/5
iv. The energy stored in C_2	d. decreases by a factor of 18/25

6. Match the column.

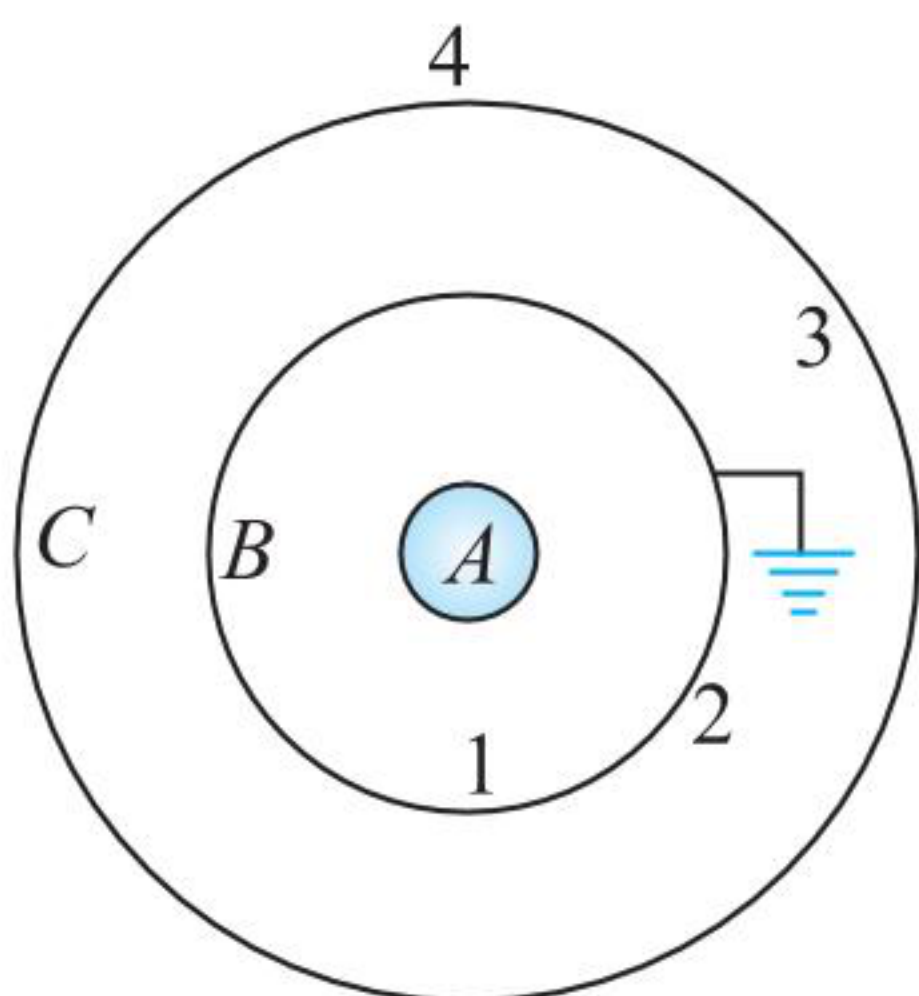
Column I	Column-II
i. If breakdown voltage of each capacitor is same, which combination can withstand largest external voltage.	a.
ii. If same charge is supplied, which combination acquires largest potential difference.	b.
iii. If same potential difference is applied, which combination can store maximum charge.	c.
iv. If capacitors are identical, which combination gives maximum equivalent capacitance.	d.

7. Two identical capacitors A and B are connected to a battery of emf ϵ as shown in figure. Now a dielectric slab is inserted between the plates of capacitor B while battery remains connected. Due to this insertion, some physical quantities may change, which are mentioned in Column I and the effect is mentioned in Column II. Match the Column I with Column II.



Column I	Column II
i. Charge on A	a. increases
ii. Charge on B	b. decreases
iii. Potential difference across A	c. remains constant
iv. Potential difference across B	d. will change

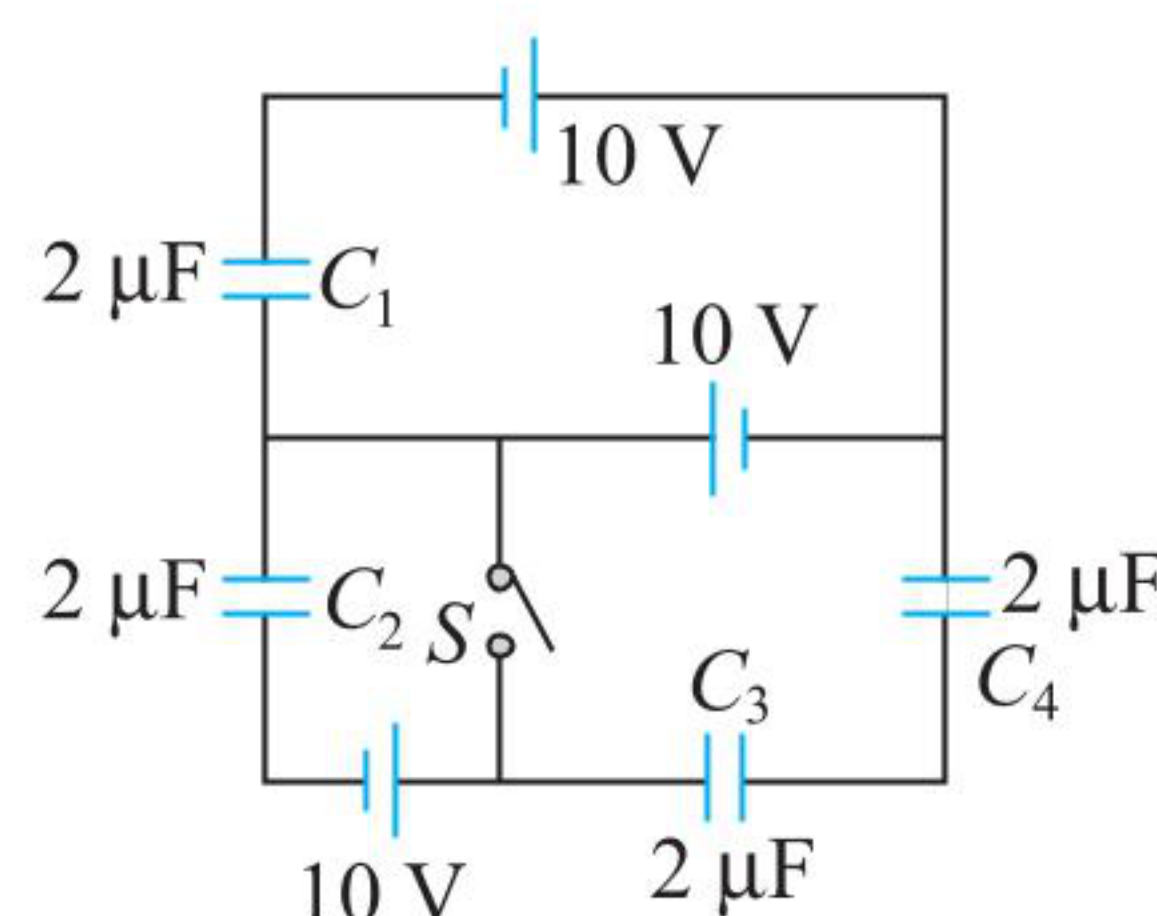
8. Figure shows three concentric thin spherical shells A , B , and C of radii R , $2R$, and $3R$. The shell B is earthed and A and C are given charges q and $2q$, respectively. If the magnitude of charge appearing on surfaces 1, 2, 3, and 4 are q_1 , q_2 , q_3 , and q_4 , respectively, then match the following columns.



Column I	Column II
i. q_1	a. q
ii. q_2	b. $\frac{4q}{3}$
iii. q_3	c. $\frac{2q}{3}$
iv. q_4	

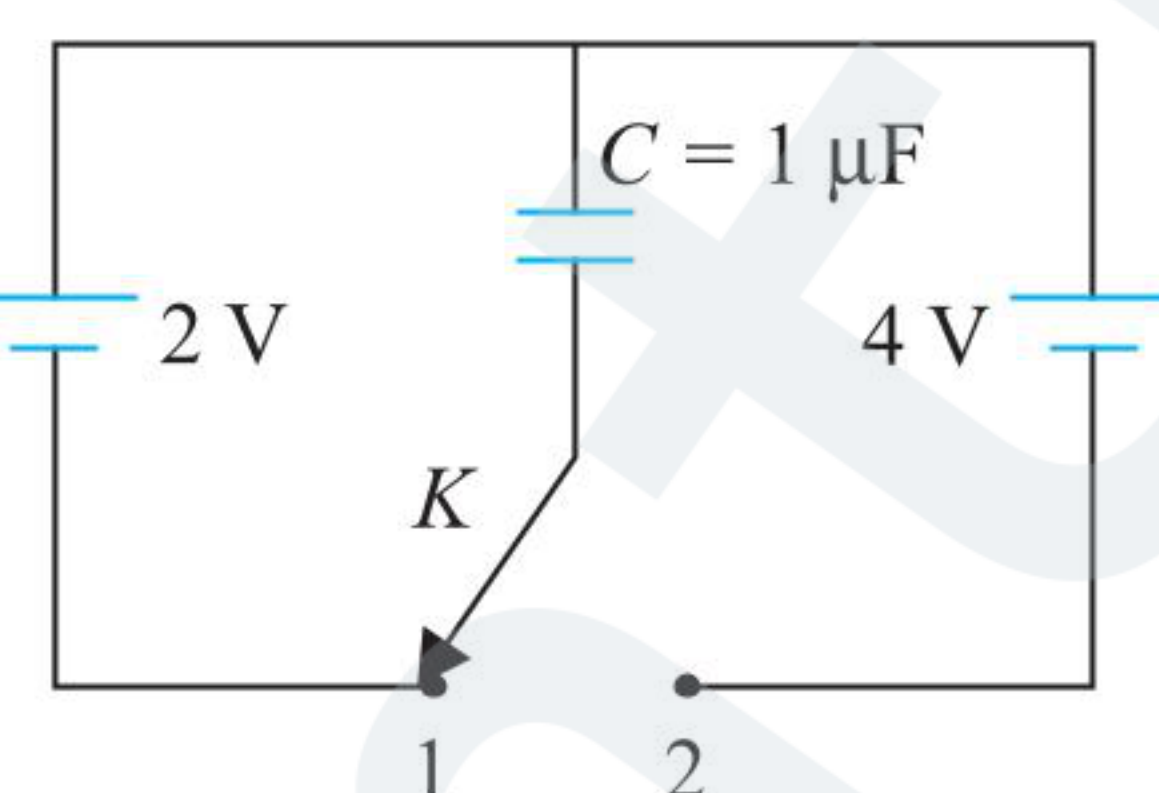
9. Before connecting the circuit shown in figure, all capacitors were uncharged. The circuit now is in steady state.

Now switch S is closed. During the time the steady state is reached again, match the following:



Column I	Column II
i. Charge on C_1	a. increases
ii. Charge on C_2	b. decreases
iii. Charge on C_3	c. remains same
iv. Charge on C_4	d. becomes zero

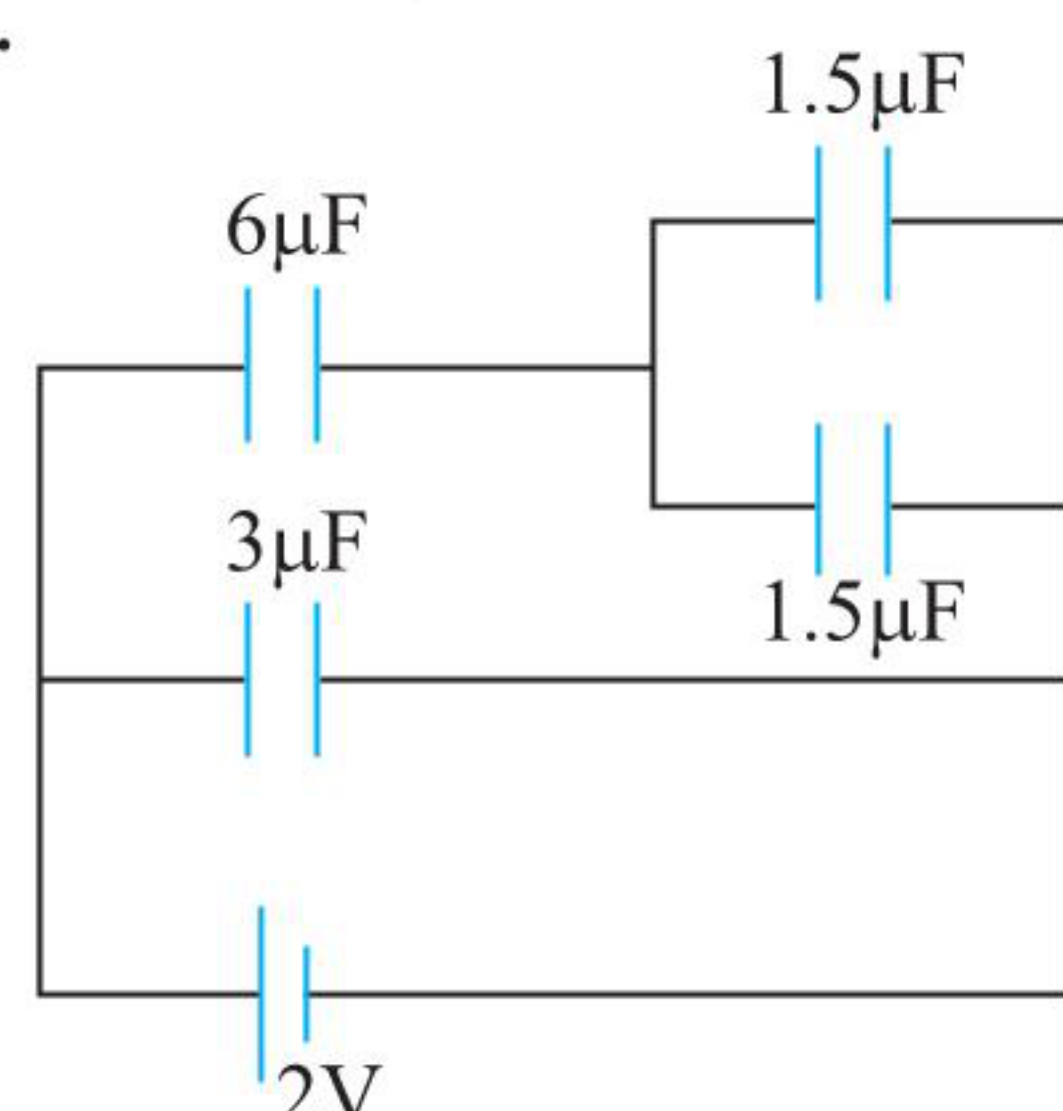
10. The circuit involves two ideal cells connected to a $1\ \mu\text{F}$ capacitor via a key K . Initially the key K is in position 1 and the capacitor is charged fully by $2\ \text{V}$ cell. The key is pushed to position 2. Column I gives physical quantities involving the circuit after the key K is pushed from position 1 to 2. Column II given corresponding results. Match the statements in Column I with the corresponding values in Column II.



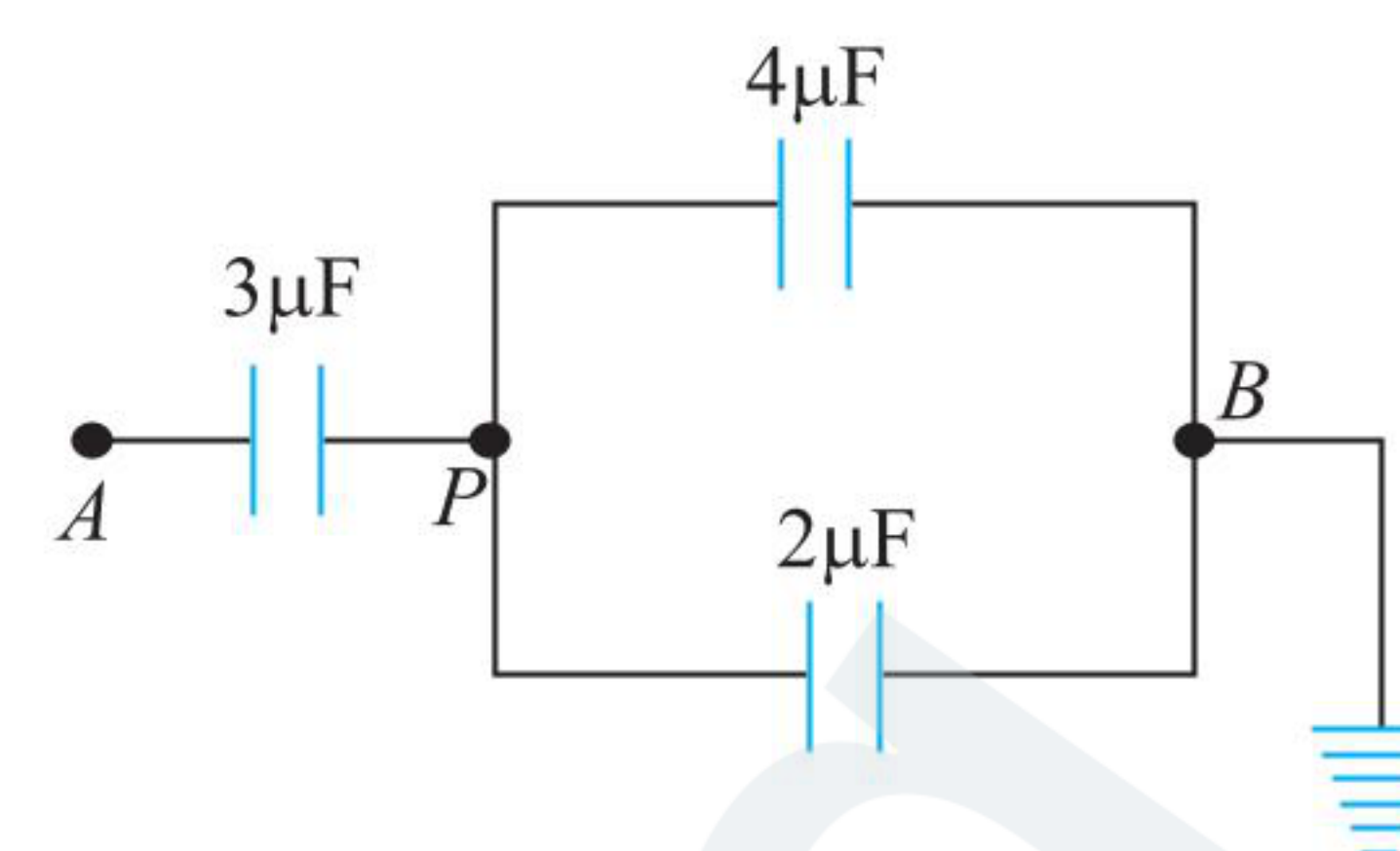
Column I	Column II
i. The net charge crossing the $4\ \text{V}$ cell (in μC) is	a. 2
ii. The magnitude of work done by $4\ \text{V}$ cell (in μJ) is	b. 6
iii. The gain in potential energy of capacitor (in μJ) is	c. 8
iv. The net heat produced in circuit (in μJ) is	d. 16

Numerical Value Type

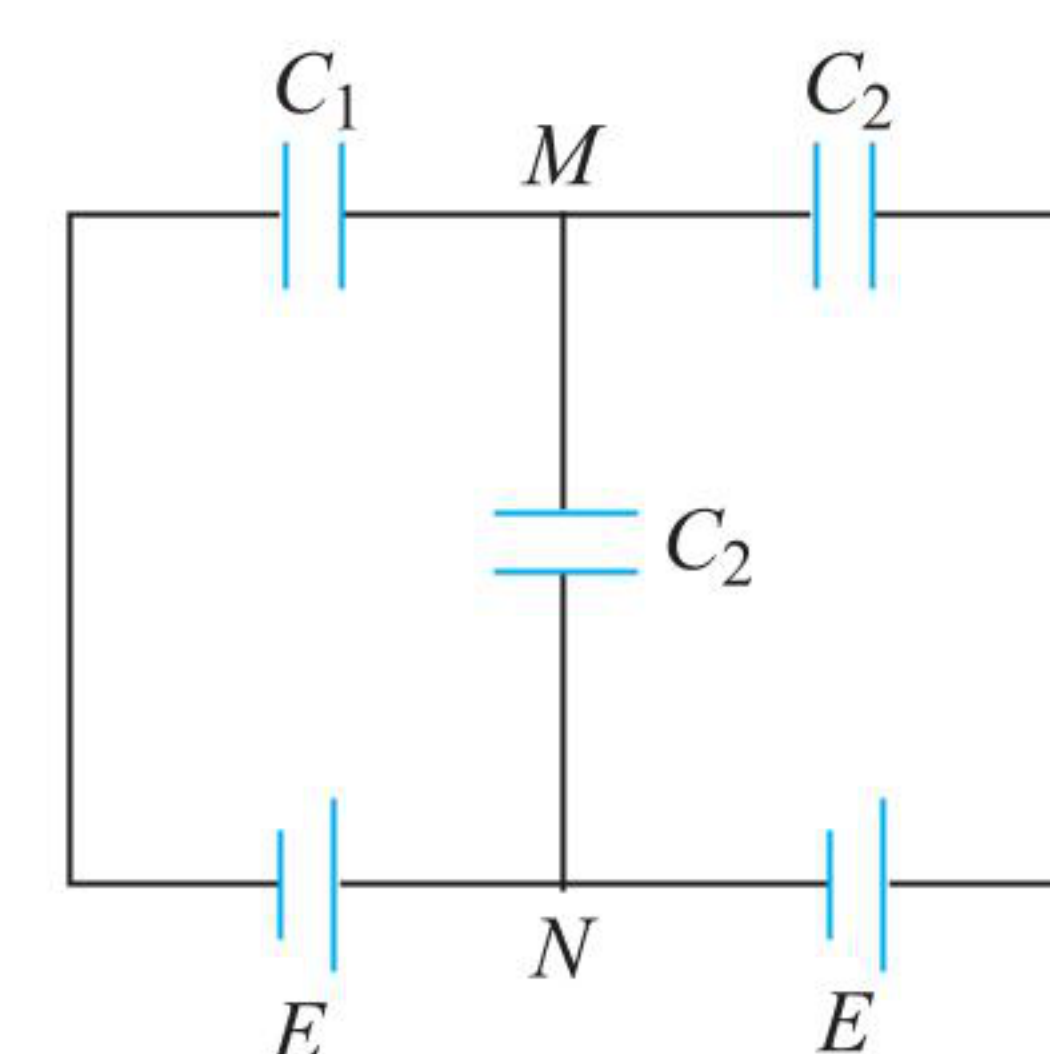
1. Each capacitance shown in figure is in μF . Find the charge on $6\ \mu\text{F}$ in μC .



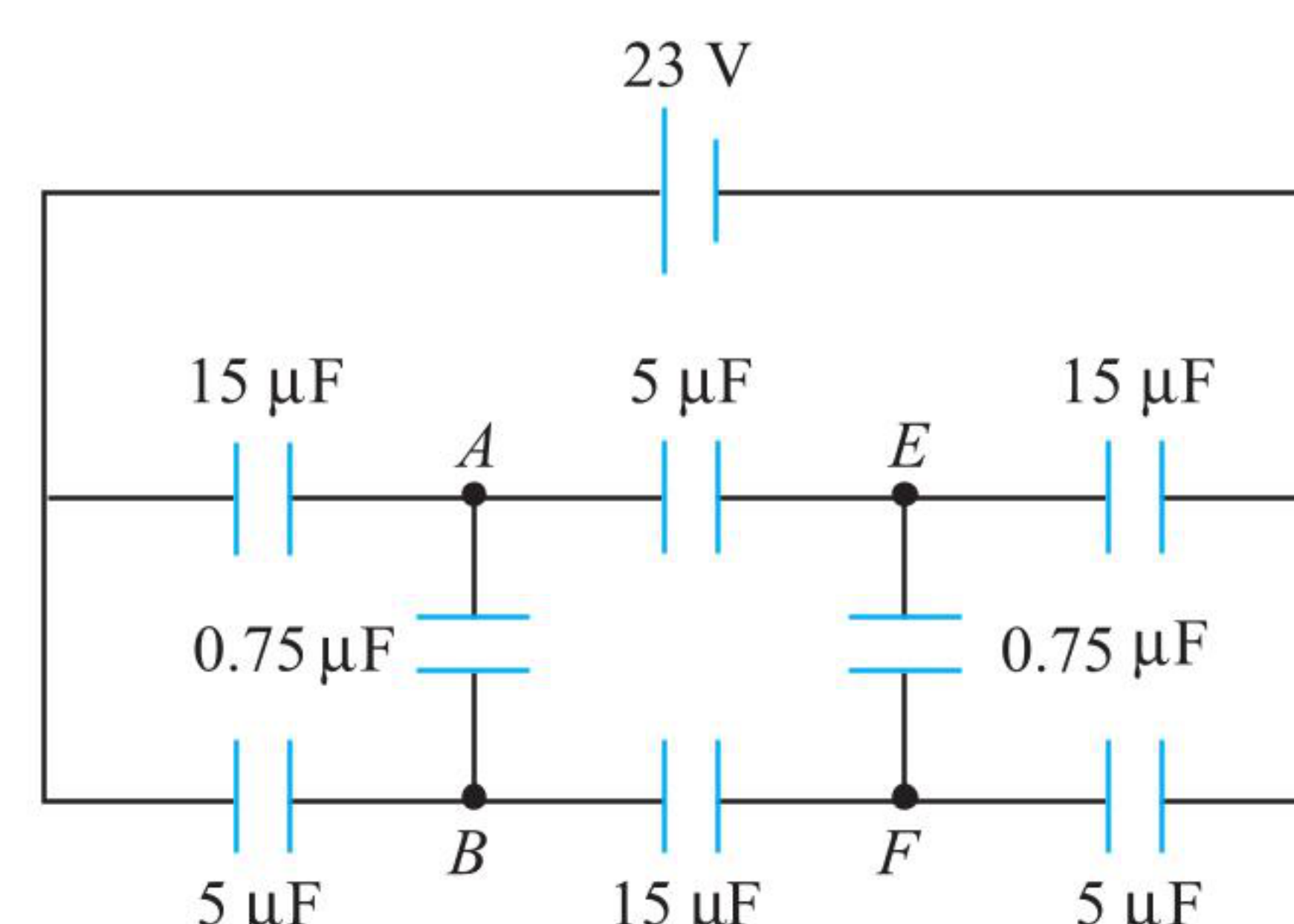
2. In figure, a potential of $+12\ \text{V}$ is given to point A , and point B is earthed. What is the potential at the point P in V ?



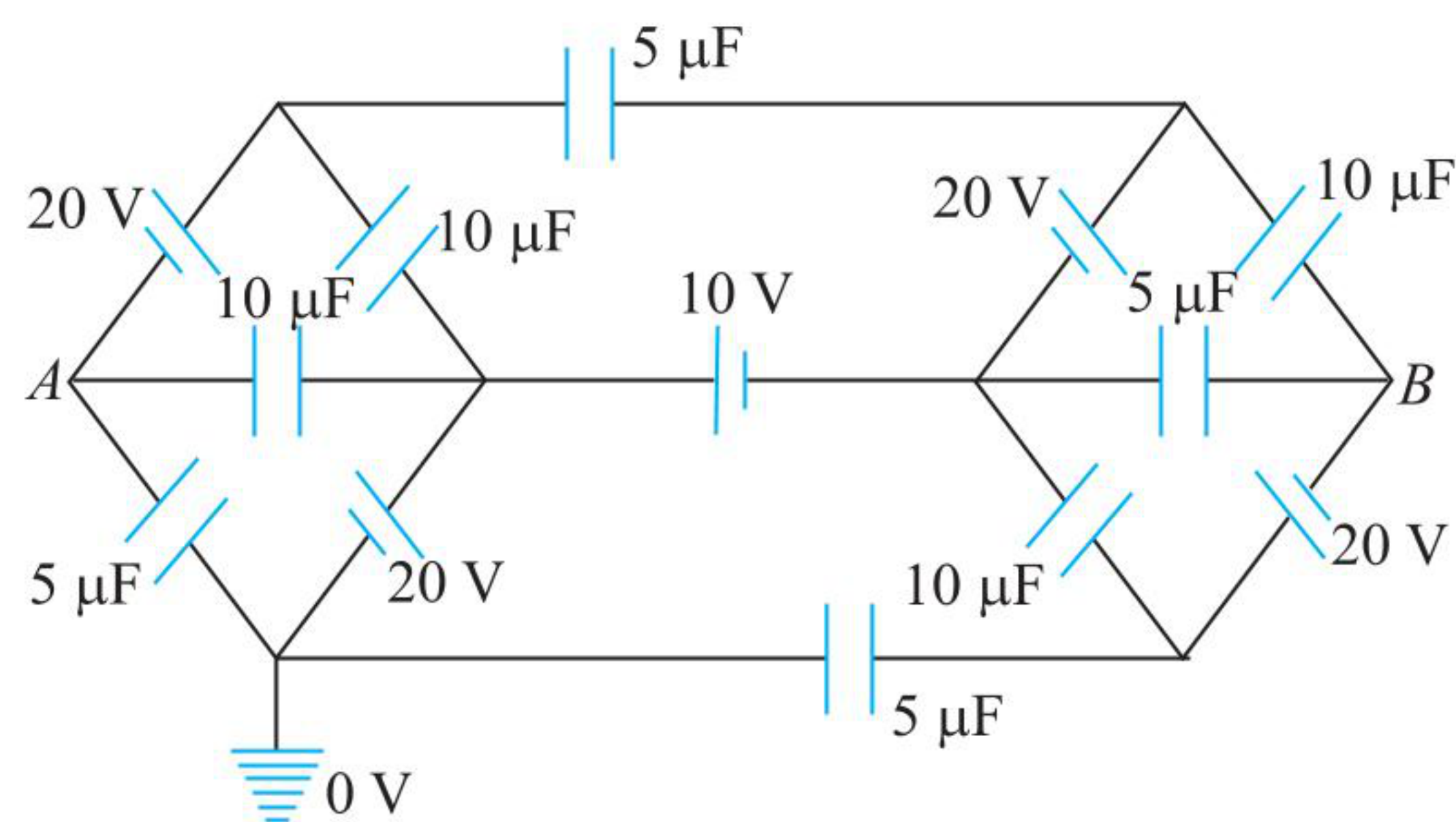
3. The capacitance of a capacitor becomes $4/3$ times its original value if a dielectric slab of thickness $t = d/2$ is inserted between the plates (d is the separation between the plates). What is the dielectric constant of the slab?
4. A spherical drop of capacitance $12\ \mu\text{F}$ is broken into eight drops of equal radius. What is the capacitance of each small drop in μF ?
5. A parallel plate capacitor of capacity C_0 is charged to a potential V_0 . E_1 is the energy stored in the capacitor when the battery is disconnected and the plate separation is doubled, and E_2 is the energy stored in the capacitor when the charging battery is kept connected and the separation between the capacitor plates is doubled. Find the ratio E_1/E_2 .
6. A capacitor of capacitance $C_1 = 1\ \mu\text{F}$ can withstand a maximum voltage of $V_1 = 6\ \text{kV}$, and another capacitor of capacitance $C_2 = 2\ \mu\text{F}$ can withstand a maximum voltage of $V_2 = 4\ \text{kV}$. If they are connected in series, what maximum voltage will the system withstand? (in kV)
7. Find the potential difference between the points M and N (in V) of the system shown in figure if the emf is equal to $E = 110\ \text{V}$ and the capacitance ratio C_2/C_1 is 23.



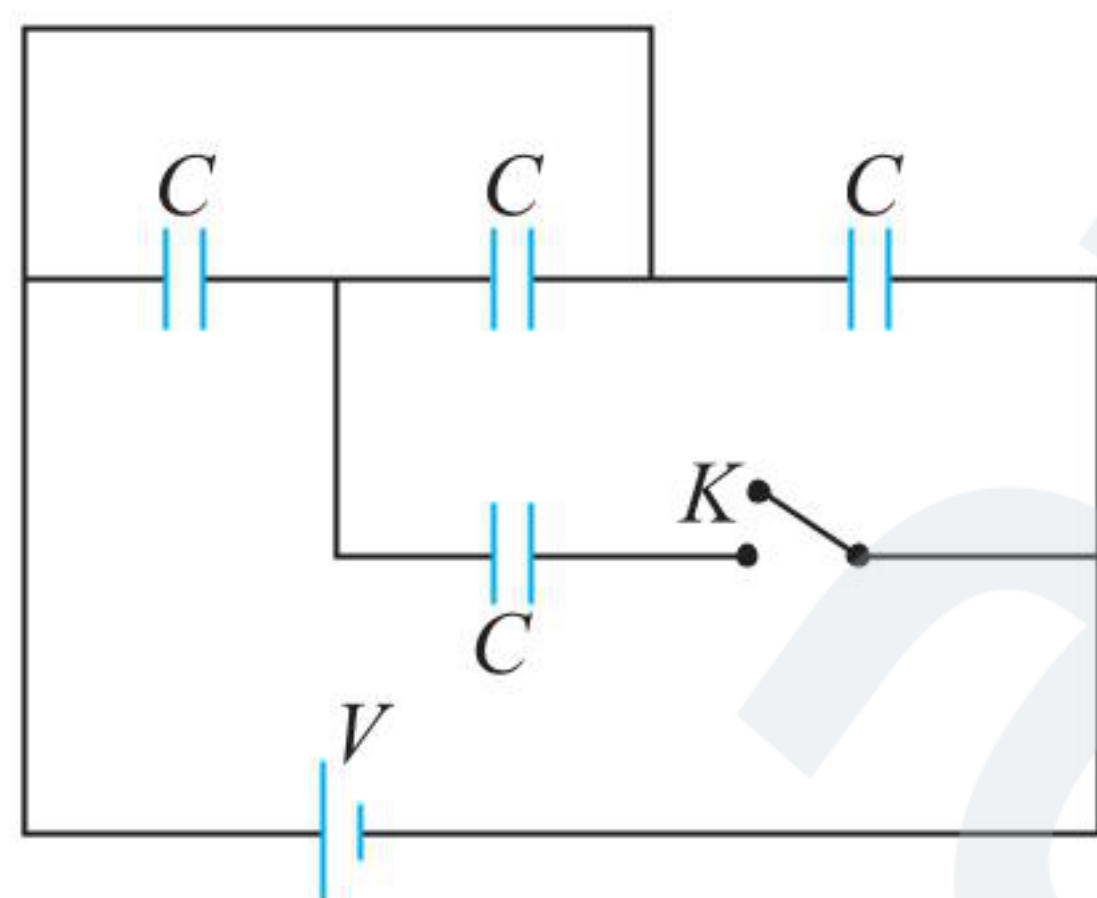
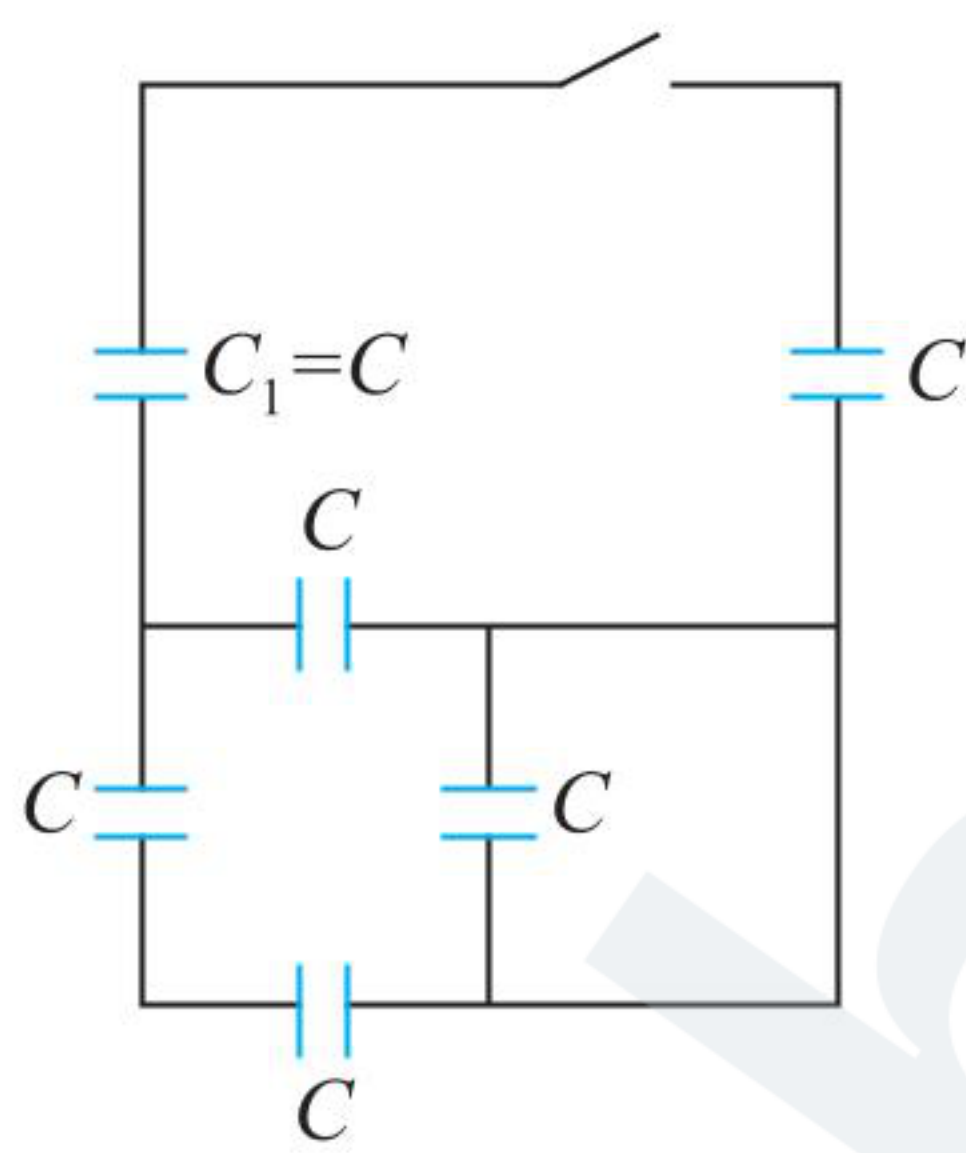
8. Find the ratio of the magnitudes of the potential difference between the points A and B and that between E and F of the circuit shown in figure.



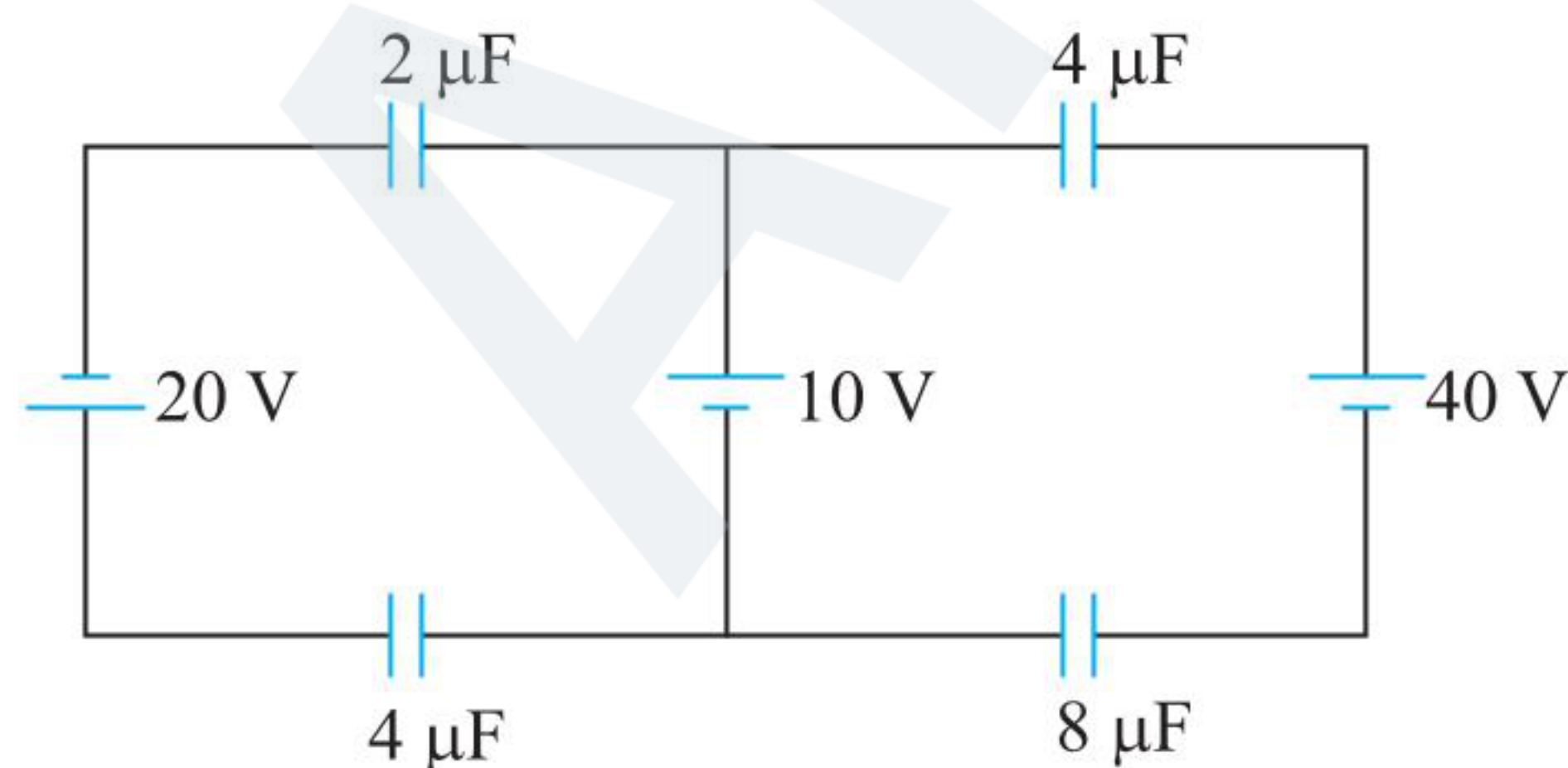
9. Find the potential difference between the points A and B (in V) in the circuit shown in figure.



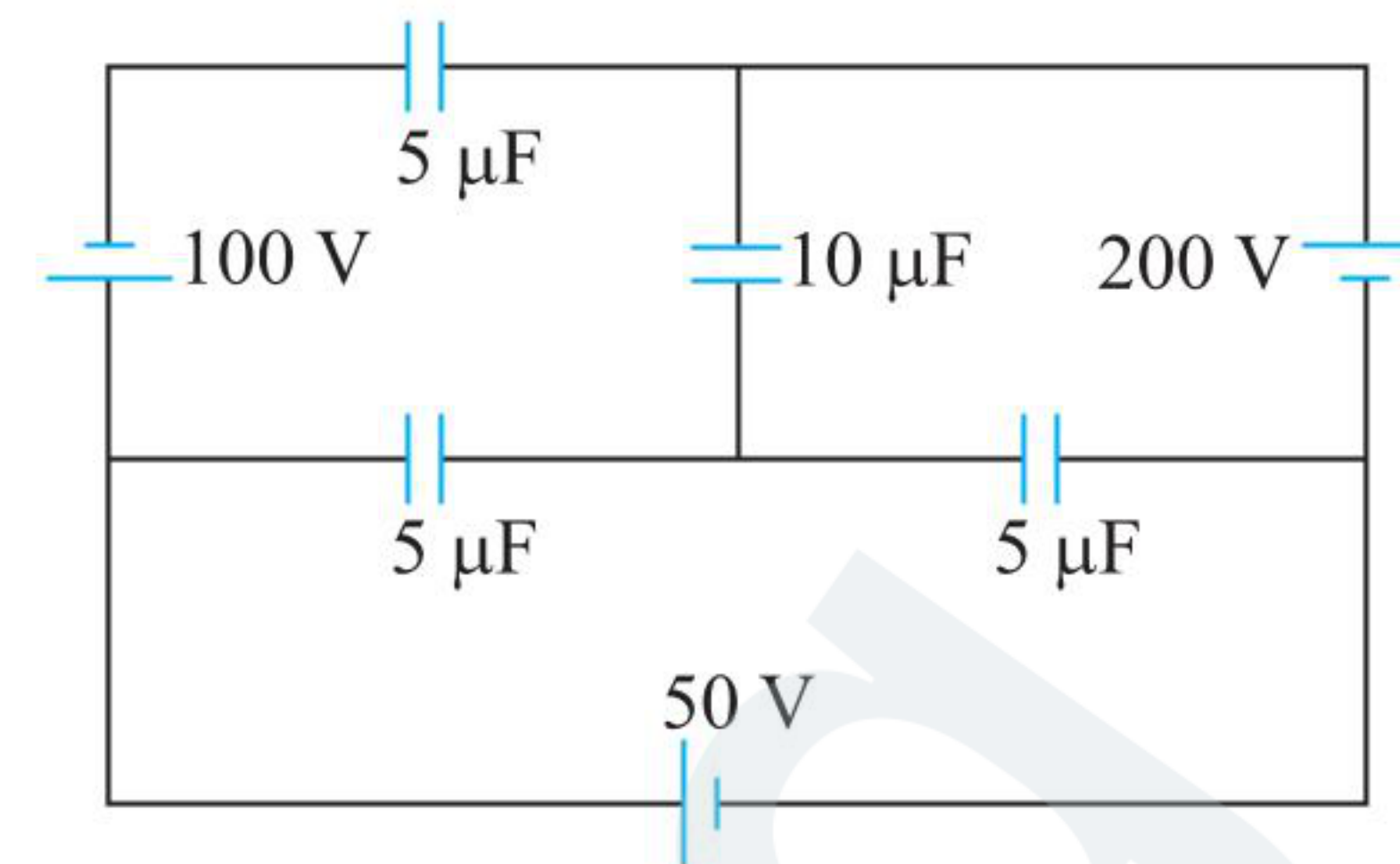
10. A charge of $20 \mu\text{C}$ is placed on the positive plate of an isolated parallel plate capacitor of capacitance $10 \mu\text{F}$. Calculate the potential difference (in V) developed between the plates.
11. A capacitor with stored energy 4.0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored (in J) in the two capacitors.
12. In the circuit shown, all capacitors are identical. Initially, the switch is open and the capacitor marked C_1 is the only one charged to a value Q_0 . After the switch is closed and the equilibrium is reestablished, the charge on the capacitor marked C_1 is Q . Find the ratio of initial charge to final charge in capacitor C_1 .
13. Find the amount by which the total energy stored in the capacitor (in μJ) will increase in the circuit shown in the figure after switch K is closed. Consider all capacitors are of capacitance $3 \mu\text{F}$ and battery voltage is 10V .



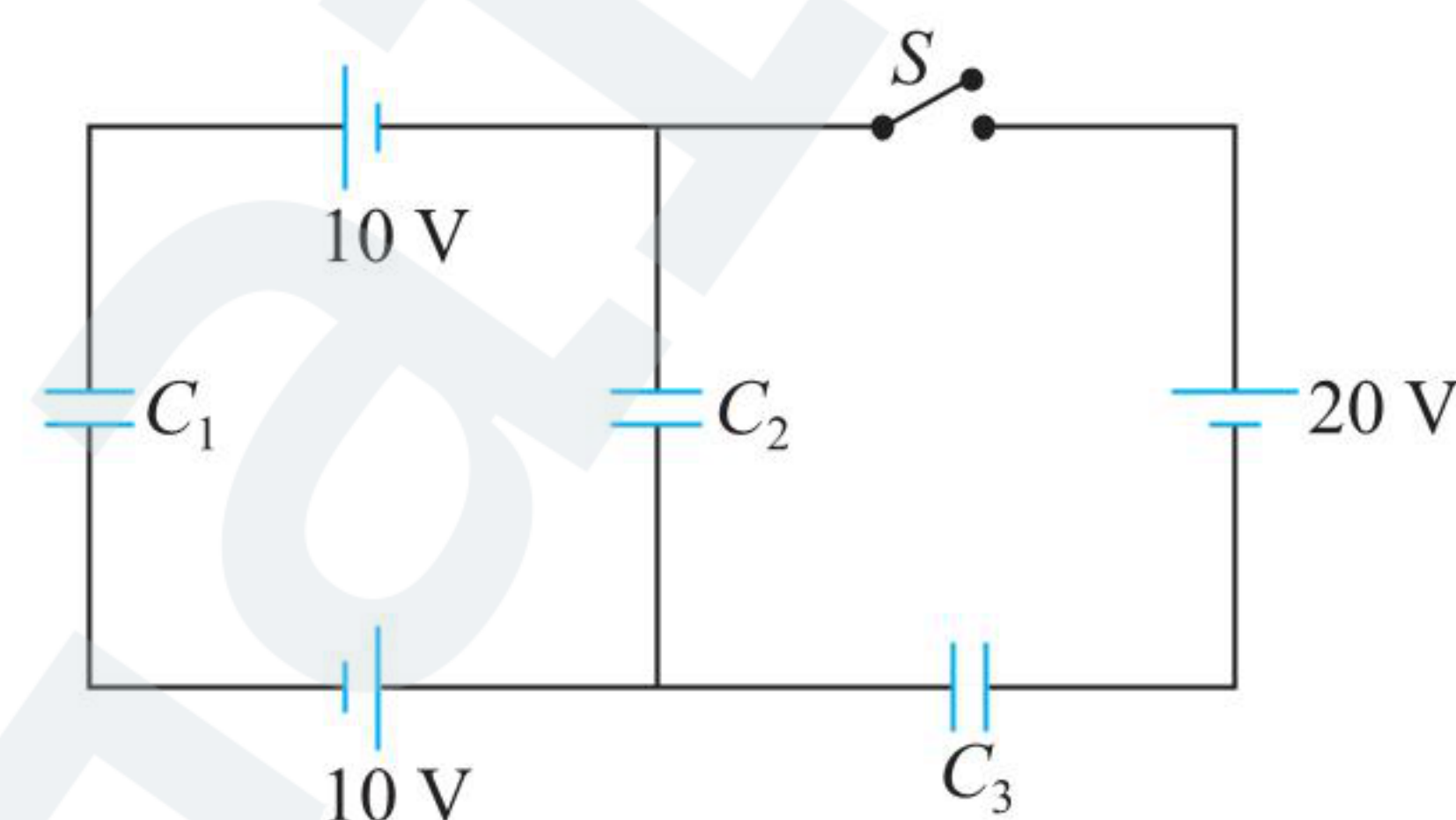
14. In the circuit shown in figure. Calculate the charge on $3 \mu\text{F}$ capacitor in steady state (in μC).



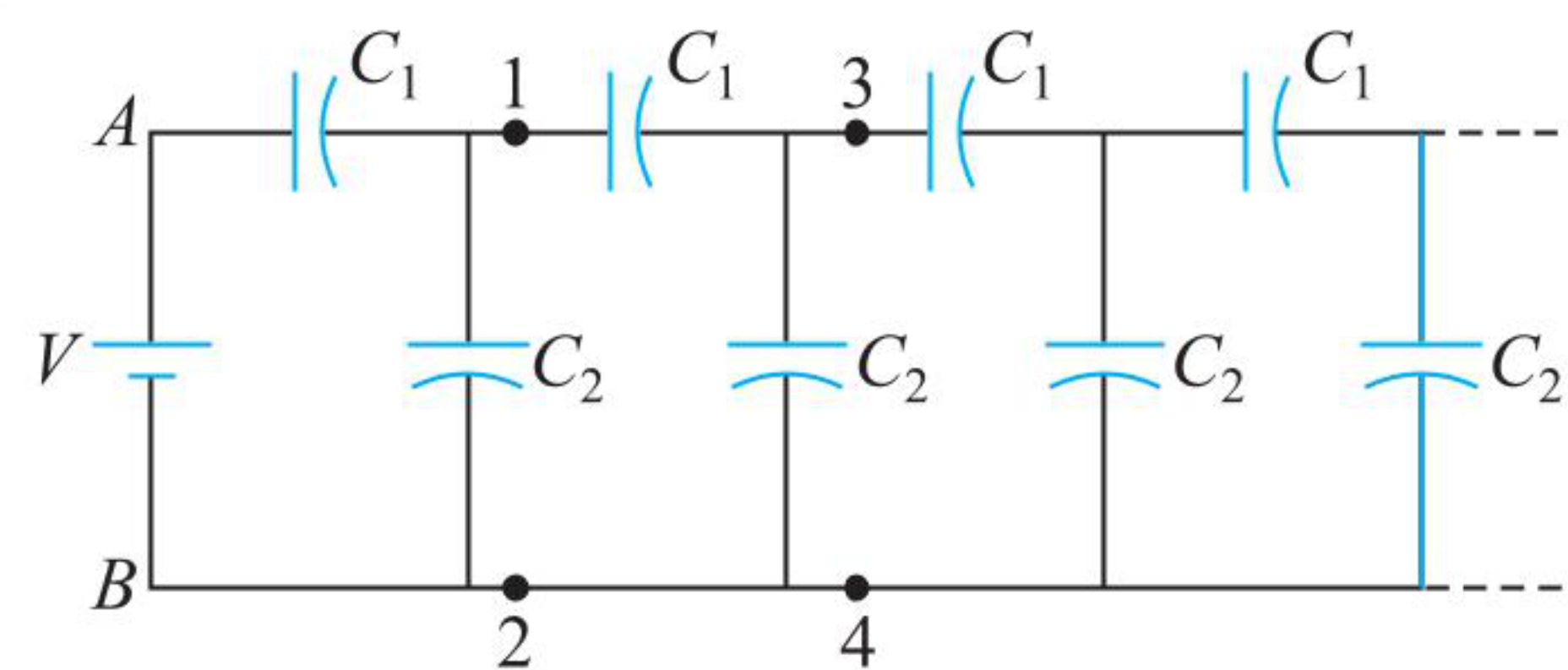
15. In the circuit shown in figure. Calculate the charge stored in $10 \mu\text{F}$ capacitor in steady state (in μC).



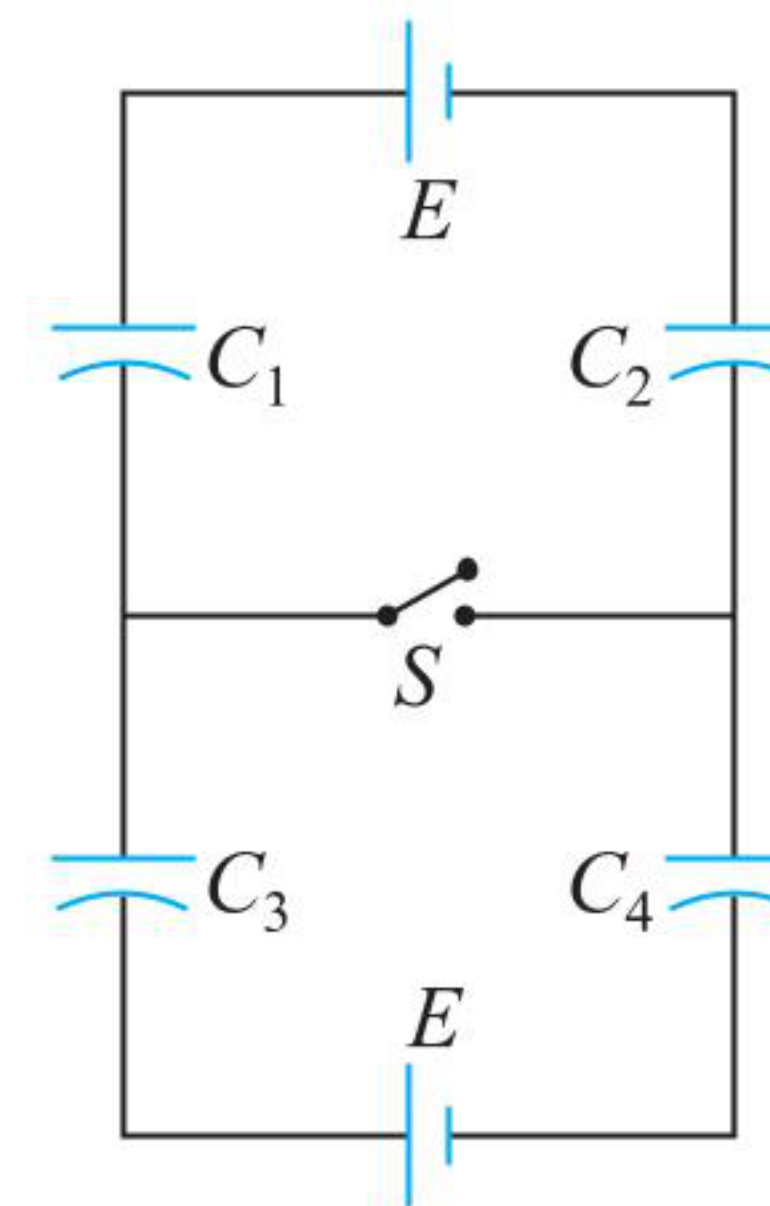
16. Find the amount of heat produced (in mJ) in the circuit shown in figure when switch is closed (μJ). Each capacitor in circuit is of capacitance $2 \mu\text{F}$.



17. A parallel plate capacitor is to be constructed which can store $q = 10 \mu\text{C}$ charge at $V = 1000 \text{ volt}$. The minimum plate area of the capacitor is required to be A_1 when space between the plates has air. If a dielectric of constant $K = 3$ is used between the plates, the minimum plate area required to make such a capacitor is A_2 . The breakdown field for the dielectric is 8 times that of air. Find A_1/A_2 .
18. In the given figure the capacitor circuit continues to infinity. The battery connected has e.m.f equal to V . The potential difference between points 1 and 2 is $V/2$, that between points 3 and 4 is $V/4$ and so on; i.e., the potential difference becomes $1/2$ after every step of the ladder. Find the ratio C_1/C_2 .



19. In the given circuit diagram, $E = 12 \text{ V}$, $C_1 = 4 \mu\text{F}$, $C_2 = 2 \mu\text{F}$, $C_3 = 6 \mu\text{F}$ and $C_4 = 3 \mu\text{F}$. Find the heat produced in the circuit (in μJ) after switch S is shorted.

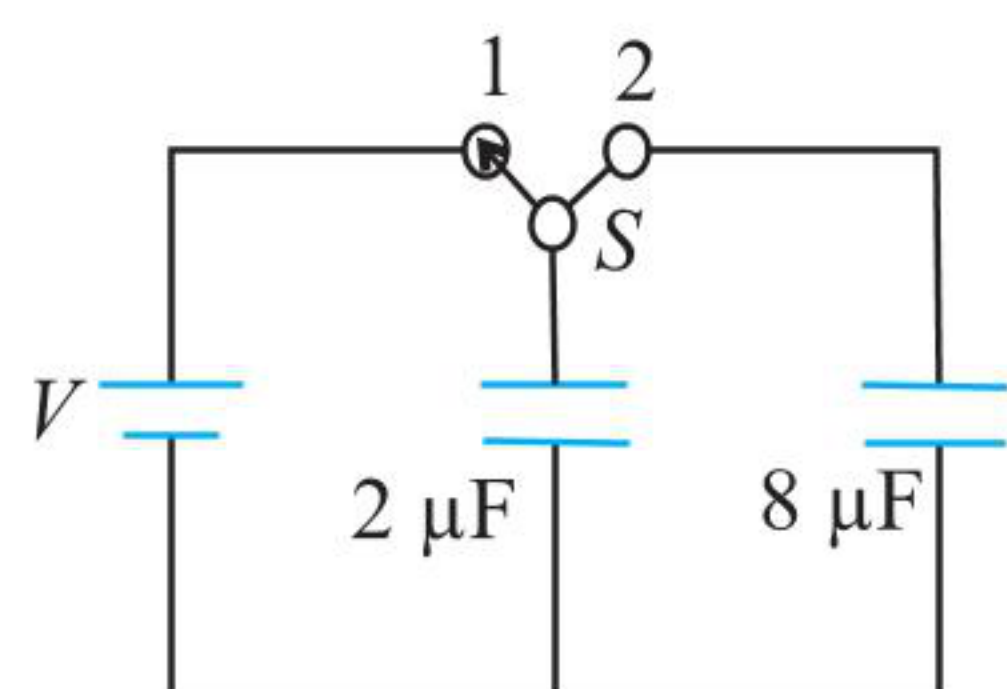


Archives

JEE ADVANCED

Single Correct Answer Type

1. A $2\ \mu\text{F}$ capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after switch S is turned to position 2 is

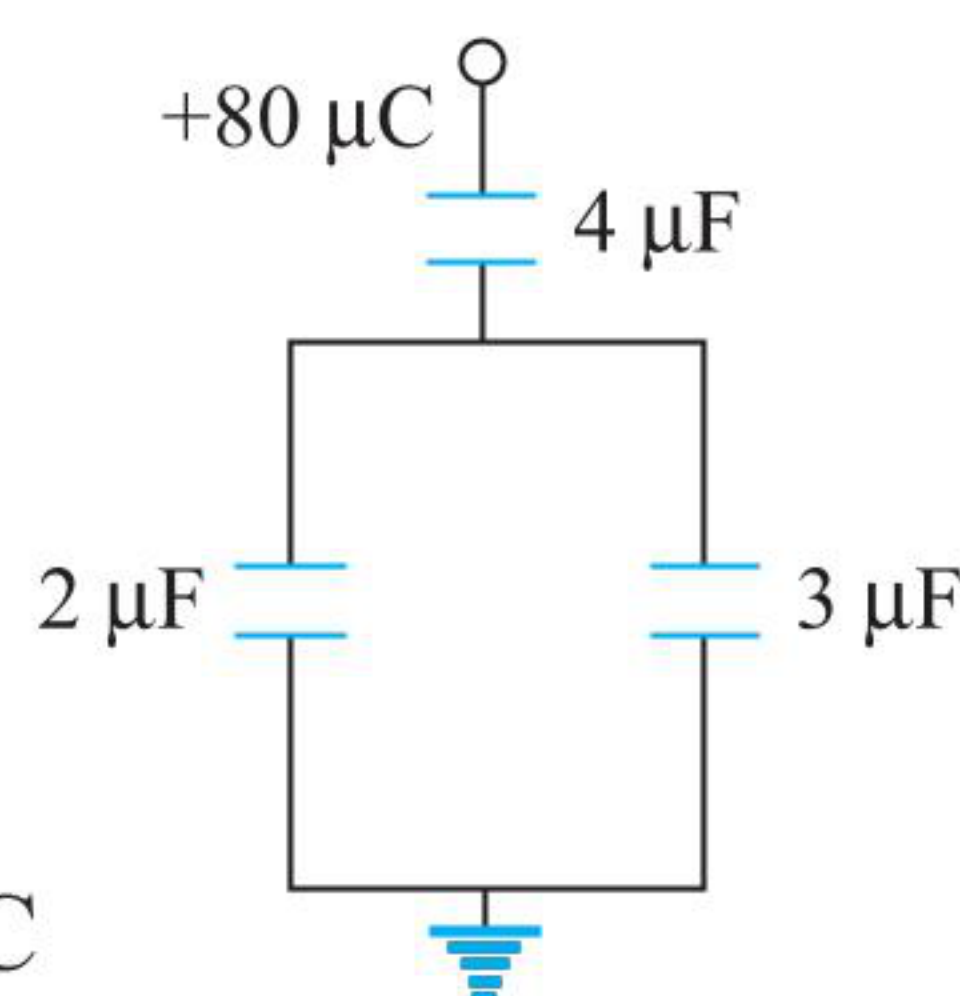


- (1) 0% (2) 20%
(3) 75% (4) 80%

(IIT-JEE 2011)

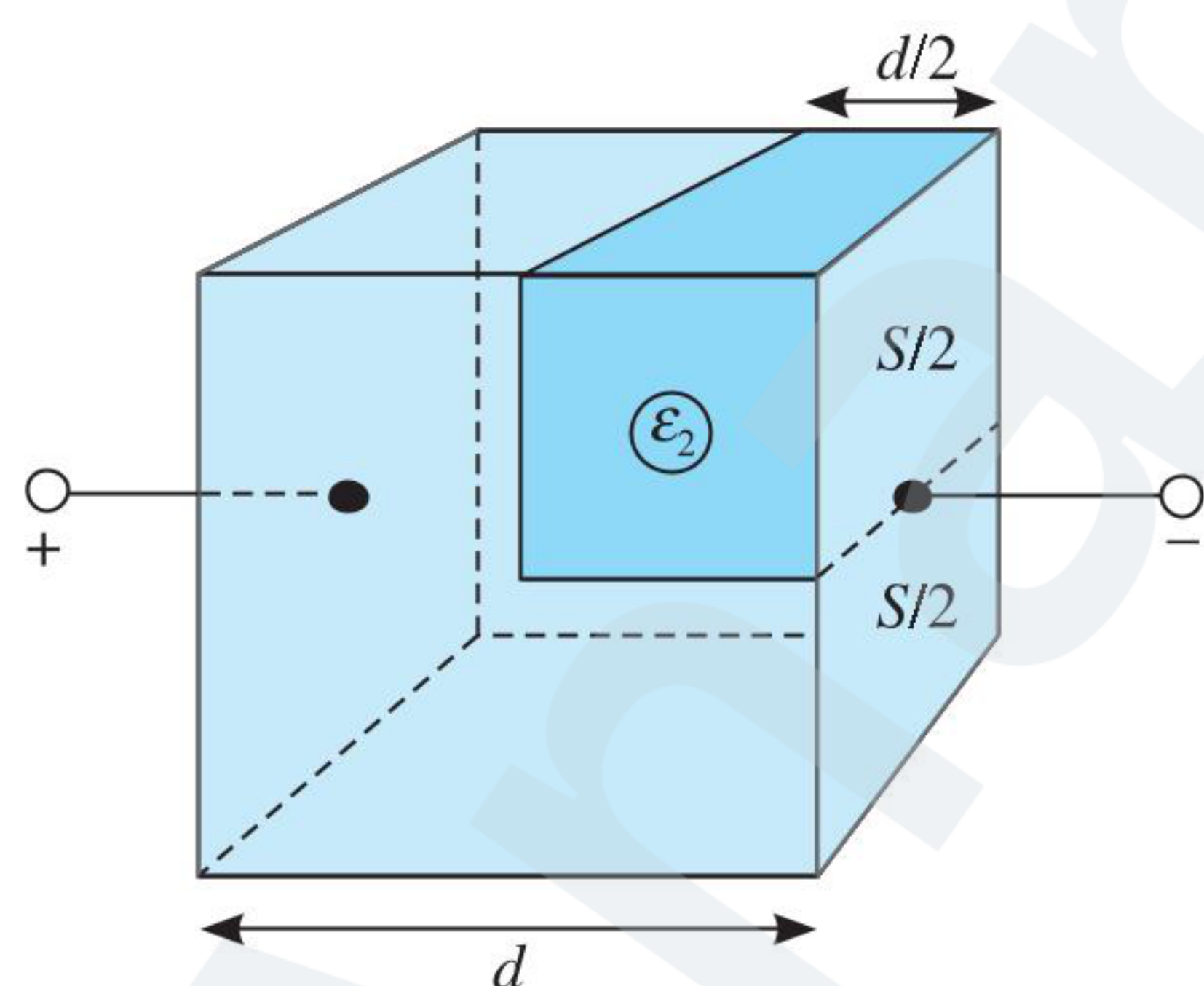
2. In the given circuit, a charge of $+80\ \mu\text{C}$ is given to the upper plate of the $4\ \mu\text{F}$ capacitor. Then in the steady state, the charge on the upper plate of the $3\ \mu\text{F}$ capacitor is

- (1) $+32\ \mu\text{C}$ (2) $+40\ \mu\text{C}$
(3) $+48\ \mu\text{C}$ (4) $+80\ \mu\text{C}$



(IIT-JEE 2012)

3. A parallel plate capacitor having plates of area S and plate separation d , has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\epsilon_1 = 2$ and $\epsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2 . The ratio $\frac{C_2}{C_1}$ is

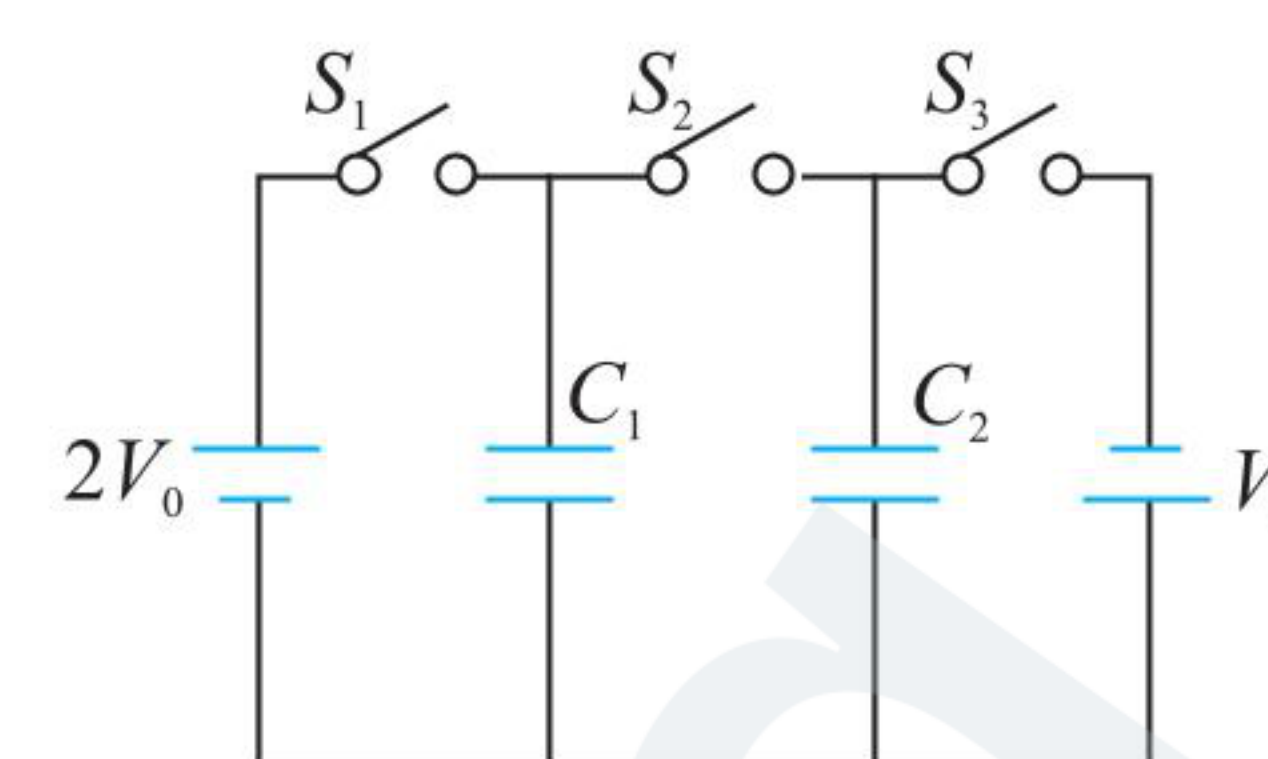


- (1) $6/5$ (2) $5/3$
(3) $7/5$ (4) $7/3$

(JEE Advanced 2015)

Multiple Correct Answers Type

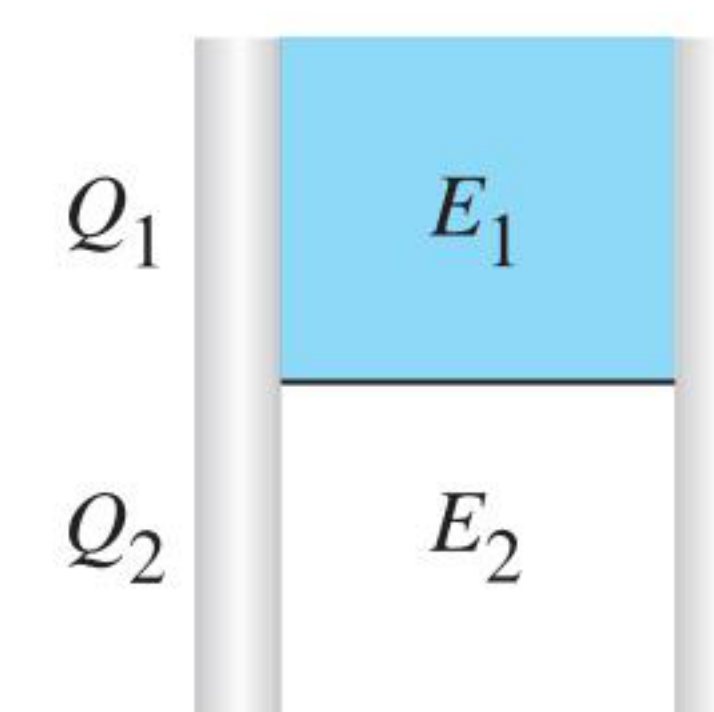
1. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C . The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S is then pressed to charge the capacitor C_2 . After some time, S_2 is released and then S is pressed. After some time,



- (1) the charge on the upper plate of C_1 is $2CV_0$
(2) the charge on the upper plate of C_1 is CV_0
(3) the charge on the upper plate of C_1 is 0
(4) the charge on the upper plate of C_2 is $-CV_0$

(IIT-JEE 2013)

2. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers $1/3$ of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1 . When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2 . The electric field in the dielectric is E_1 and that in the other portion is E_2 . Choose the correct option/ options, ignoring edge effects.

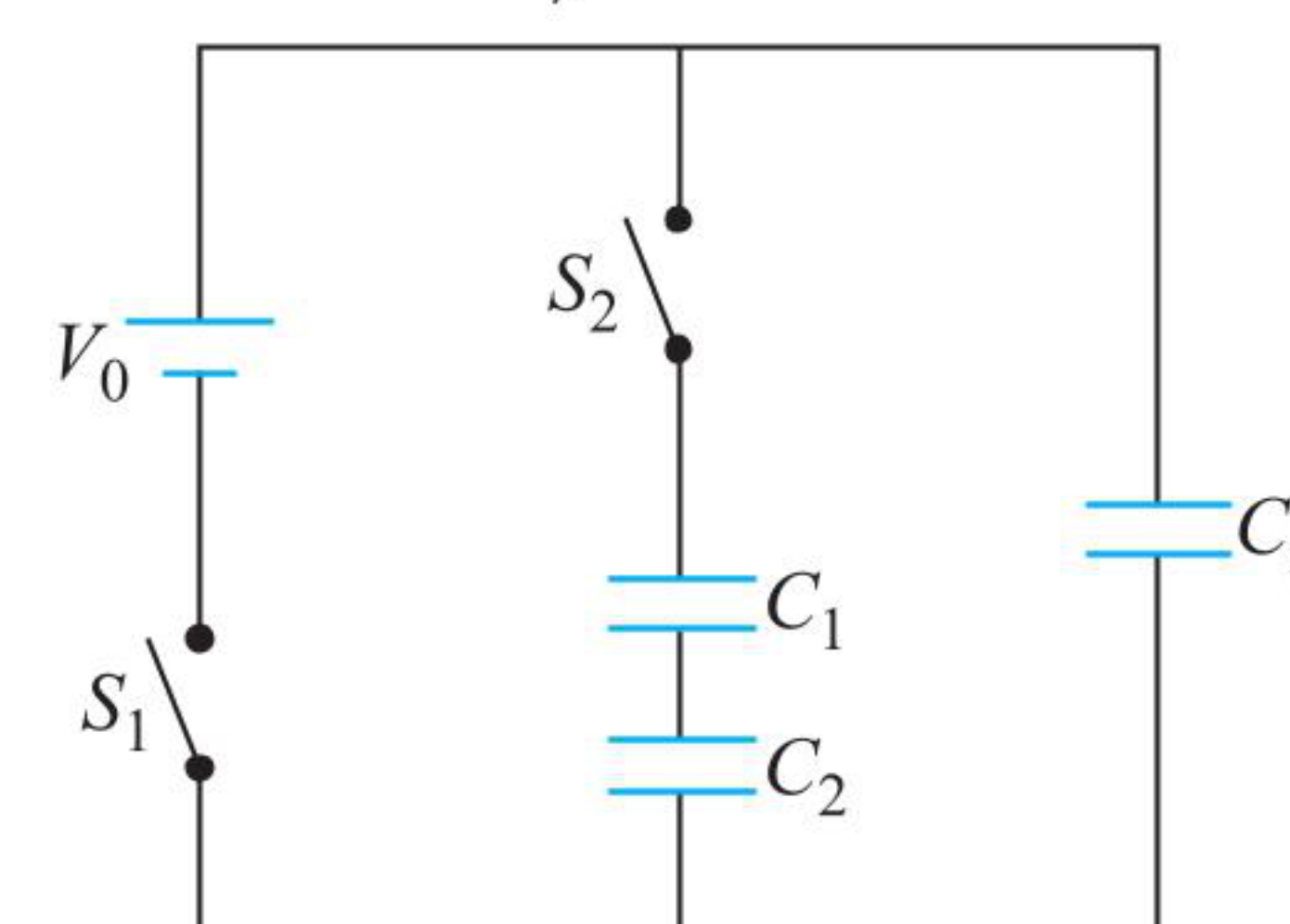


- (1) $\frac{E_1}{E_2} = 1$ (2) $\frac{E_1}{E_2} = \frac{1}{K}$
(3) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (4) $\frac{C}{C_1} = \frac{2+K}{K}$

(JEE Advanced 2014)

Numerical Value Type

1. Three identical capacitors C_1 , C_2 and C_3 have a capacitance of $1.0\ \mu\text{F}$ each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ϵ_r . The cell electromotive force (emf) $V_0 = 8\text{V}$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be $5\ \mu\text{C}$. The value of ϵ_r is



(JEE Advanced 2018)

2. A parallel plate capacitor of capacitance C has spacing d between two plates having area A . The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K \left(1 + \frac{m}{N} \right)$.

For a very large $N(>10^3)$, the capacitance C is $\alpha \left(\frac{K \epsilon_0 A}{d \ln 2} \right)$.
The value of α will be _____.
[ϵ_0 is the permittivity of free space]

(JEE Advanced 2019)

Answers Key

EXERCISES

Single Correct Answer Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (4) | 2. (1) | 3. (3) | 4. (2) | 5. (4) |
| 6. (2) | 7. (4) | 8. (4) | 9. (3) | 10. (2) |
| 11. (1) | 12. (3) | 13. (3) | 14. (1) | 15. (1) |
| 16. (4) | 17. (3) | 18. (2) | 19. (1) | 20. (3) |
| 21. (3) | 22. (1) | 23. (2) | 24. (4) | 25. (2) |
| 26. (3) | 27. (2) | 28. (2) | 29. (1) | 30. (3) |
| 31. (3) | 32. (3) | 33. (1) | 34. (2) | 35. (1) |
| 36. (4) | 37. (3) | 38. (2) | 39. (3) | 40. (1) |
| 41. (2) | 42. (4) | 43. (4) | 44. (2) | 45. (3) |
| 46. (3) | 47. (1) | 48. (2) | 49. (4) | 50. (1) |
| 51. (1) | 52. (4) | 53. (2) | 54. (1) | 55. (4) |
| 56. (3) | 57. (4) | 58. (2) | 59. (1) | 60. (1) |
| 61. (2) | 62. (4) | 63. (2) | 64. (1) | 65. (1) |
| 66. (2) | 67. (4) | 68. (3) | 69. (3) | 70. (1) |
| 71. (2) | 72. (1) | 73. (1) | 74. (2) | 75. (3) |

Multiple Correct Answers Type

- | | | |
|-----------------|----------------|-----------------|
| 1. (2),(3),(4) | 2. (2),(3),(4) | 3. (1),(2),(4) |
| 4. (1),(3),(4) | 5. (1),(2),(3) | 6. (1),(3) |
| 7. (2),(3) | 8. (1),(3),(4) | 9. (1),(3),(4) |
| 10. (1),(2),(4) | 11. (1),(2) | 12. (1),(4) |
| 13. (1),(4) | 14. (1),(3) | 15. (1),(2),(3) |
| 16. (2),(3) | 17. (1),(4) | 18. (2),(4) |
| 19. (1),(3),(4) | 20. (2),(4) | 21. (1),(2),(3) |
| 22. (1),(3) | 23. (1),(2) | 24. (1),(2) |
| 25. (2),(3),(4) | | |

Linked Comprehension Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (2) | 2. (4) | 3. (3) | 4. (2) | 5. (1) |
| 6. (2) | 7. (1) | 8. (4) | 9. (4) | 10. (1) |
| 11. (2) | 12. (3) | 13. (1) | 14. (4) | 15. (2) |
| 16. (3) | 17. (2) | 18. (1) | 19. (2) | 20. (4) |

- | | | | | |
|---------|---------|---------|---------|---------|
| 21. (1) | 22. (2) | 23. (4) | 24. (4) | 25. (2) |
| 26. (4) | 27. (1) | 28. (2) | 29. (3) | 30. (4) |
| 31. (4) | 32. (1) | 33. (3) | 34. (1) | |

Matrix Match Type

- i. $\rightarrow$ a, c.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a, c.; iv. $\rightarrow$ d.
- i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.; iv. $\rightarrow$ d.
- i. $\rightarrow$ b.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.
- i. $\rightarrow$ b., d.; ii. $\rightarrow$ a., c., d.; iii. $\rightarrow$ a., c., d.; iv. $\rightarrow$ a., c., d.
- i. $\rightarrow$ a., c.; ii. $\rightarrow$ b.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ b.
- i. $\rightarrow$ d.; ii. $\rightarrow$ d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ a.
- i. $\rightarrow$ a., d.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ b., d.
- i. $\rightarrow$ a.; ii. $\rightarrow$ b.; iii. $\rightarrow$ b.; iv. $\rightarrow$ c.
- i. $\rightarrow$ c.; ii. $\rightarrow$ a.; iii. $\rightarrow$ b.; iv. $\rightarrow$ b.
- i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ a.

Numerical Value Type

- | | | | | |
|------------|------------|-----------|-----------|-----------|
| 1. (4) | 2. (4) | 3. (2) | 4. (6) | 5. (4) |
| 6. (9) | 7. (22) | 8. (5) | 9. (10) | 10. (1) |
| 11. (2) | 12. (1.60) | 13. (100) | 14. (40) | 15. (700) |
| 16. (0.30) | 17. (24) | 18. (2) | 19. (240) | |

ARCHIVES

JEE Advanced

Single Correct Answer Type

- | | | |
|--------|--------|--------|
| 1. (4) | 2. (3) | 3. (4) |
|--------|--------|--------|

Multiple Correct Answers Type

- | | |
|------------|------------|
| 1. (2),(4) | 2. (1),(4) |
|------------|------------|

Numerical Value Type

- | | |
|-----------|-----------|
| 1. (1.50) | 2. (1.00) |
|-----------|-----------|

Chapter 4

Concept Application Exercises

Exercise 4.1

1. (a) $U = Q^2 / 2C$

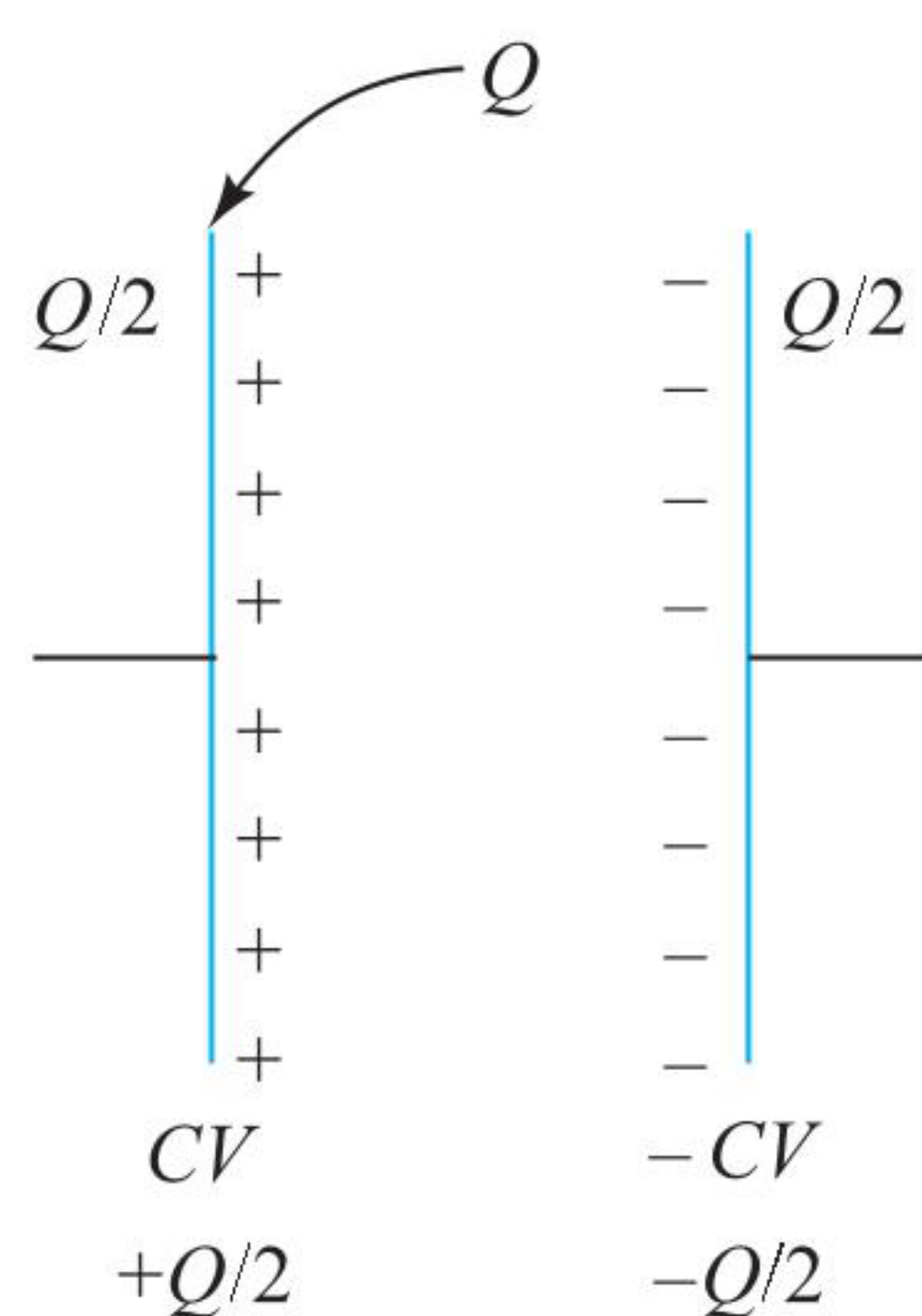
$$Q = \sqrt{2UC} = \sqrt{2(25.0 \text{ J})(5.00 \times 10^{-9} \text{ F})} = 5.00 \times 10^{-4} \text{ C}$$

The number of electrons N that must be removed from one plate and added to the other is

$$N = Q/e = (5.00 \times 10^{-4} \text{ C}) / (1.6 \times 10^{-19} \text{ C}) = 3.125 \times 10^{15} \text{ electrons}$$

(b) To double U while keeping Q constant, decrease C by a factor of 2. $C = \epsilon_0 A/d$, half the plate area or double the plate separation.

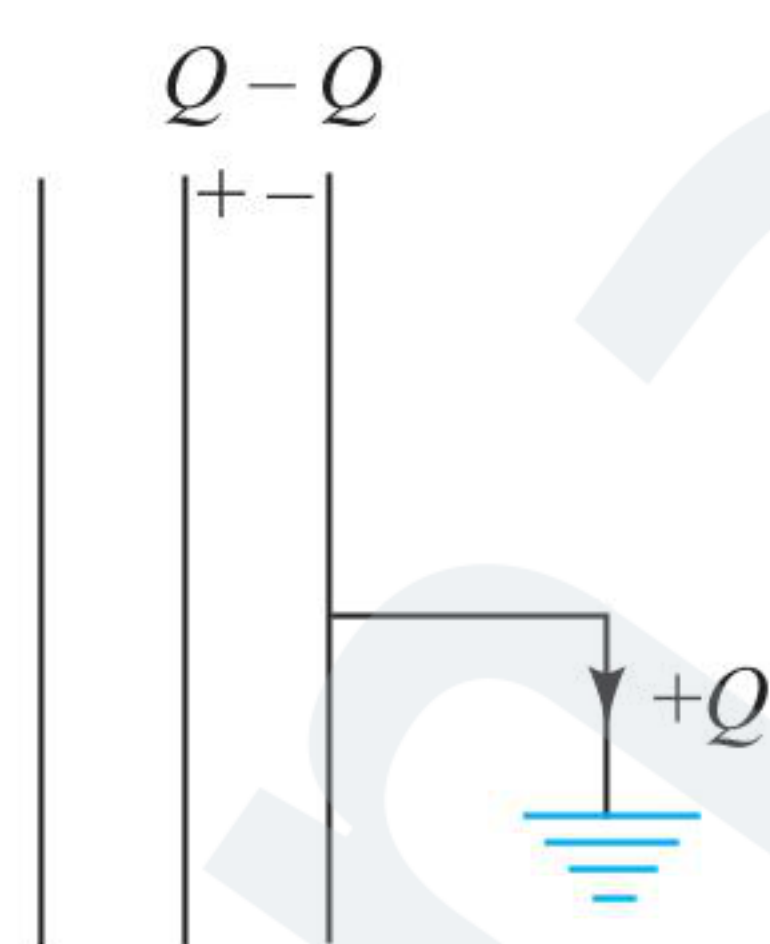
2. Positive plate is given a charge Q ; the charge distribution on different surface are shown in figure.



Hence, new charge on capacitor is $\left(CV + \frac{Q}{2}\right)$. The potential difference between the plates will be

$$V' = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

3. On connecting with earth, the charge on central plate will shift to its right face. $-Q$ charge will be induced on the left face of the earthed plate. Thus, $+Q$ charge will flow to the earth.



4. As $q = CV$, C decreases in both the cases.

- In the first case, V remains the same; hence, q decreases, due to which force of attraction will decrease.
- But in the second case, q will remain the same; hence, force of attraction remains the same. So more work will be done in the second case.

5. (a) True

(b) True

(c) True

6. From the given graphs, find the voltages, V_A and V_B , on capacitors A and B corresponding to charge Q on each of the capacitors. Clearly,

$$V_A = \frac{Q}{C_A} \text{ and } V_B = \frac{Q}{C_B}$$

$$\text{or } \frac{V_B}{V_A} = \frac{Q/C_B}{Q/C_A} = \frac{C_A}{C_B}$$

Since $V_B > V_A$, $C_A > C_B$, i.e., the capacitor A has the higher capacitance.

7. Since slope of the graph is $q/V = C$, and the line A has a larger slope than the line B , it represents the capacitor of larger capacitance. Further, as $C \propto A$ and the plate area of C_2 is double that of C_1 , C_2 has the larger capacitance. Hence, the line A (with larger slope) represents capacitor C_2 , and the line B represents capacitor C_1 .

8. (a) $Q = CV_0$

(b) They must have equal potential difference, and their combined charge must add up to the original charge. Therefore,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \text{ and also } Q_1 + Q_2 = CV_0$$

$$C_1 = C \text{ and } C_2 = \frac{C}{2},$$

$$\text{So } \frac{Q_1}{C} = \frac{Q_2}{(C/2)}$$

$$\text{or } Q_2 = \frac{Q_1}{2}$$

$$\text{or } Q = \frac{3}{2}Q_1 \text{ or } Q_1 = \frac{2}{3}Q$$

$$\text{So } V = \frac{Q_1}{C_1} = \frac{2}{3} \frac{Q}{C} = \frac{2}{3}V_0$$

$$\begin{aligned} \text{(c) } U &= \frac{1}{2} \left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} \right) = \frac{1}{2} \left[\frac{(\frac{2}{3}Q)^2}{C} + \frac{2(\frac{1}{3}Q)^2}{C} \right] \\ &= \frac{1}{3} \frac{Q^2}{C} = \frac{1}{3} CV_0^2 \end{aligned}$$

$$\text{(d) The original } U \text{ was } U = \frac{1}{2} CV_0^2 \text{ or } \Delta U = \frac{-1}{6} CV_0^2.$$

(e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

$$9. \text{ (a) } U_0 = \frac{Q^2}{2C} = \frac{xQ^2}{2\epsilon_0 A}$$

(b) Increase the separation by dx .

$$U = \frac{(x+dx)Q^2}{2\epsilon_0 A} = U_0(1+dx/x)$$

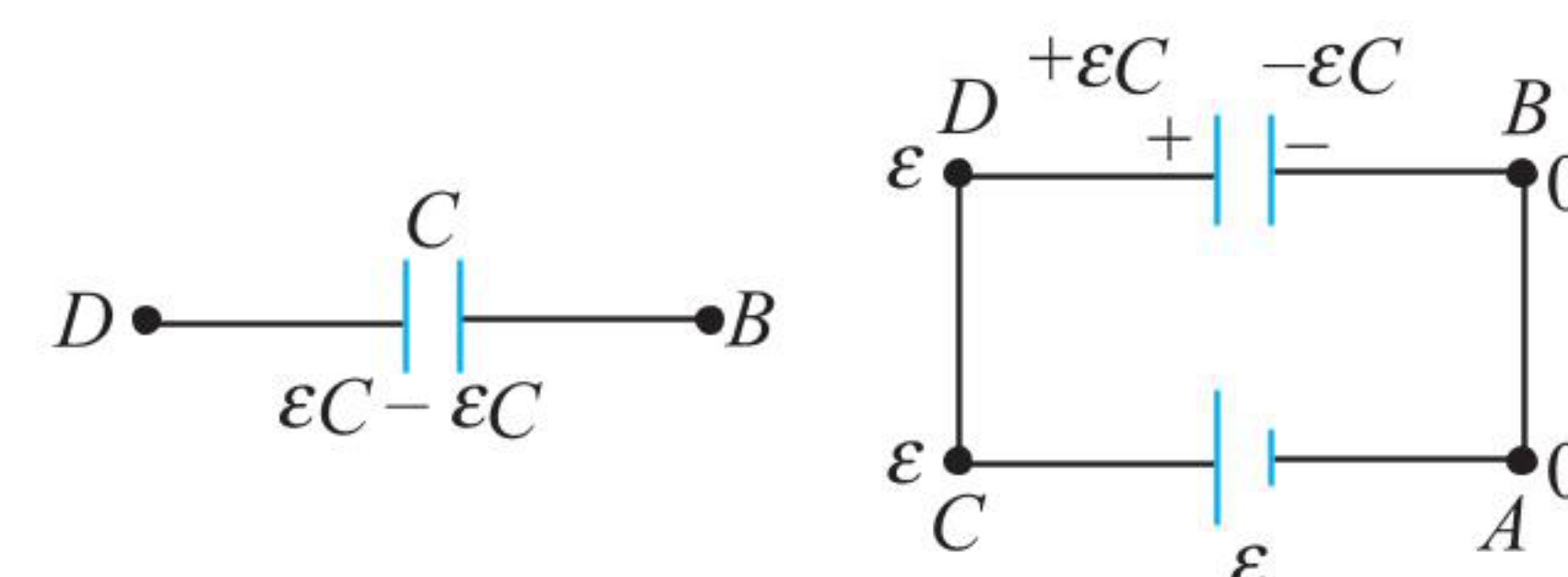
The change is then $(Q^2/2\epsilon_0 A)dx$.

(c) The work done in increasing the separation is given by

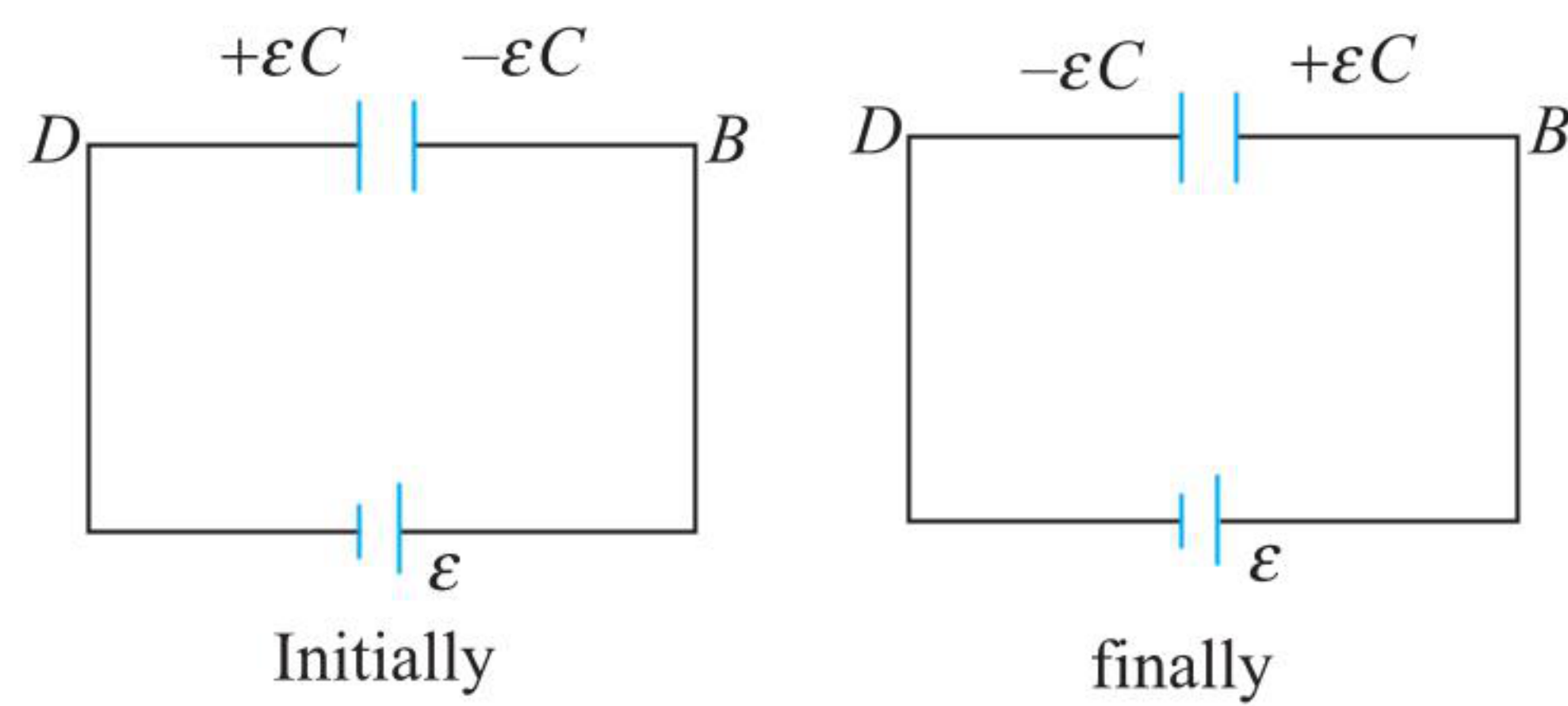
$$dW = U - U_0 = \frac{dxQ^2}{2\epsilon_0 A} = Fdx \text{ or } F = \frac{Q^2}{2\epsilon_0 A}$$

(d) The reason for the difference is that E is the field due to both plates. The force is QE if E is the field due to one plate and Q is the charge on the other plate.

10. Since the initial and final charges on the capacitor are same before and after connection, no charge will flow in the circuit; so heat loss is 0.



11.



From figure, net charge flow through battery is

$$Q_{\text{final}} - Q_{\text{initial}} = \epsilon C - (-\epsilon C) = 2\epsilon C$$

Thus, work done by battery (W) is $Q \times V = 2\epsilon C \times \epsilon = 2\epsilon^2 C$ or heat produced is $2\epsilon^2 C$.

12. Capacitance of sphere

$$C = 4\pi\epsilon_0 R = \frac{0.5 \times 10^{-3}}{9 \times 10^9} = \frac{1}{18} \times 10^{-12} \text{ F}$$

Rate to escape of charge from surface

$$= \frac{80}{100} \times 6.25 \times 10^{10} \times 1.6 \times 10^{-19} = 8 \times 10^{-9} \text{ C/s}$$

therefore $q = (8 \times 10^{-9})t$ and

$$q = CV \Rightarrow 8 \times 10^{-9} t = \frac{1}{18} \times 10^{-12} \times 1$$

$$\Rightarrow t = \frac{10^{-12}}{8 \times 10^{-9} \times 18} = \frac{10^{-3}}{144} = 6.95 \mu\text{s}$$

13. Energy losses $\Delta U = \frac{1}{2} CV_0^2 - \frac{1}{2} CV^2$

Fractional energy loss

$$\frac{\Delta U}{U_0} = \frac{\frac{1}{2} CV_0^2 - \frac{1}{2} CV^2}{\frac{1}{2} CV_0^2} = \frac{V_0^2 - V^2}{V_0^2}$$

$$= \frac{(100)^2 - (80)^2}{(100)^2} = \frac{20 \times 180}{(100)^2} = \frac{9}{25}$$

14. As the plates were moved very slowly, there will not loss of energy in the form of heat.

(a) The charge in capacitor, $Q_0 = C_0 V_0$

$$\text{Initial energy stored, } U_i = \frac{1}{2} C_0 V_0^2$$

After the separation is made half new capacitance, $C = 2C_0$

The charge will remain constant,

$$\text{Final energy stored, } U_f = \frac{Q_0^2}{2(C)} = \frac{C_0^2 V_0^2}{2(2C_0)} = \frac{1}{4} C_0 V_0^2$$

$$\text{Work done by the external agent, } W = \Delta U = U_f - U_i = -\frac{1}{4} C_0 V_0^2$$

The two plates attract each other. The external agent applies force that is opposite to the displacement. Hence, work done is negative,

(b) This time the potential difference across the capacitor remains constant.

$$\text{Initial energy stored, } U_i = \frac{1}{2} C_0 V_0^2$$

$$\text{Final energy stored, } U_f = \frac{1}{2} (2C_0) V_0^2$$

$$\text{Change in potential energy, } \Delta U = \frac{1}{2} C_0 V_0^2$$

The additional charge supplied by battery,

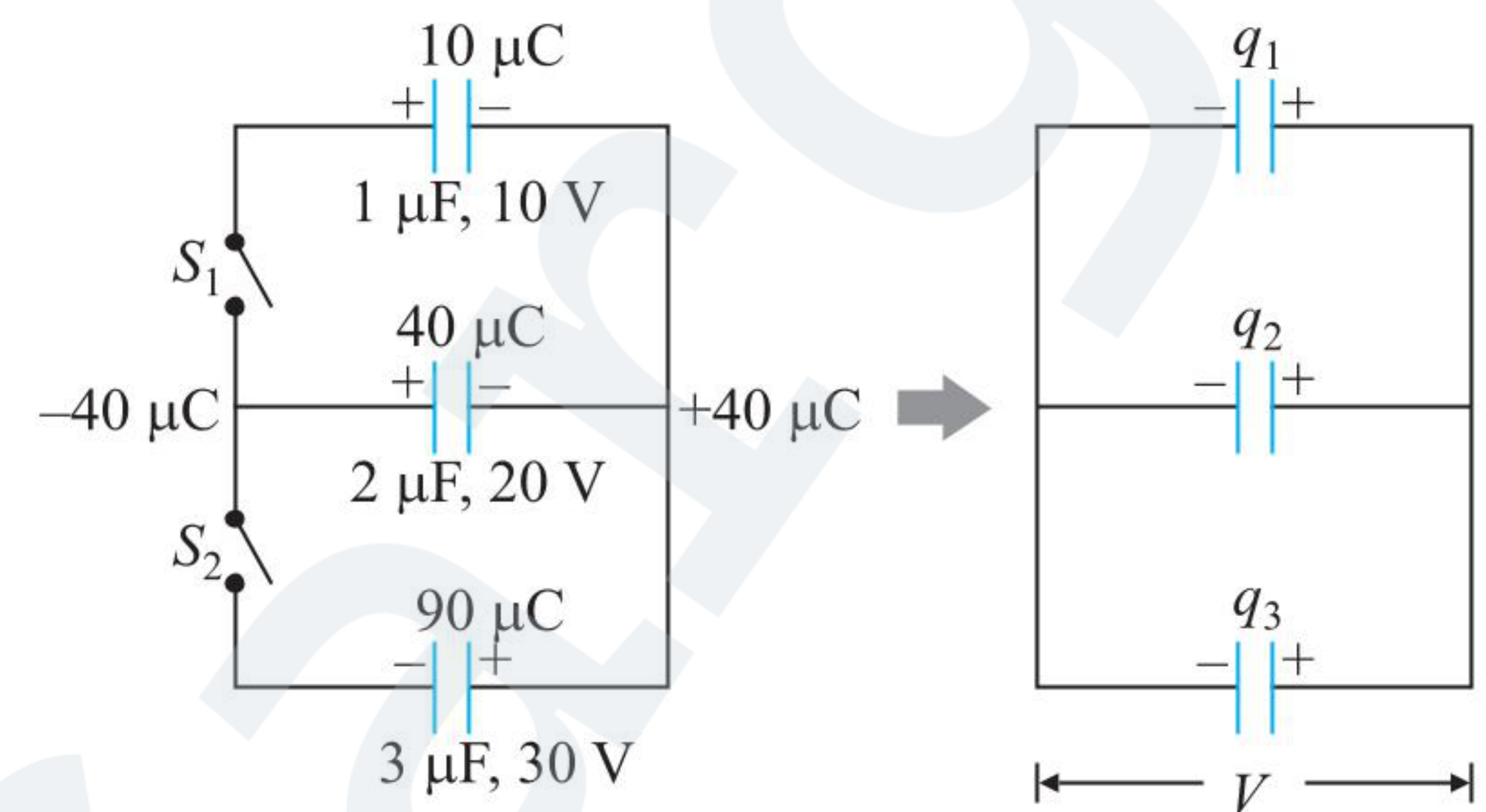
$$\Delta Q = 2C_0 V_0 - C_0 V_0 = C_0 V_0$$

Work done by the battery, $W_{\text{battery}} = \Delta Q V_0 = C_0 V_0^2$

$$W_{\text{external}} + W_{\text{battery}} = \Delta U$$

$$\therefore W_{\text{external}} = \frac{1}{2} C_0 V_0^2 - C_0 V_0^2 = -\frac{1}{2} C_0 V_0^2$$

15. The situation is shown in figure



Total charge on all three capacitors = $(90 - 40 - 10) \mu\text{C} = 40 \mu\text{C}$

Total capacity = $(1 + 2 + 3) \mu\text{F} = 6 \mu\text{F}$

(a) Common potential,

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{40 \mu\text{C}}{6 \mu\text{F}} = \frac{20}{3} \text{ volt}$$

$$(b) \quad q_1 = C_1 V = (1 \mu\text{F}) \left(\frac{20}{3} \right) = \frac{20}{3} \mu\text{C}$$

$$q_2 = C_2 V = (2 \mu\text{F}) \left(\frac{20}{3} \right) = \frac{40}{3} \mu\text{C}$$

$$q_3 = C_3 V = (3 \mu\text{F}) \left(\frac{20}{3} \right) = 20 \mu\text{C}$$

Exercise 4.2

1. Since E inside the conductor is zero, the induced electric field $\vec{E}_{\text{ind}}$ exactly nullifies the external electric field; $\vec{E}_{\text{ext}} = \vec{E}_0$. Outside the plate the equal and opposite surface charges contribute no net field. Hence, outside the plate, the net field is equal to the applied (external) field E_0 .

Since, $\vec{E}_{\text{outside}} = \frac{\sigma}{\epsilon_0} \hat{n}$ and $\vec{E}_{\text{outside}} = E_0$, we can write, $\sigma = \epsilon_0 E_0$

2. (i) The charge q remains constant

$$\text{Hence, } \frac{U}{U_0} = \frac{C_0}{C} = \frac{1}{\epsilon_r}$$

$$\text{Or } U = \frac{U_0}{\epsilon_r} = \frac{10^{-3}}{100} = 10^{-5} \text{ J}$$

(ii) Since $U < U_0$, $\Delta U = -U_0 \left(1 - \frac{1}{\epsilon_r} \right)$; this extra energy goes to work done by the external agent who introduces the dielectric slab into the space between capacitor plates.

$$3. (i) \quad \frac{U}{U_0} = \frac{\left(\frac{1}{2} CV^2 \right)}{\left(\frac{1}{2} C_0 V^2 \right)} = \frac{C}{C_0} = \epsilon_r$$

$$\text{Or } U = U_0 \epsilon_r = (2 \times 10^{-4})(150) = 0.03 \text{ J}$$

The excess energy $\Delta U = U_0(\epsilon_r - 1) > 0$. This extra energy is drawn from the supply.

$$(ii) \Delta q = (\Delta C)V = (\epsilon_r - 1)C_0 = 149 \times 4 \times 10^{-8} = 5.96 \times 10^{-6} \text{ C}$$

$$(iii) \text{ The charge induced in dielectric, } q_{\text{induced}} = q_f \left(1 - \frac{1}{\epsilon_r}\right) = \frac{149}{150} q_f,$$

$$\Rightarrow \frac{q_{\text{induced}}}{q_f} = \frac{149}{150}$$

4. Let the charge on the capacitor be Q . The surface charge density should be $\sigma = \frac{Q}{A}$ and the electric field $E = \frac{Q}{KA\epsilon_0}$.

This electric field should not exceed the dielectric strength

$$E_{\text{break}} = 1.9 \times 10^7 \text{ V/m}$$

$\therefore$ if the maximum charge which can be given is Q then

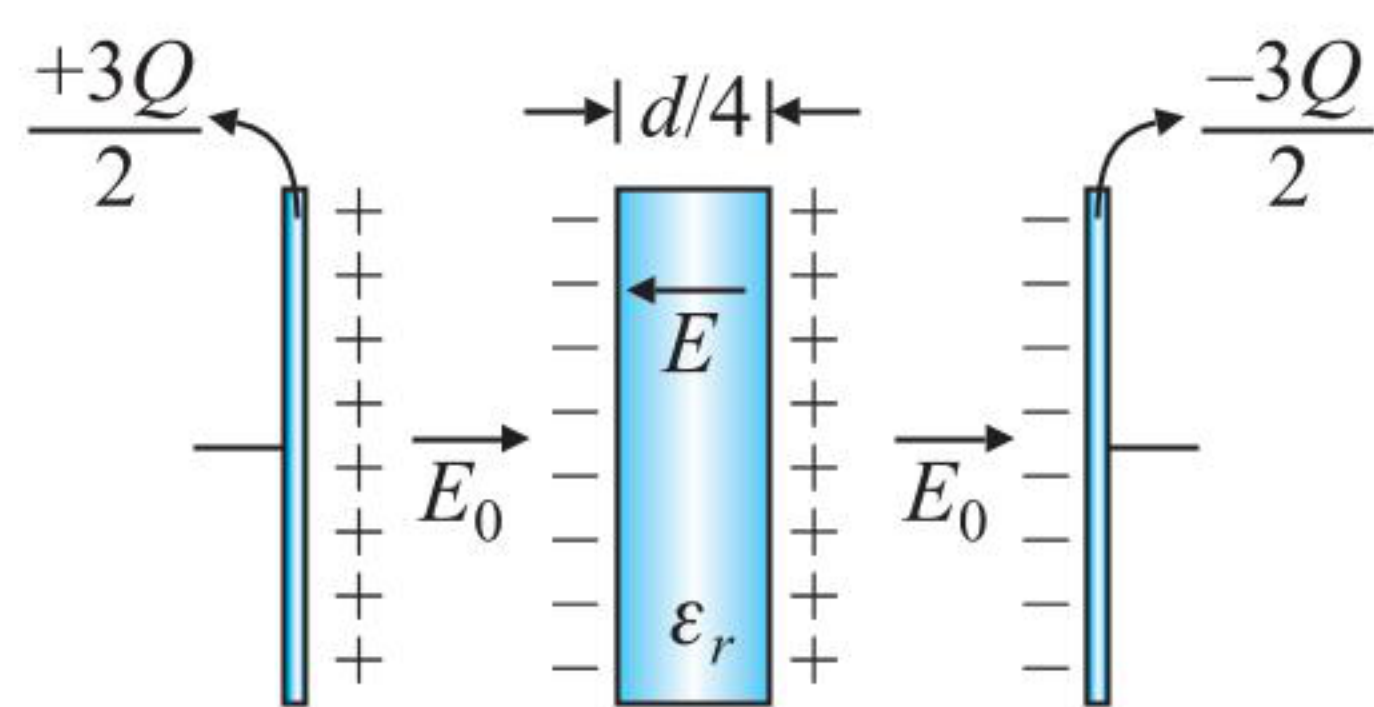
$$\frac{Q}{KA\epsilon_0} = E_{\text{break}}$$

$$Q = E_{\text{break}} \times KA\epsilon_0 = 1.9 \times 10^7 \times (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12}) \\ = 8.4 \times 10^{-6} \text{ C.}$$

5. (a) The charge density in the inner side of conductors is given as

$$\sigma = \left[\frac{Q - (-2Q)}{2} \right] = \frac{3Q}{2A}$$

- (b) The charge density on the dielectric slab is



$$\sigma' = \sigma \left(1 - \frac{1}{\epsilon_r}\right) = \sigma \left(1 - \frac{1}{2}\right) = \frac{\sigma}{2} = \frac{3Q}{4A}$$

- (c) The electric field in the dielectric is

$$E = \frac{E_0}{\epsilon_r} = \frac{\left(\frac{\sigma}{\epsilon_r}\right)}{\epsilon_r} = \frac{3Q}{2\epsilon_0\epsilon_r A} = \frac{3Q}{4\epsilon_0 A}$$

- (d) The potential difference between the conductor is

$$V = E_0 \left(d - \frac{d}{4}\right) + E \frac{d}{4} = \frac{3Q}{2\epsilon_0 A} \cdot \frac{3d}{4} + \frac{3Q}{4\epsilon_0 A} \cdot \frac{d}{4} = \frac{21Qd}{16\epsilon_0 A}$$

- (e) The capacitance is

$$C = \frac{\left(\frac{3Q}{2}\right)}{V} = \frac{\left(\frac{3Q}{2}\right)}{\left(\frac{21Qd}{16\epsilon_0 A}\right)} = \frac{8\epsilon_0 A}{7d}$$

6. (a) (i) In case of a capacitor as $E = (V/d)$, the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5 \text{ KV}$$

(ii) Now as after charging battery is removed, capacitor is isolated so $q = \text{constant}$. If C' and V' are the capacity and potential after the introduction of plate

$$q = CV = C'V' \text{ i.e., } V' = \frac{C}{C'}V$$

$$\text{And as } C = \frac{\epsilon_0 A}{d}$$

$$\text{and } C' = \frac{\epsilon_0 A}{(d-t) + (t/K)}, V' = \frac{(d-t) + (t/K)}{d} \times V$$

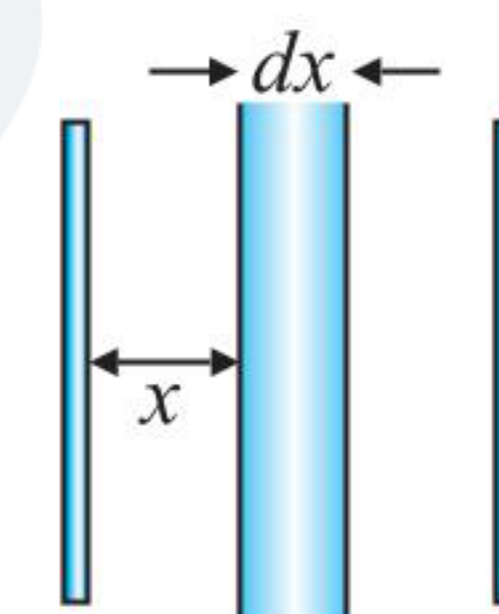
So in case of metal plate as

$$K = \infty, V_M = \left[\frac{d-t}{d} \right] \times V = \left[\frac{0.05 - 0.01}{0.05} \right] \times 1.5 = 1.2 \text{ KV}$$

- (b) And if instead of metal plate, dielectric with $K = 2$ is introduced

$$V_D = \frac{(d-t) + (t/K)}{d} \times V \\ = \left[\frac{(0.05 - 0.01) + (0.01/2)}{0.05} \right] \times 1.5 = 1.35 \text{ KV}$$

7. (a) The capacitance of the system can be thought of as the series combination of elementary capacitance dC



$$\frac{1}{C_{\text{eq}}} = \int \frac{1}{dC} = \int \frac{1}{\left(\frac{\epsilon_0 \epsilon_r A}{dx}\right)} = \frac{d}{\epsilon_0 A} \int_0^d \frac{dx}{\epsilon_r}$$

$$= \frac{1}{\epsilon_0 A} \int_0^d \frac{dx}{a + bx} = \frac{1}{\epsilon_0 Ab} \ln \left(\frac{a + bd}{a} \right)$$

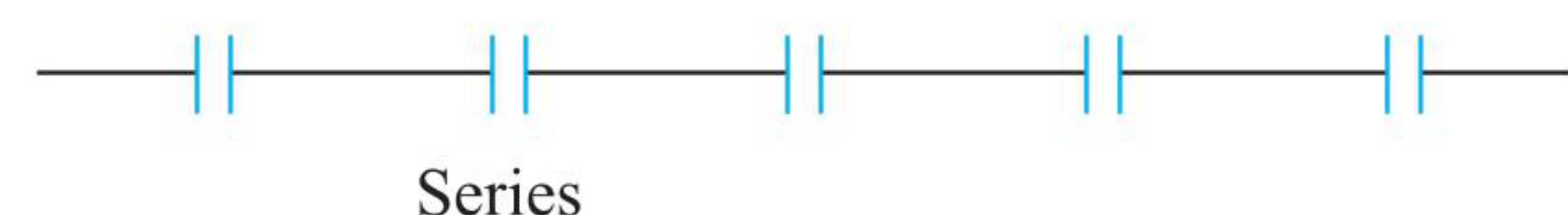
$$\text{Or } C_{\text{eq}} = \frac{\epsilon_0 Ab}{\ln \left(\frac{a + bd}{a} \right)}$$

$$(b) V = \int E dx = \int \frac{Q}{\epsilon_0 A \epsilon_r} dx = \frac{Q}{\epsilon_0 A} \int_0^x \frac{dx}{\epsilon_r}$$

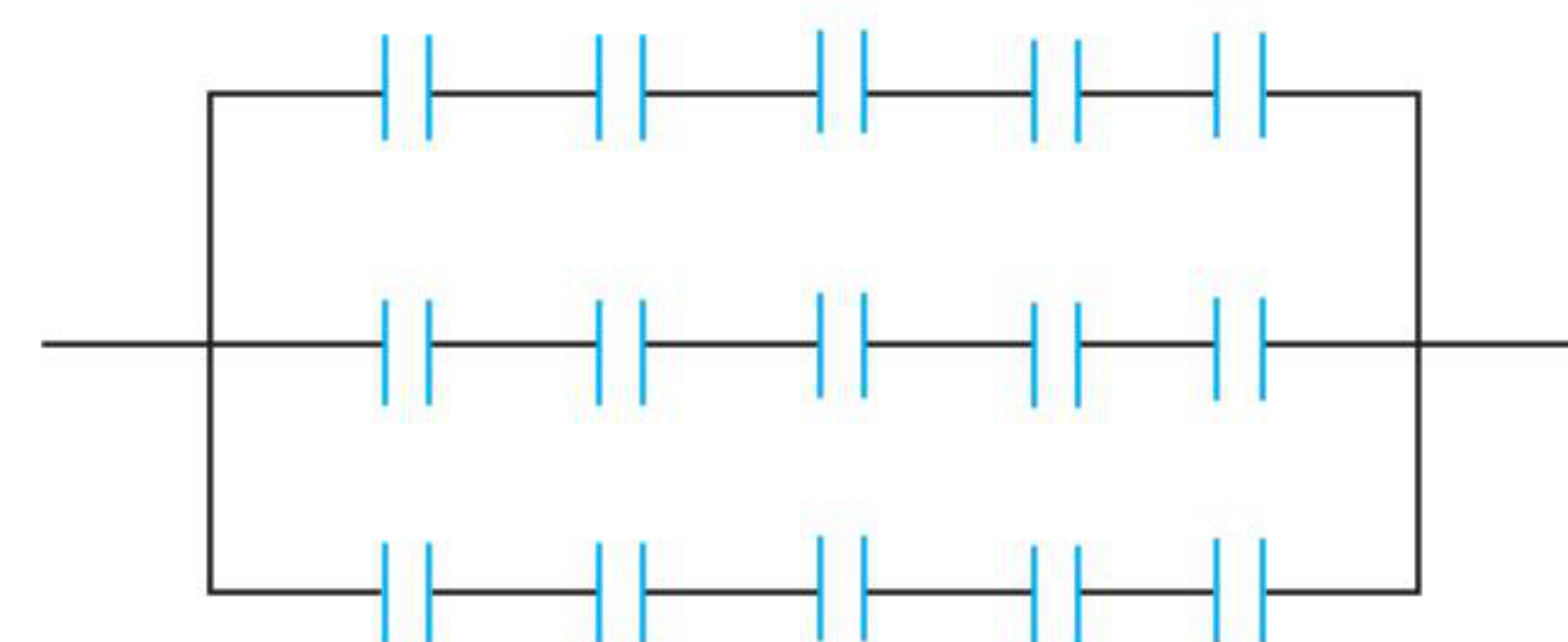
$$= \frac{Q}{\epsilon_0 A} \int_0^x \frac{dx}{a + bx} = \frac{Q}{\epsilon_0 Ab} \ln \left(\frac{a + bx}{a} \right)$$

Exercise 4.3

1. (a) Five capacitors in series



- (b) Three rows in parallel with each row containing five capacitors



Series and parallel common grouping

2. In series, potentials of all capacitors will be added, so net potential will be NV .
3. Capacitors 4 and 5 are short-circuited. So, they are useless. '1', '2', and '3' will be in parallel and then 6 will be in series with them. $C_{\text{eq}} = 3C/4$

4. $V = 10 \text{ V}$

$$C_{\text{eq}} = C_1 + C_2 \quad [C_1 \text{ and } C_2 \text{ are in parallel}]$$

$$= 6 + 5 = 11 \mu\text{F};$$

$$Q = CV = 11 \times 10 = 110 \mu\text{C}$$

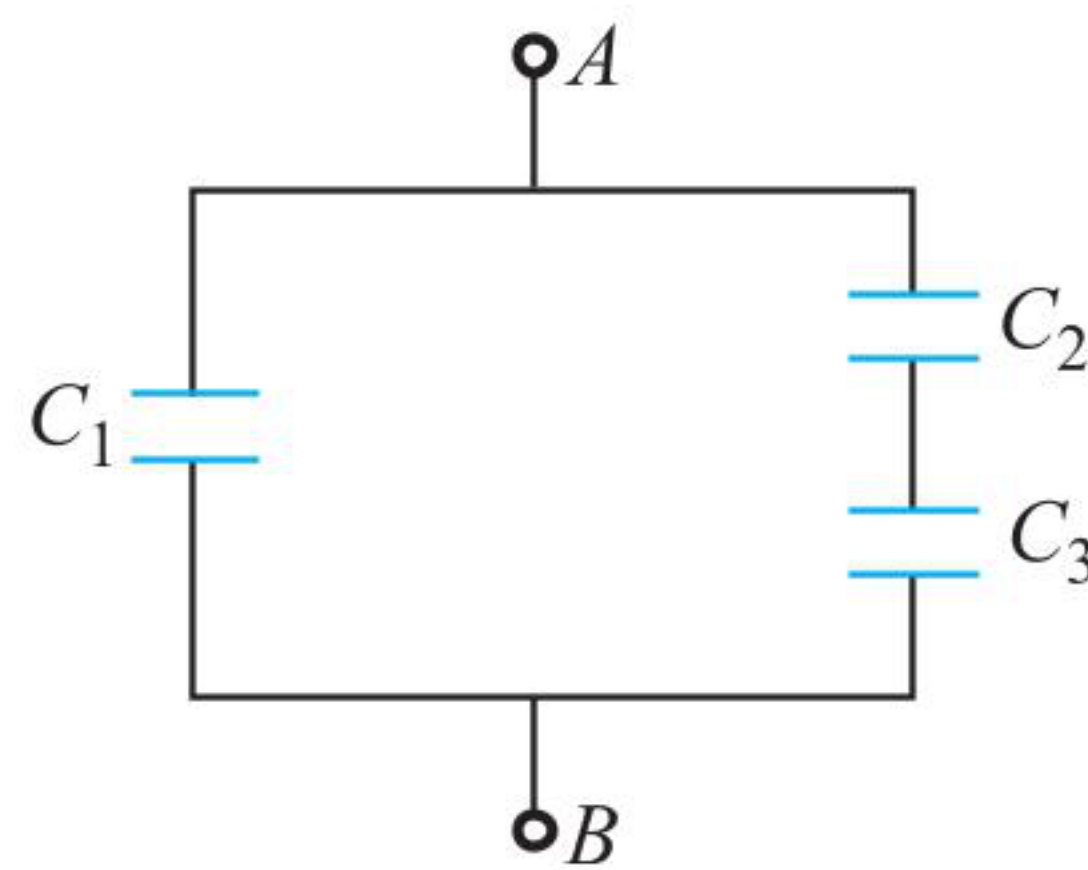
5. It is equivalent to

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

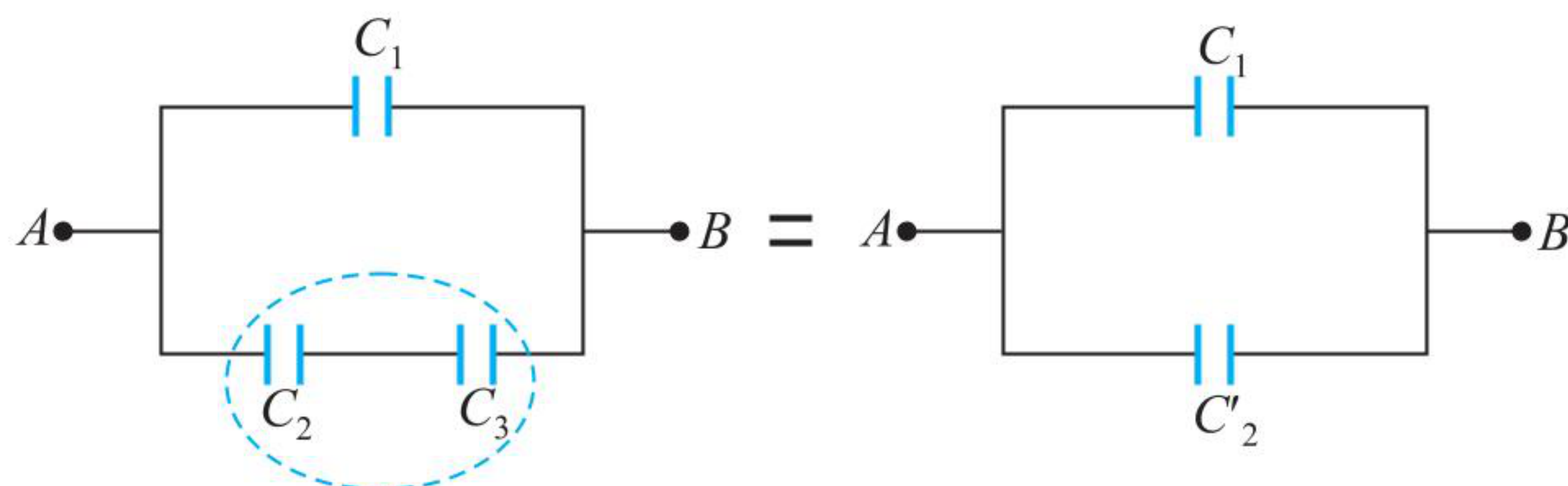
$$= \frac{A_1 K_1 \epsilon_0}{d_1 + d_2} + \frac{\frac{A_2 K_2 \epsilon_0}{d_1} \cdot \frac{A_2 K_3 \epsilon_0}{d_2}}{\frac{A_2 K_2 \epsilon_0}{d_1} + \frac{A_2 K_3 \epsilon_0}{d_2}}$$

$$= \frac{A_1 K_1 \epsilon_0}{d_1 + d_2} + \frac{A_2^2 K_2 K_3 \epsilon_0^2}{A_2 K_2 \epsilon_0 d_2 + A_2 K_3 \epsilon_0 d_1}$$

$$= \frac{A_1 K_1 \epsilon_0}{d_1 + d_2} + \frac{A_2 K_2 K_3 \epsilon_0}{K_2 d_2 + K_3 d_1}$$



6. The given system can be redrawn as shown in figure.



Here $C_1 = \frac{\epsilon_0 \epsilon_r (A/2)}{d} = \frac{\epsilon_0 A}{2d}$

$$C_2 = \frac{\epsilon_0 \epsilon_r (A/2)}{(d/2)} = \frac{2\epsilon_0 A}{d}$$

and $C_3 = \frac{\epsilon_0 \epsilon_r (A/2)}{(d/2)} = \frac{3\epsilon_0 A}{d}$

The equivalent capacitance across A and B ,

$$C_{AB} = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{17\epsilon_0 A}{10d}$$

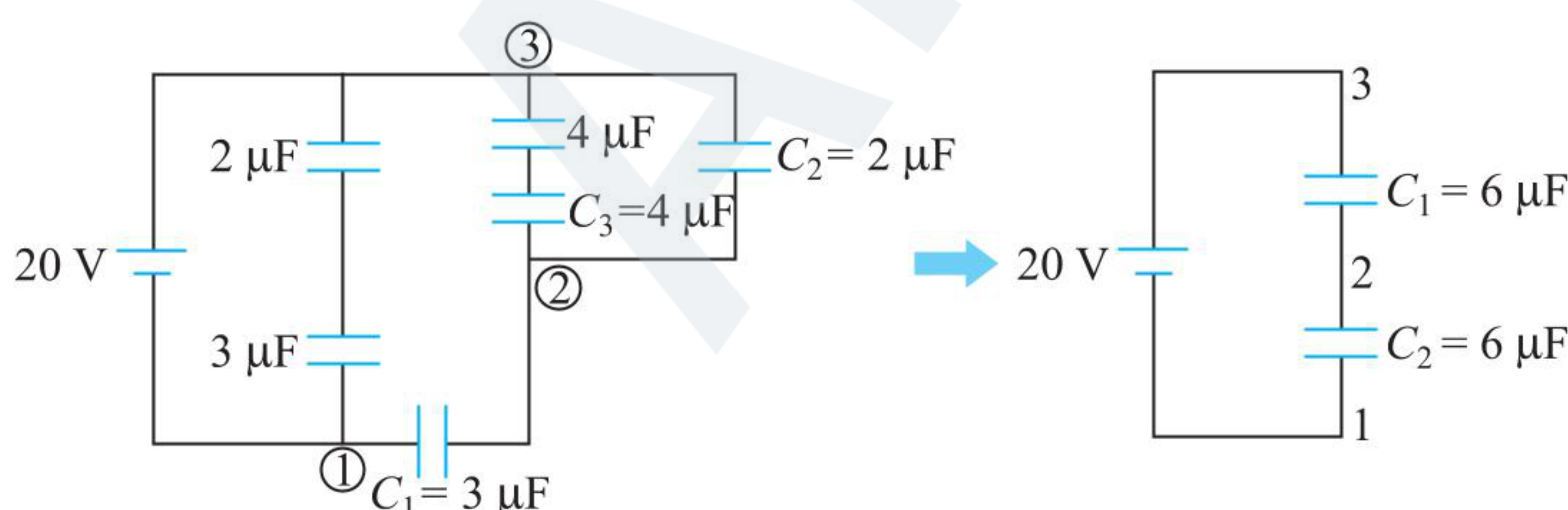
7. $\frac{V_1}{V_{1.5}} = \frac{1.5}{1}$

$$\therefore V_1 = \left(\frac{1.5}{1.5+1} \right) (30) = 18 \text{ V}$$

$$\frac{V_{2.5}}{V_{0.5}} = \frac{0.5}{2.5} = \frac{1}{5} \therefore V_{2.5} = \left(\frac{1}{1+5} \right) (30) = 5 \text{ V}$$

Now, $|V_{ab}| = V_1 - V_{2.5} = 13 \text{ V}$

8. (a) Simple circuit is as shown below.



(b) Hence equivalent capacitance $C_{\text{eq}} \Rightarrow \frac{6 \times 6}{6+6} = 3 \mu\text{F}$.

(c) Upper network and lower network both have same capacitance $= 6 \mu\text{F}$

$$V_1 - V_2 = \frac{20}{2} = 10 \text{ V} \Rightarrow V_{C_1} = 10 \text{ V}$$

$$q_{C_1} = (C_1)(V_{C_1}) = 30 \mu\text{C}$$

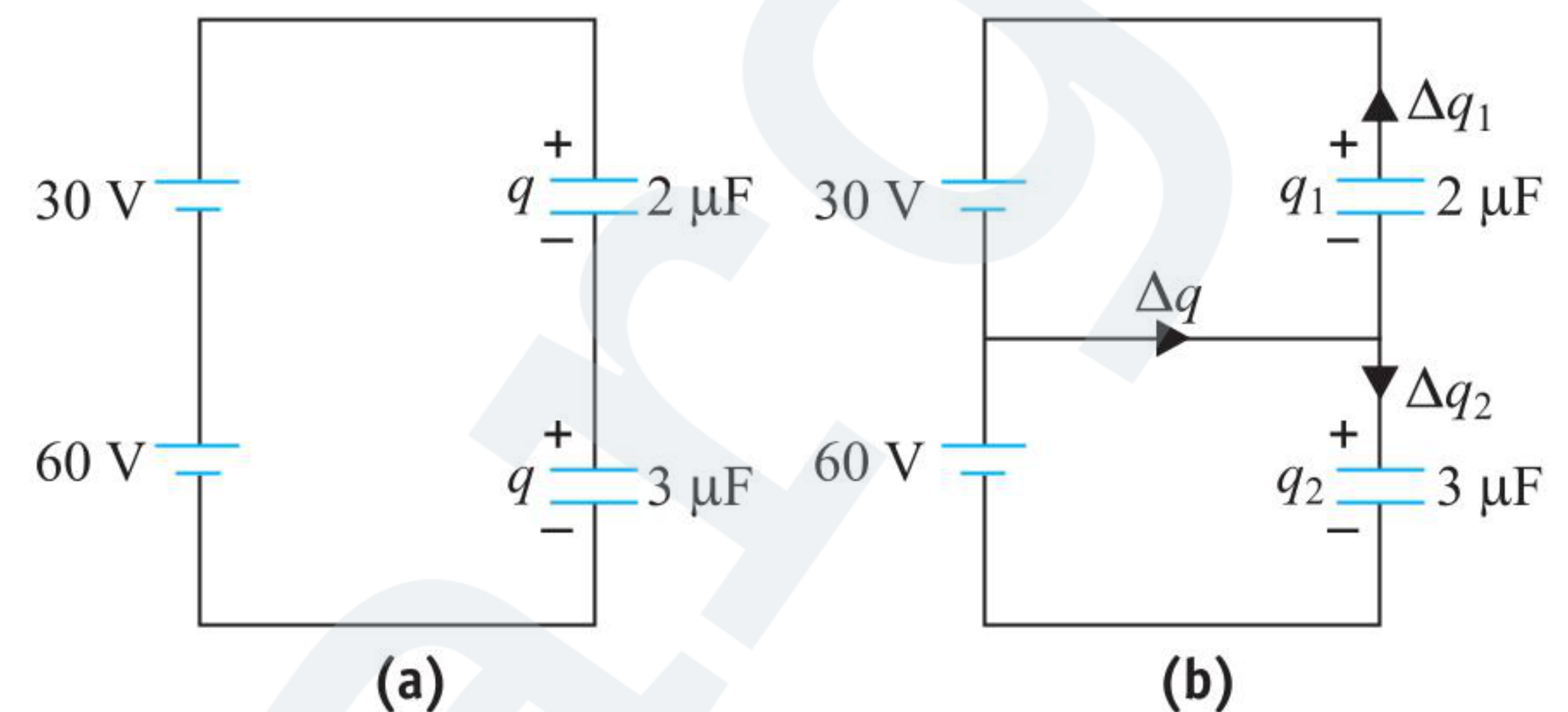
(d) $V_{C_2} = 10 \text{ V}, \therefore q_{C_2} = (C_2)(V_{C_2}) = 20 \mu\text{C}$

(e) $V_{C_3} = 5 \text{ V}, \therefore q_{C_3} = (C_3)(V_{C_3}) = 20 \mu\text{C}$

9. When switch is opened, $2 \mu\text{F}$ and $3 \mu\text{F}$ capacitors are in series. So,

$$C_{\text{eq}} = \frac{2 \times 3}{5} = \frac{6}{5} \mu\text{F}$$

Hence, charge on each capacitor, $q = CV = \frac{6}{5} \times 90 = 108 \mu\text{C}$



When switch S is closed, let q_1 and q_2 be charge on the two capacitors as Fig. (b). So,

$$q_1 = 2 \times 30 = 60 \mu\text{C}$$

$$q_2 = 3 \times 60 = 180 \mu\text{C}$$

From figure it is clear that, after closing the switch the charge flowing out of $2 \mu\text{F}$ capacitor,

$$\Delta q_1 = 108 - 60 = 48 \mu\text{C}$$

And the charge flown into $3 \mu\text{F}$ capacitor, $\Delta q_2 = 180 - 108 = 72 \mu\text{C}$

So the net charge flown through the switch,

$$\Delta q = \Delta q_1 + \Delta q_2 = 48 + 72 = 120 \mu\text{C}$$

10. Charge across $5 \mu\text{F}$ capacitor $10 \mu\text{C}$.

Potential difference across $5 \mu\text{F}$ capacitor is

$$\frac{q}{C} = \frac{10 \times 10^{-6}}{5 \times 10^{-6}} = 2 \text{ volt}$$

As 3, 4, and $5 \mu\text{F}$ capacitors are in parallel, so potential difference across each of the them equals to 2 V .

Charge on $3 \mu\text{F}$ capacitor $= CV = 3 \times 10^{-6} \times 2 = 6 \mu\text{C}$

Charge on $4 \mu\text{F}$ capacitor $4 \times 10^{-6} \times 2 = 8 \mu\text{C}$

Total charge flowing in upper branch of circuit is

$$10 \mu\text{C} + 6 \mu\text{C} + 8 \mu\text{C} = 24 \mu\text{C}$$

Potential difference across C_2 is $\frac{24 \mu\text{C}}{4 \mu\text{C}} = 6 \text{ V}$

Total potential difference across AB is $= 6 + 2 = 8 \text{ V}$.

Equivalent capacitance of lower branch of circuit is

$$\frac{6 \times 3}{6+3} = 2 \mu\text{F}$$

So, charge flowing is $= 2 \times 10^{-6} \times 8 = 16 \mu\text{C}$

Therefore, potential difference between A and C is

$$\frac{16 \mu\text{C}}{3 \mu\text{F}} = \frac{16}{3} = 5.33 \text{ V}$$

11. Before closing the switch.

$$C_{\text{eq}} = \frac{2C \times C}{2C + C} = \frac{2}{3}C = \frac{2}{3} \times 2 = \frac{4}{3} \mu\text{F}$$

$$\therefore q_i = C_{\text{eq}} V = \frac{4}{3} \times 30 = 40 \mu\text{C}$$

The energy stored is $U_i = \frac{1}{2} C_{\text{eq}} V^2 = 6 \times 10^{-4} \text{ J}$

after closing the switch $C'_{\text{eq}} = C$ $\{ \because$ Left capacitors are short circuited $\}$.

$$\therefore V = CV = 2 \times 30 = 60 \mu\text{C}$$

$$\text{and } U_f = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} CV^2 = 9 \times 10^{-4} \text{ J}$$

$$(a) \Delta q = q_f - q_i = 60 - 40 = 20 \mu\text{C}$$

$$(b) \text{ and } (c) W_{\text{cell}} = \Delta U + \Delta H \text{ or } V\Delta q = U_f - U_i + \Delta H$$

$$\therefore \Delta U = 9 \times 10^{-4} - 6 \times 10^{-4} = 3 \times 10^{-4} \text{ J} = 0.3 \text{ mJ}$$

$$W_{\text{cell}} = V\Delta q = 30 \times 20 \times 10^{-6} = 6 \times 10^{-4} = 0.6 \text{ mJ}$$

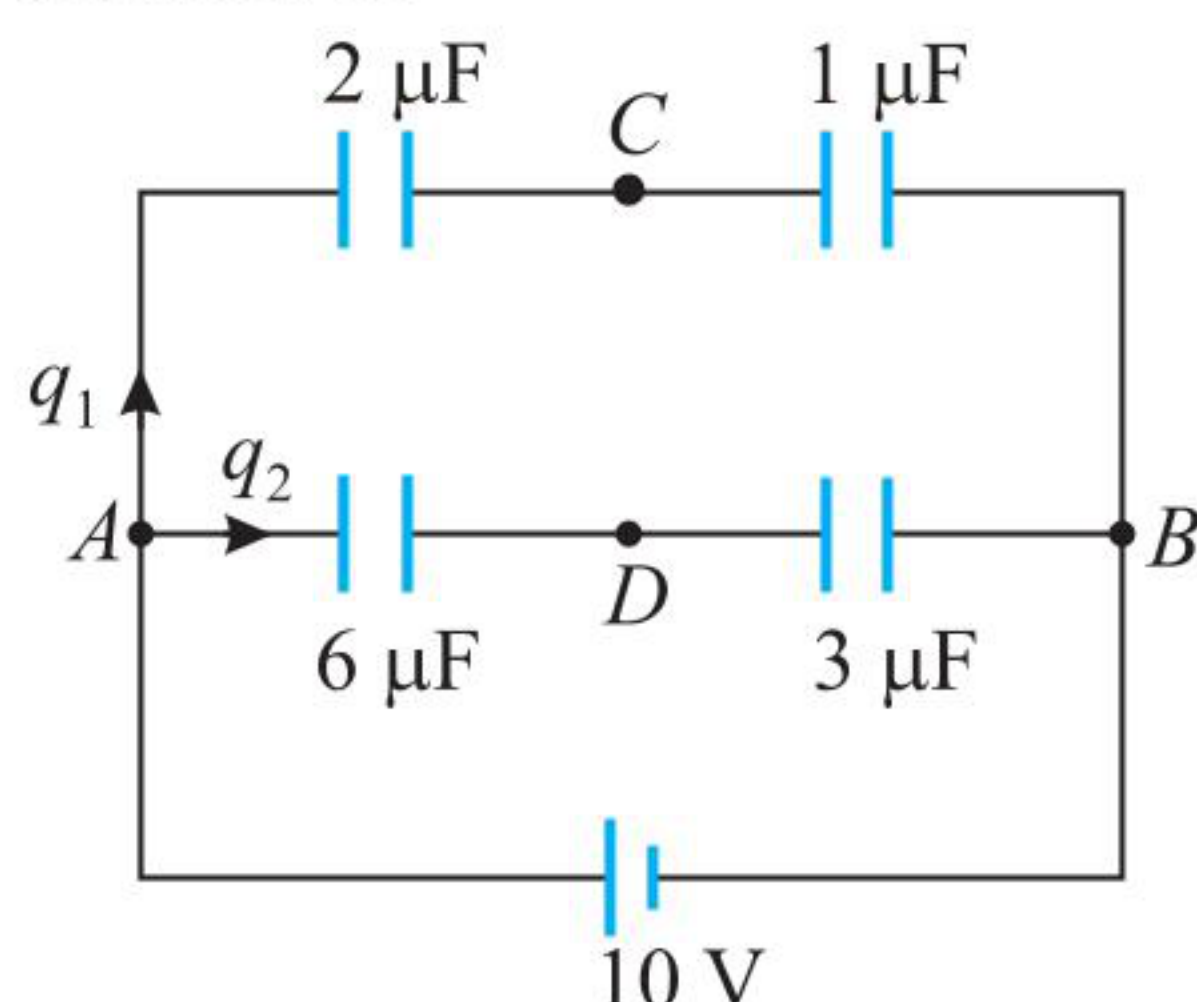
$$\text{Heat generated } \Delta H = W_{\text{battery}} - \Delta U \\ = 0.6 - 0.3 = 0.3 \text{ mJ}$$

$$(d) \text{ The charge flow through switch} = \text{charge supplied by battery} + \\ \text{charges flowing for neutralization of leftward capacitors} \\ = 20 + 20 + 20 = 60 \mu\text{C}$$

{ $\because$ Charges on each leftward capacitors are $20 \mu\text{C}$ before short circuiting}.

Exercise 4.4

1. Given circuit is balanced Wheatstone bridge. Hence $V_C = V_D = 0 \text{ V}$.
Now the circuit reduces to



$$\text{Hence } V_{AC} = 10 \left(\frac{1}{2+1} \right) = \frac{10}{3} \text{ V}$$

$$\text{and } V_{AD} = 10 \left(\frac{3}{6+3} \right) = \frac{10}{3} \text{ V}$$

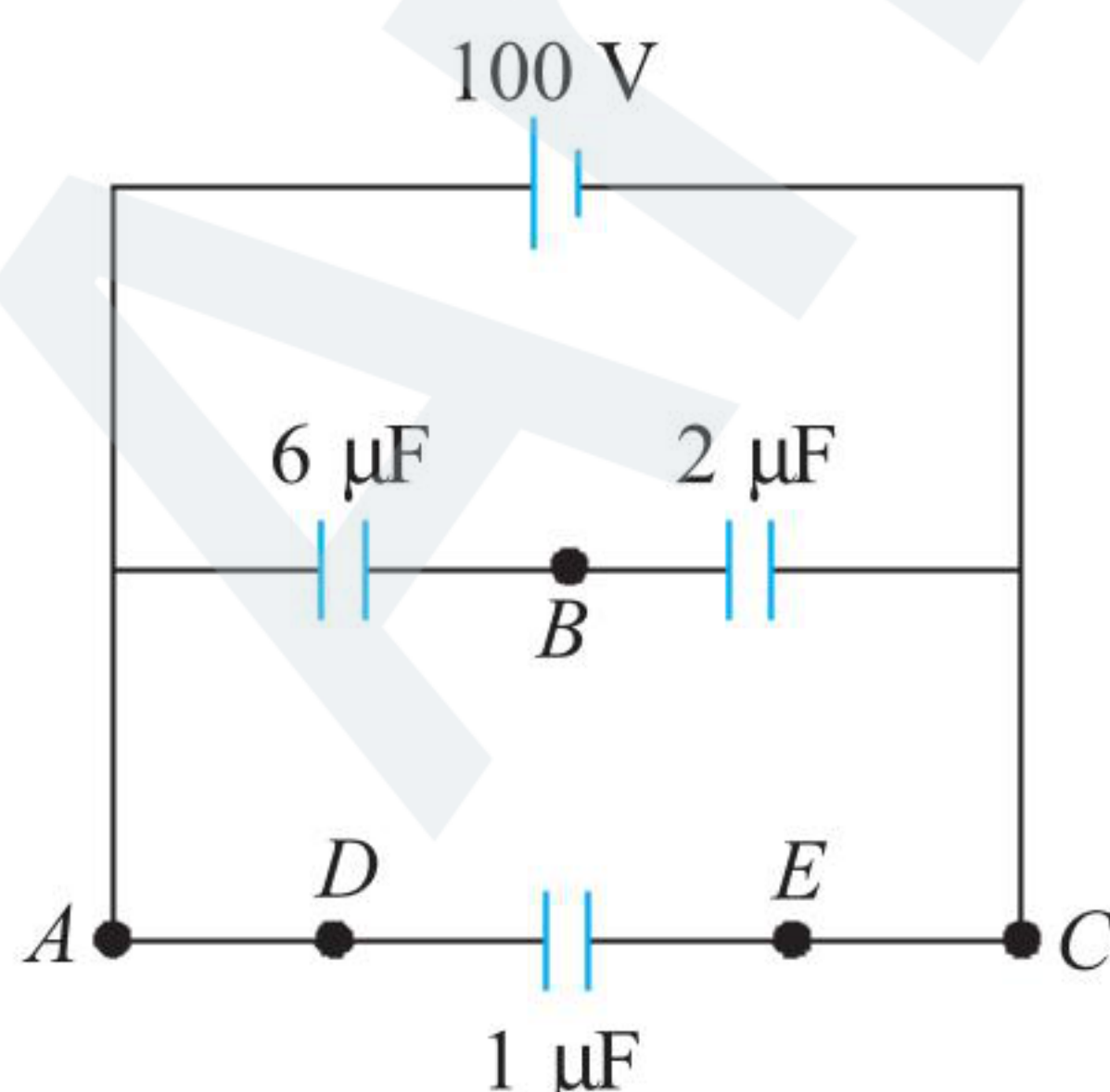
$$\text{It means } V_A = \frac{10}{3} \text{ V}$$

$$\text{As } V_A - V_B = 10 \text{ V}$$

$$\frac{10}{3} - V_B = 10 \Rightarrow V_B = -\frac{20}{3} \text{ V}$$

$$\text{Charge on } 2 \mu\text{F capacitor } q_{C_2} = 2 \times \frac{10}{3} = \frac{20}{3} \mu\text{C}$$

2. Both $3 \mu\text{F}$ capacitors will be in parallel, and similarly upper two $1 \mu\text{F}$ capacitors will be in parallel. The circuit may be redrawn as follows:



$$(a) V_A - V_B = \frac{100 \times 2}{6+2} = 25 \text{ V}$$

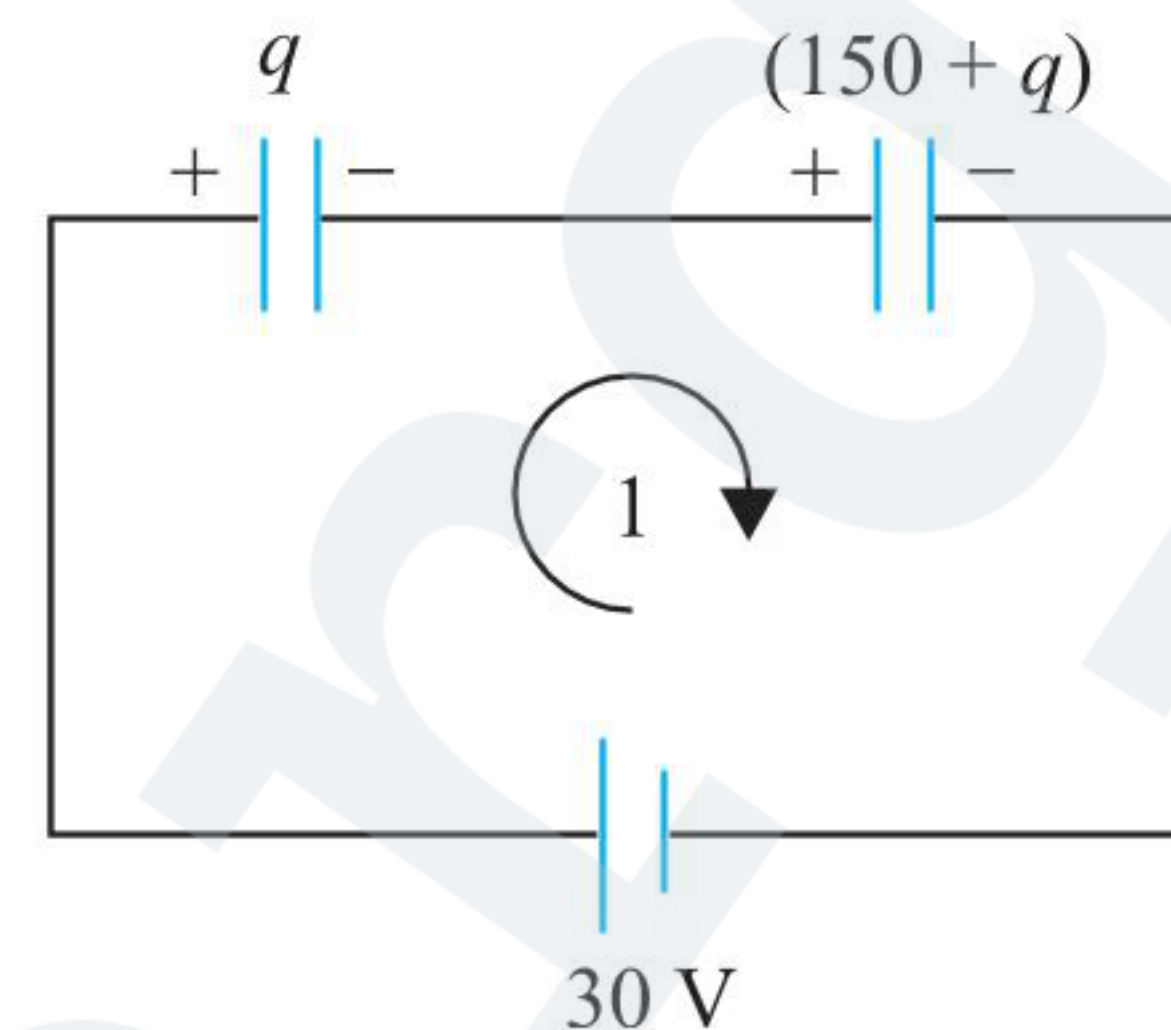
$$(b) V_B - V_C = \frac{100 \times 6}{6+2} = 75 \text{ V}$$

$$(c) V_D - V_E = 100 \text{ V}$$

$$(d) C_{eq} = \frac{6 \times 2}{6+2} + 1 = \frac{5}{2} \mu\text{F}$$

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \left(\frac{5}{2} \times 10^{-6} \right) \times (100)^2 \\ = 125 \times 10^{-4} \text{ J}$$

3. The initial charge on the larger capacitor is



$$Q = C\Delta V = 10 \mu\text{F} (15 \text{ V}) = 150 \mu\text{C}$$

An additional charge is pushed through the 30 V battery, giving the smaller capacitor charge q and the larger charge $(150 + q)$.

In close loop (1)

$$30 - \frac{q}{5} - \frac{(150 + q)}{10} = 0 \text{ or } q = 50 \mu\text{C}$$

Potential difference across C_1

$$V_1 = \frac{q}{C_1} = \frac{50}{5} = 10 \text{ V} \text{ and } V_2 = \frac{200}{10} = 20 \text{ V}$$

4. (a) Since a switch is open for long time. The capacitor is in series. Hence equivalent capacity is $C_{eq} = C/2$. Initial charge on each capacitor is $q_0 = C\epsilon/2$. When switch is closed, final charge on capacitor C_1 , is $q_1 = C\epsilon$ and final charge in C_2 , is $q_2 = 0$. Hence, when the switch is closed, charge flow through battery is

$$\Delta q = \left(C\epsilon - \frac{C\epsilon}{2} \right) = \frac{C\epsilon}{2}$$

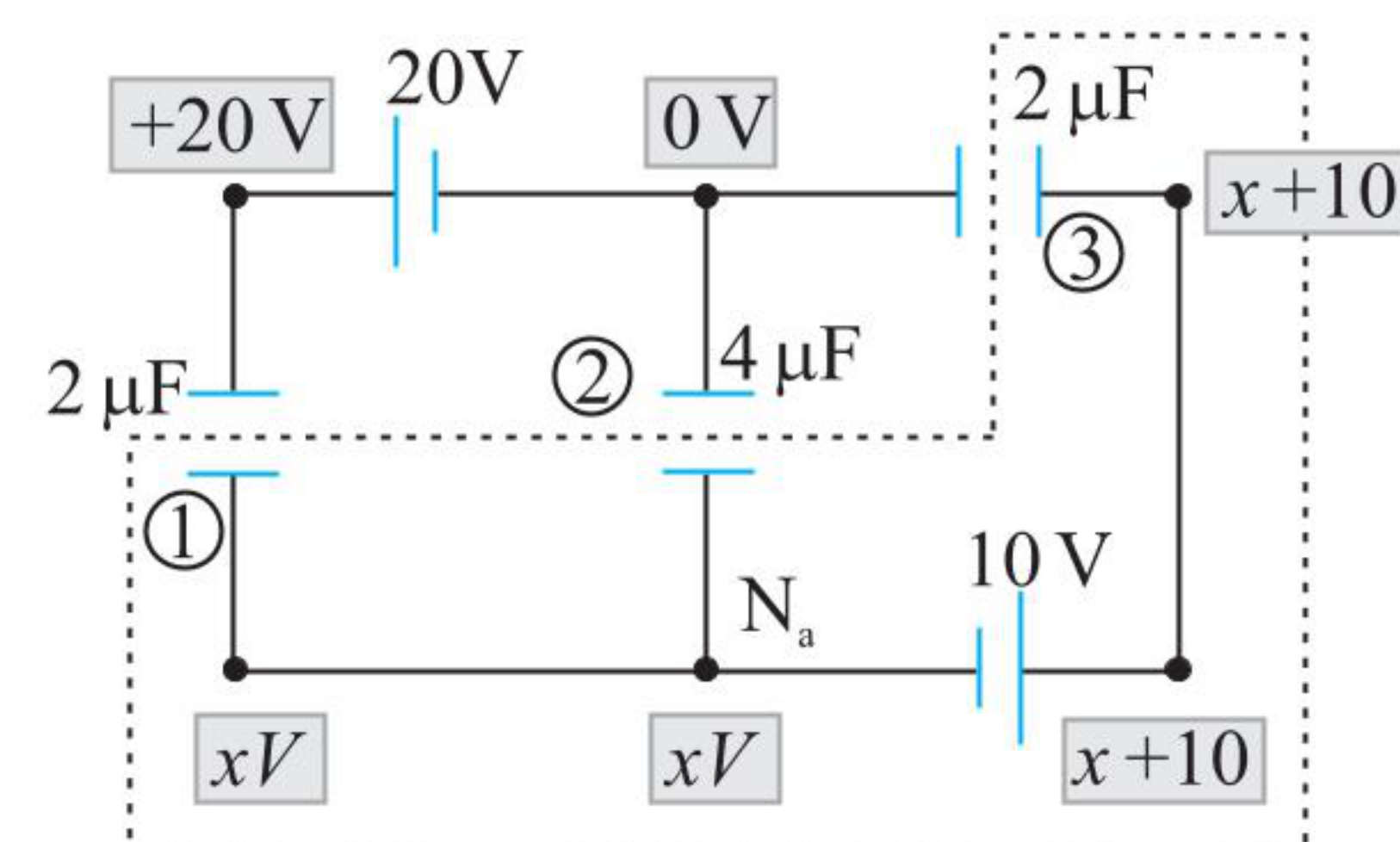
$$(b) \text{ Work done by battery is } \Delta q \cdot \epsilon = \frac{C\epsilon}{2} \times \epsilon = \frac{C\epsilon^2}{2}$$

(c) Change in energy stored is

$$\frac{1}{2} C\epsilon^2 - \frac{1}{2} \times \left(\frac{C}{2} \right) \times \epsilon^2 = \frac{1}{4} C\epsilon^2$$

$$(d) \text{ Heat developed is } W_{\text{battery}} - \Delta U = \frac{C\epsilon^2}{2} - \frac{C\epsilon^2}{4} = \frac{C\epsilon^2}{4}$$

5. Figure shows an isolated system.



For isolated system, net charge is zero.

$$2(x - 20) + 4x + (x + 10)2 = 0$$

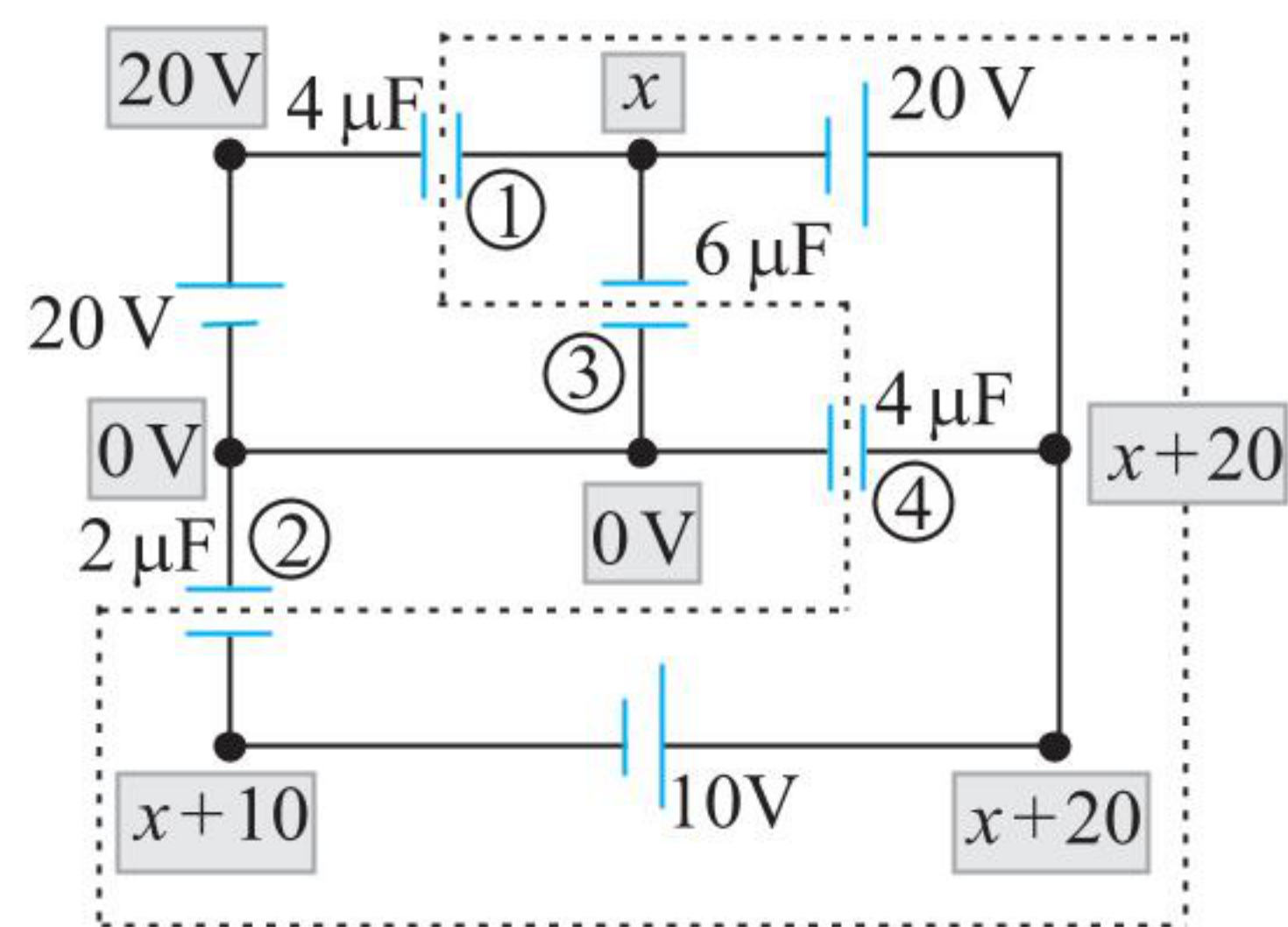
$$\text{or } 8x = 20 \text{ or } x = 2.5 \text{ V}$$

Thus, charge on capacitor (1) is $35 \mu\text{C}$

Thus, charge on capacitor (2) is $10 \mu\text{C}$

Thus, charge on capacitor (3) is $25 \mu\text{C}$

6. The figure shows an isolated system that includes all the capacitor plates isolated from other parts of circuit. We can conclude net sum of charge should be zero.



$$2(x+10) + 4(x+20) = 6(0-x) + 4(x-20) = 0$$

$$\text{or } 4x + 20 = 0 \text{ or } x = -5 \text{ V}$$

$$\text{Hence } q_1 = 4[20 - (-5)] = 100 \mu\text{C}$$

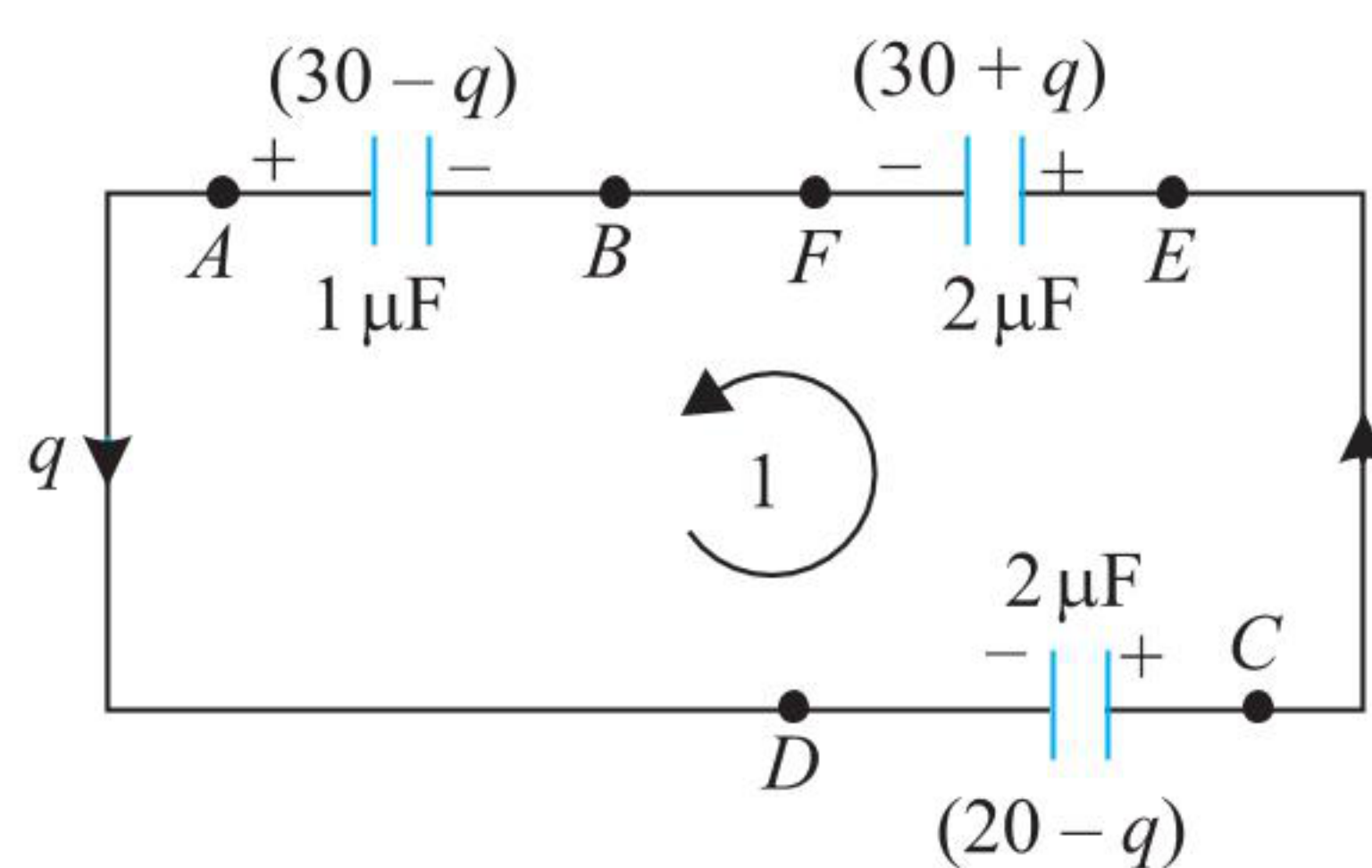
$$q_2 = 2[(-5) + 10 - 0] = 10 \mu\text{C}$$

$$q_3 = 6[0 - (-5)] = 30 \mu\text{C}$$

$$q_4 = 4[(-5) + 20 - 0] = 60 \mu\text{C}$$

7. Let charge flow be q .

Now applying Kirchhoff's voltage law, we get



$$\frac{(20-q)}{2} - \frac{(30+q)}{2} + \frac{(30-q)}{1} = 0$$

$$\text{or } -2q = -25 \text{ or } q = 12.5 \mu\text{C}$$

$$\text{Charge on } C_1 \text{ is } q_1 = 30 - 12.5 = 17.5 \mu\text{C}$$

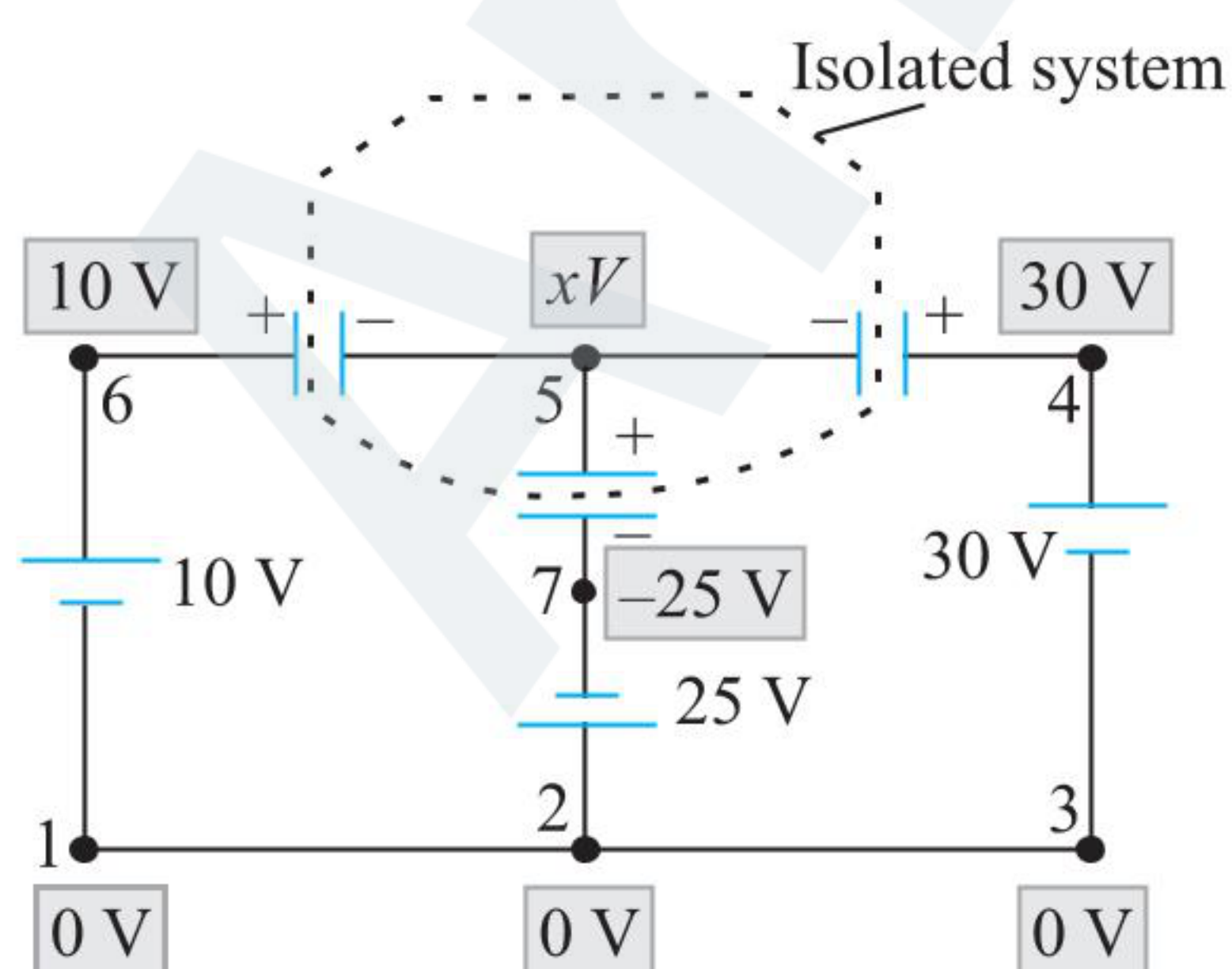
$$\text{Charge on } C_2 \text{ is } q_2 = 30 + 12.5 = 42.5 \mu\text{C}$$

$$\text{Charge on } C_3 \text{ is } q_3 = 20 - 12.5 = 7.5 \mu\text{C}$$

8. Let potential at 1 is 0, so at 4 it is 30 V, at 6 it is 10 V, and at point 7 potential is -25 V. Let us assume potential at point 5 be x V. Let us take isolated system as shown in the figure. (Total charge of all the plates connected to '5' must be same as before, i.e., 0.)

$$\therefore 1(x-10) + 2(x-30) + 2(x+25) = 0$$

$$\text{or } 5x = 20 \text{ or } x = 4$$



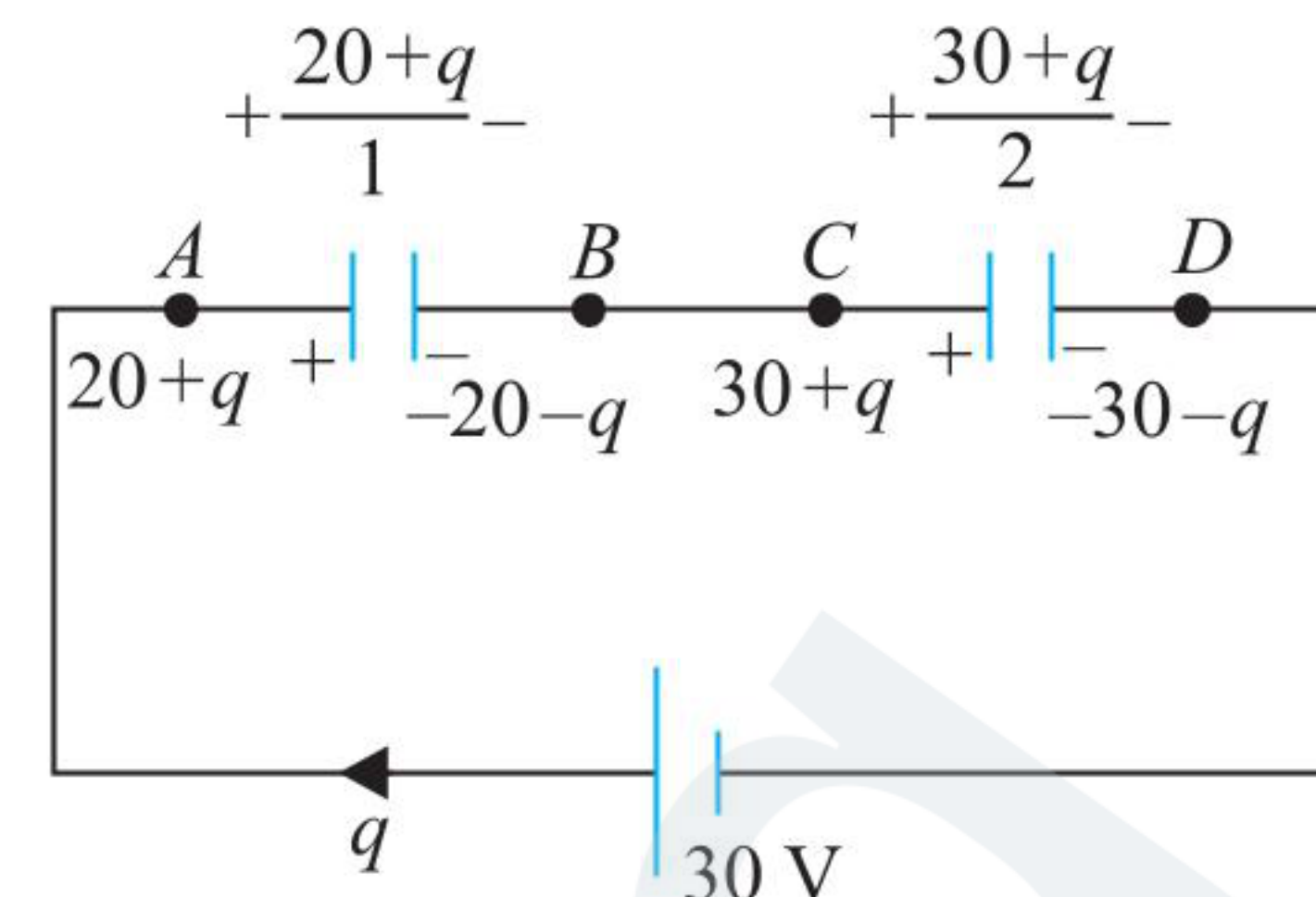
Final charges:

$$\text{On } C_1, Q_1 \text{ is } (30 - 4)2 = 52 \mu\text{C}$$

$$\text{On } C_2, Q_2 \text{ is } 2(4 - (-25)) = 58 \mu\text{C}$$

$$\text{On } C_3, Q_3 \text{ is } 1(10 - 4) = 6 \mu\text{C}$$

9. Now applying Kirchhoff's voltage law



$$\text{or } \frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$$\text{or } -40 - 2q - 30 - q = -60$$

$$\text{or } 3q = -10$$

$$\text{Charge flow is } -10/3 \mu\text{C}$$

$$\text{Charge on capacitor of capacitance } 1 \mu\text{F is } 20 + q = \frac{50}{3} \mu\text{C}$$

$$\text{Charge on capacitor of capacitance } 2 \mu\text{F is } 30 + q = \frac{80}{3} \mu\text{C}$$

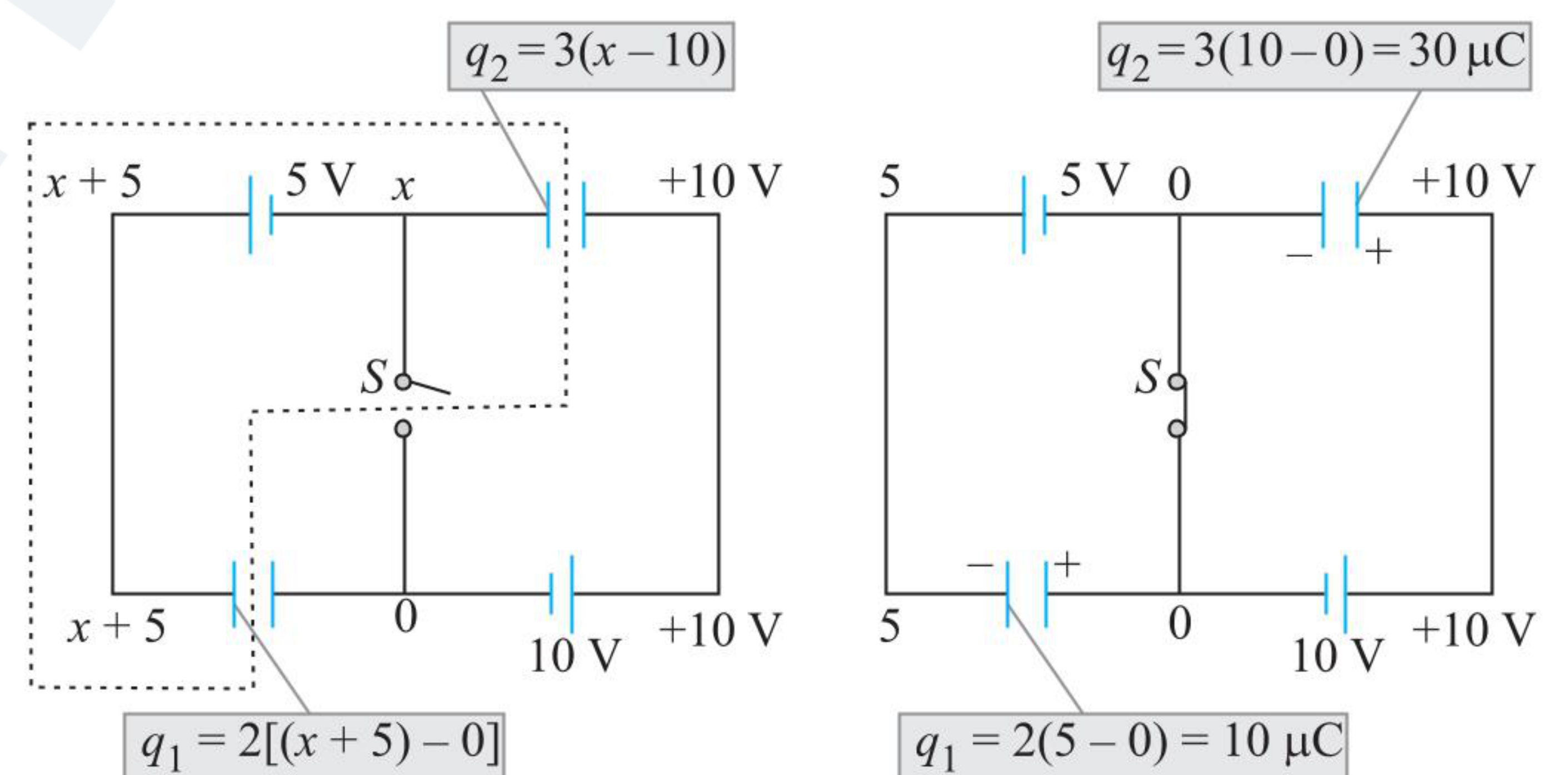
10. When switch is opened, let us consider an isolated system as shown. In this isolated system, net charge should be zero.

$$3(x-10) + 2(x+5) = 0 \text{ or } x = 4 \text{ V}$$

$$\text{Hence charge on } C_1 \text{ is } q_1 = 2(4 + 5) = 18 \mu\text{C}$$

$$\text{And charge on } C_2 \text{ is } q_2 = 3(4 - 10) = -18 \mu\text{C}, \text{ or left plate is negative.}$$

$$\text{Hence, charge on } C_2 \text{ is } q_2 = 18 \mu\text{C} \text{ (right plate is positive)}$$

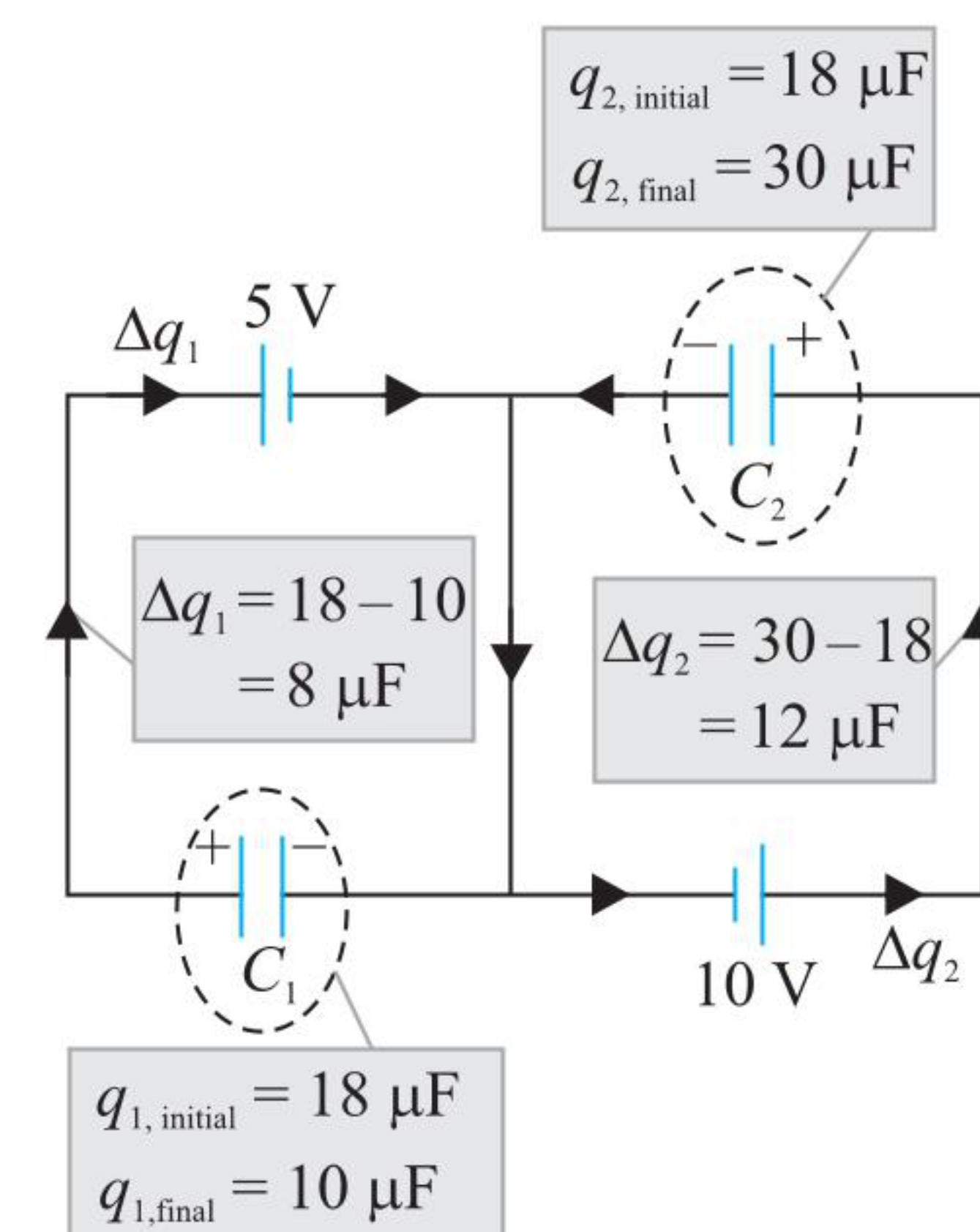


When switch is open

When switch is closed

When switch is closed, the final charge on the capacitors is shown in the figure. Note that $\Delta q_1 = 8 \mu\text{C}$ is charging battery; hence, work done by 5 V battery will be negative.

$$W_{5V} = -(8 \mu\text{C})(5 \text{ V}) = -40 \mu\text{J}$$



Δq_2 is moving from positive terminal of 10 V battery, hence work done by 10 V battery is $(120 \mu\text{C})(10 \text{ V}) = 120 \mu\text{J}$.

Net work done by batteries is

$$W_{\text{battery}} = -40 + 120 = 80 \mu\text{J}$$

Change in potential energy of the capacitors

$$\Delta U = U_f - U_i = \left(\frac{1}{2} 3[10]^2 + \frac{1}{2} 2[5]^2 \right) - \left(\frac{1}{2} 3[6]^2 + \frac{1}{2} 2[9]^2 \right) = 40 \mu\text{J}$$

Heat developed in the circuit is

$$H = W_{\text{battery}} - \Delta U = 80 - 40 = 40 \mu\text{J}$$

11. When switch S is open, for isolated system 1,

$$6(x - 200) + 3(x - 0) = 0$$

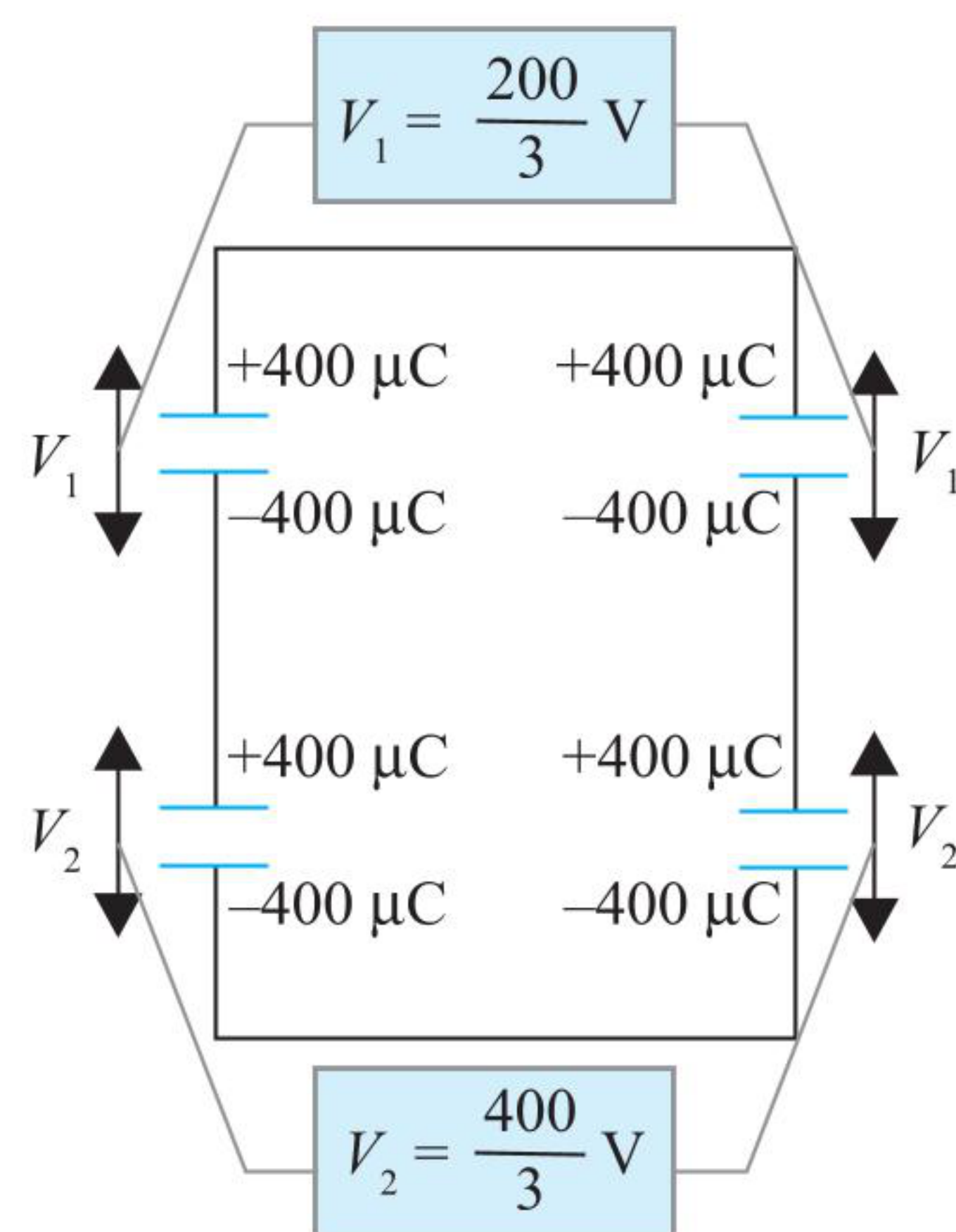
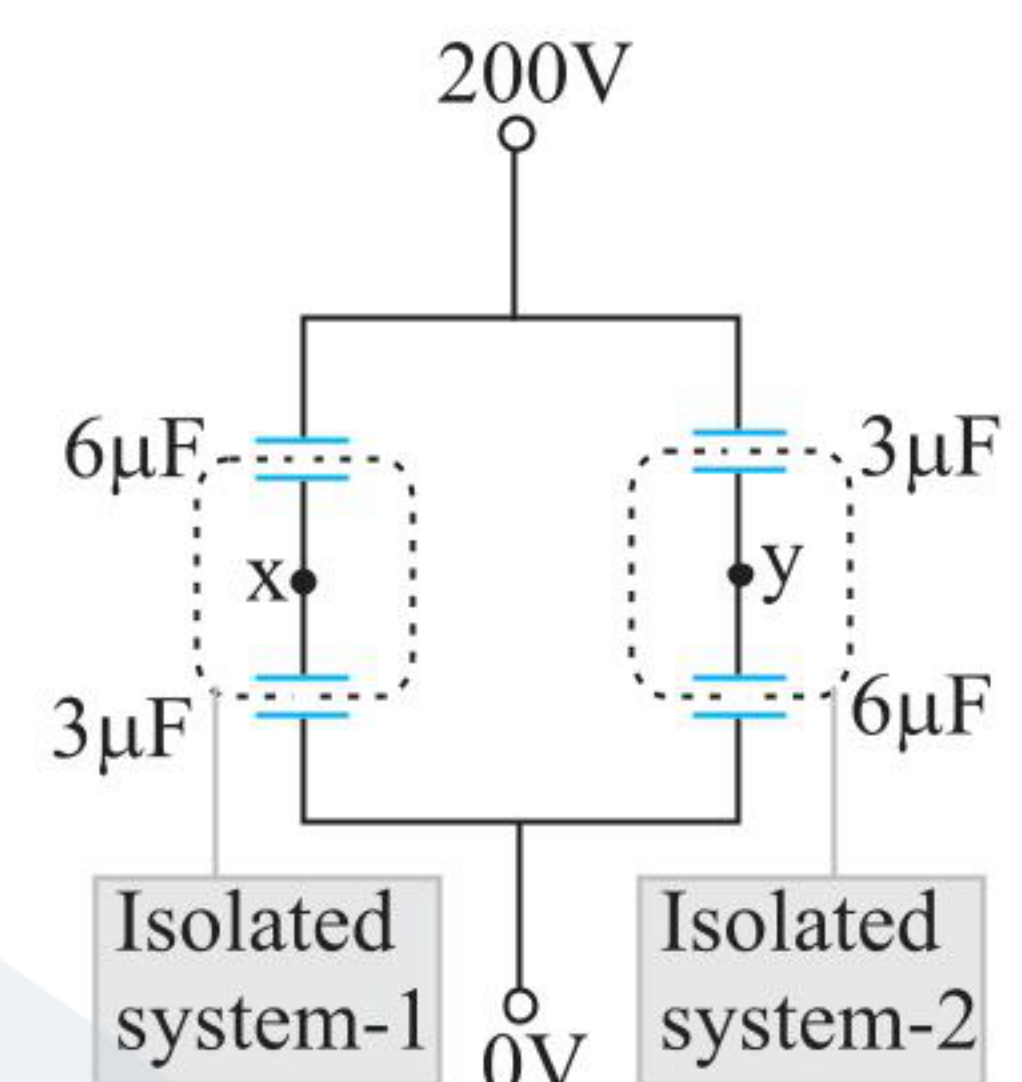
$$\text{or } x = \frac{400}{3} \text{ V}$$

Similarly for isolated system 2

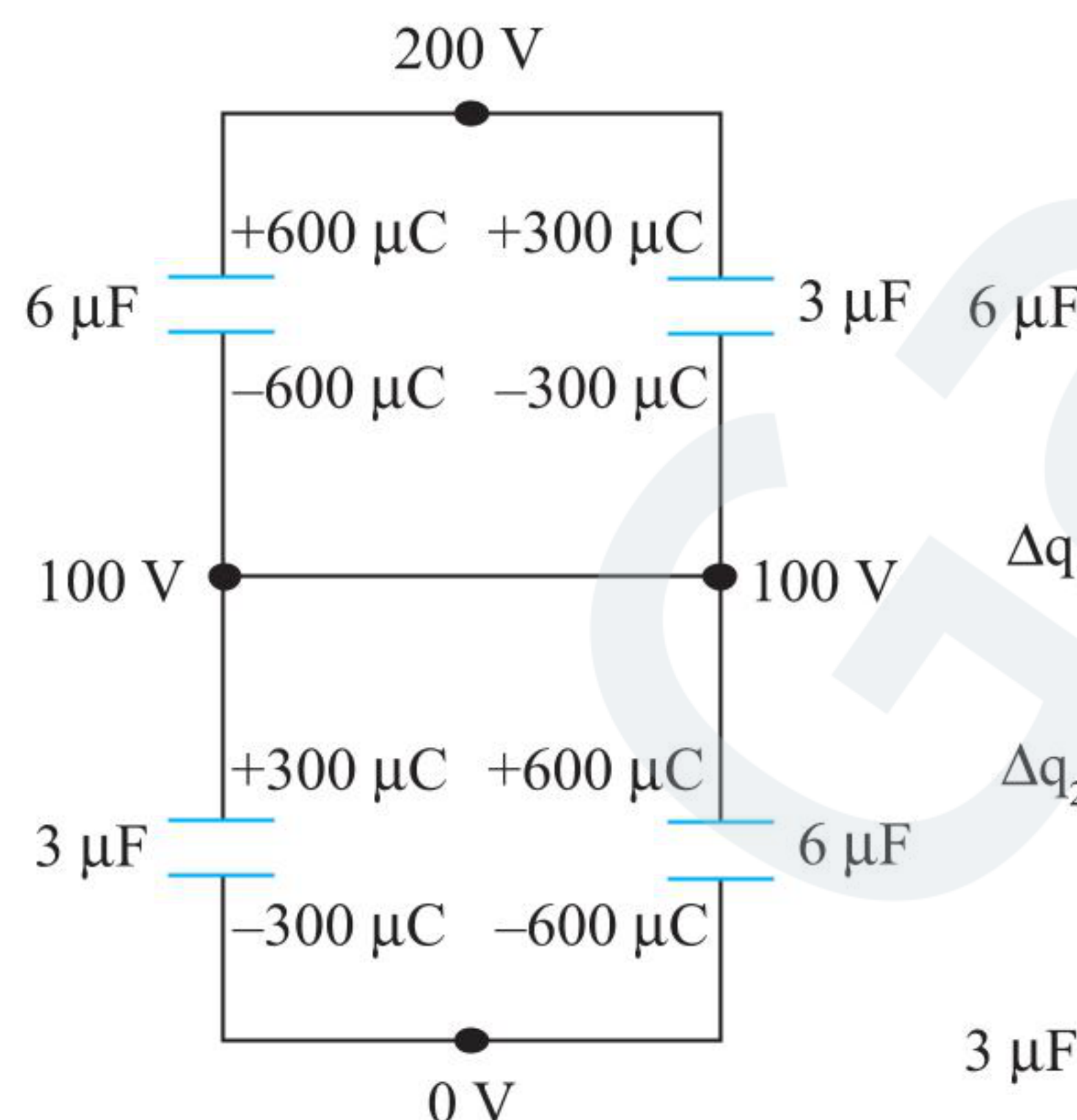
$$3(y - 200) + 6(y - 0) = 0$$

$$\text{or } y = \frac{200}{3} \text{ V}$$

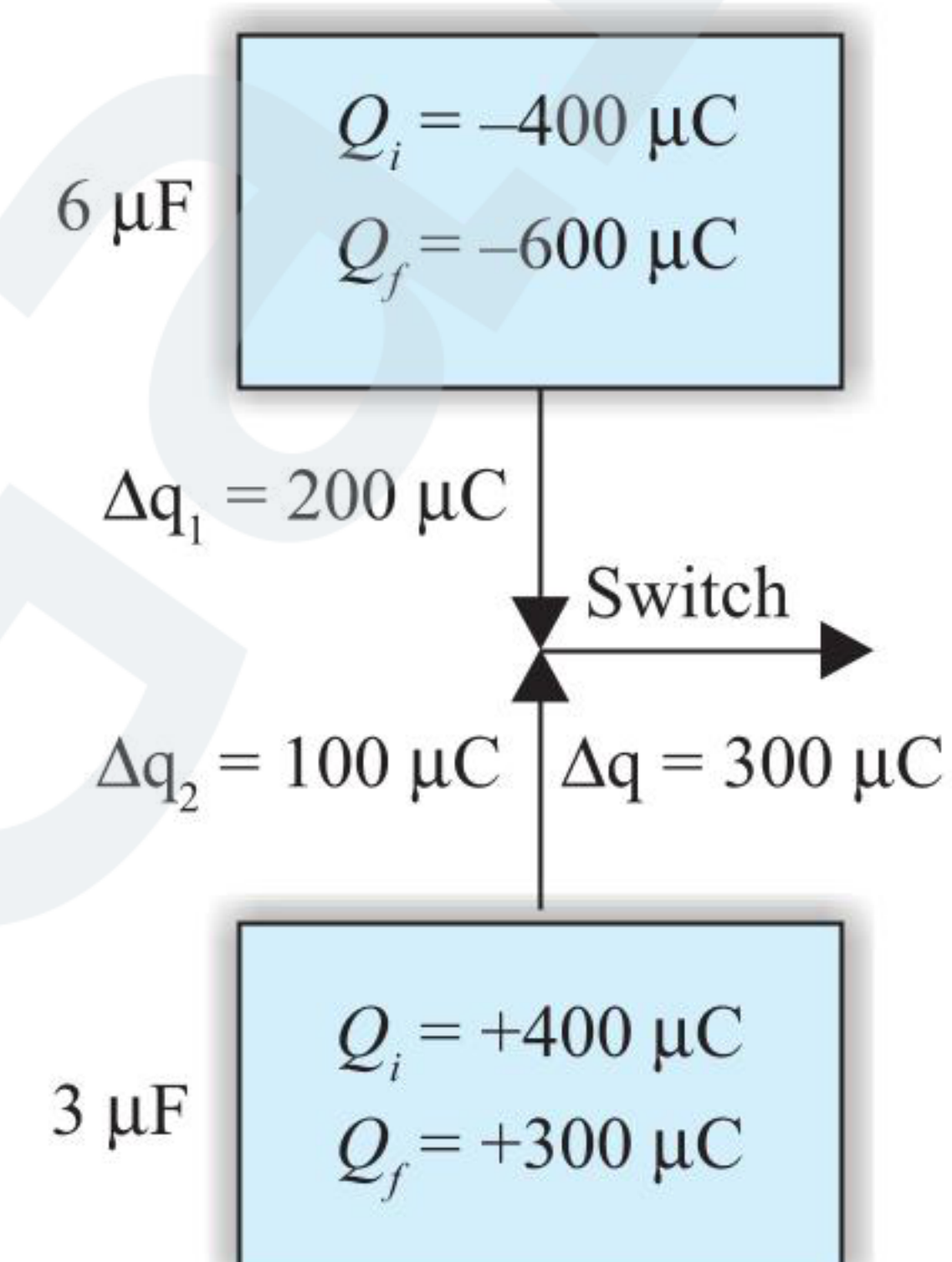
$$\text{Thus } V_a - V_b = x - y = \frac{400}{3} - \frac{200}{3} = \frac{200}{3} \text{ V}$$



Before closing the switch



When switch is closed



When switch is closed, the potentials of a and b will be equal to 100 V. Thus, initial charge and final charge distribution on all the plates is shown in figure. Figure shows charge transfer on plates connected by switch. Charge flowing through switch is $300 \mu\text{C}$.

Exercise 4.5

1. (a) $q_1 = 80 \mu\text{C}$, $q_2 = 20 \mu\text{C}$

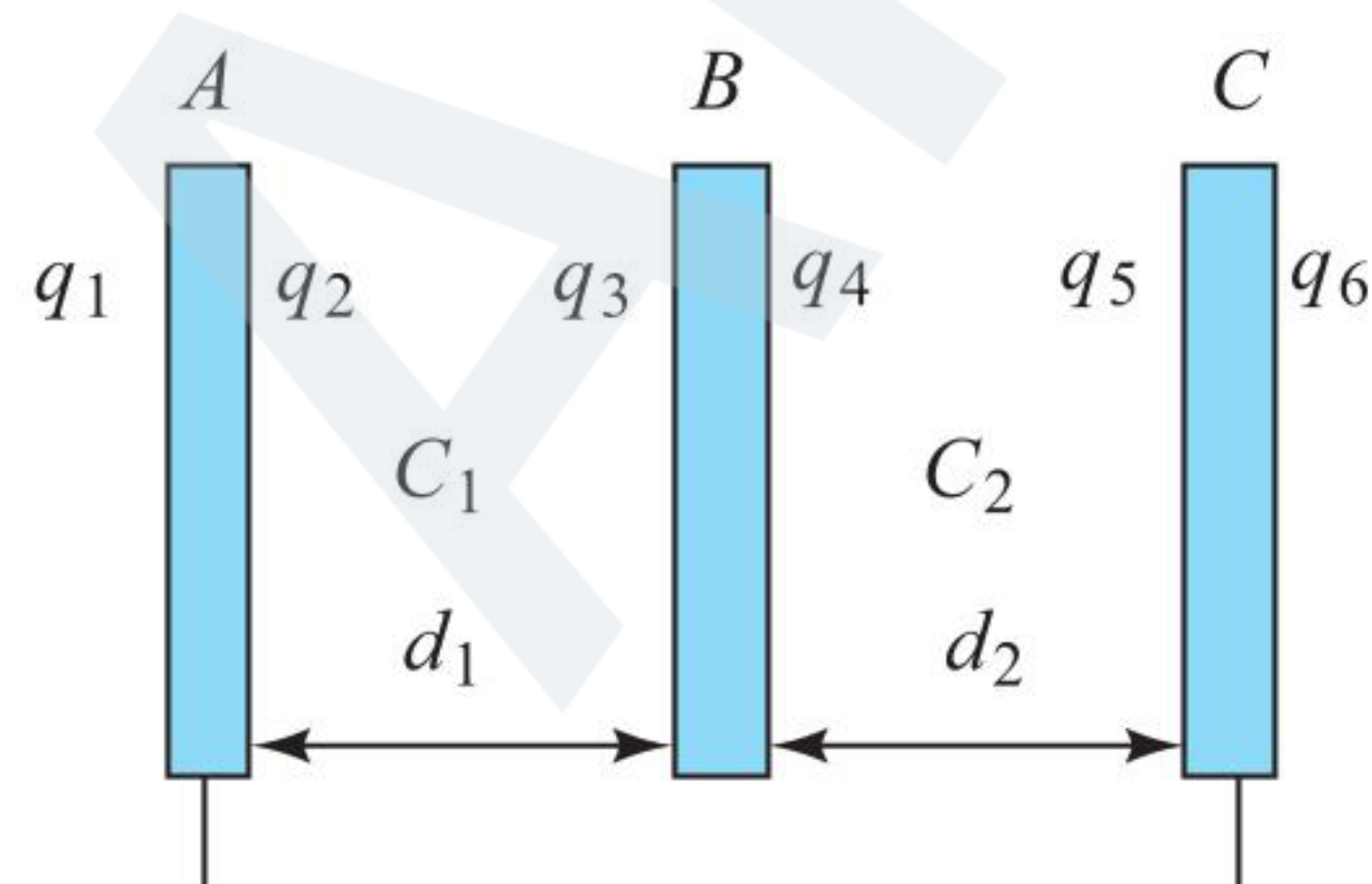
$$q_3 = -20 \mu\text{C}, q_4 = 80 \mu\text{C}$$

(b) $q_1 = 0$, $q_2 = -60 \mu\text{C}$

$$q_3 = 60 \mu\text{C}, q_4 = 0$$

(c) $q_1 = q_2 = q_3 = q_4 = 0$

2. Since charges on outer surfaces will be half of the total charge of all the three plates, so $q_1 = q_6 = 60/2 = 30 \mu\text{C}$.



$$q_3 = -q_2, q_4 = -q_5$$

$$\Rightarrow V_{AB} = V_{CB}$$

$$\text{or } \frac{q_2}{C_1} = \frac{q_5}{C_2}$$

$$q_2 d_1 = q_5 d_2 \Rightarrow q_2 2d_2 = q_5 d_2$$

$$q_5 = 2q_2 \Rightarrow q_4 = 2q_3$$

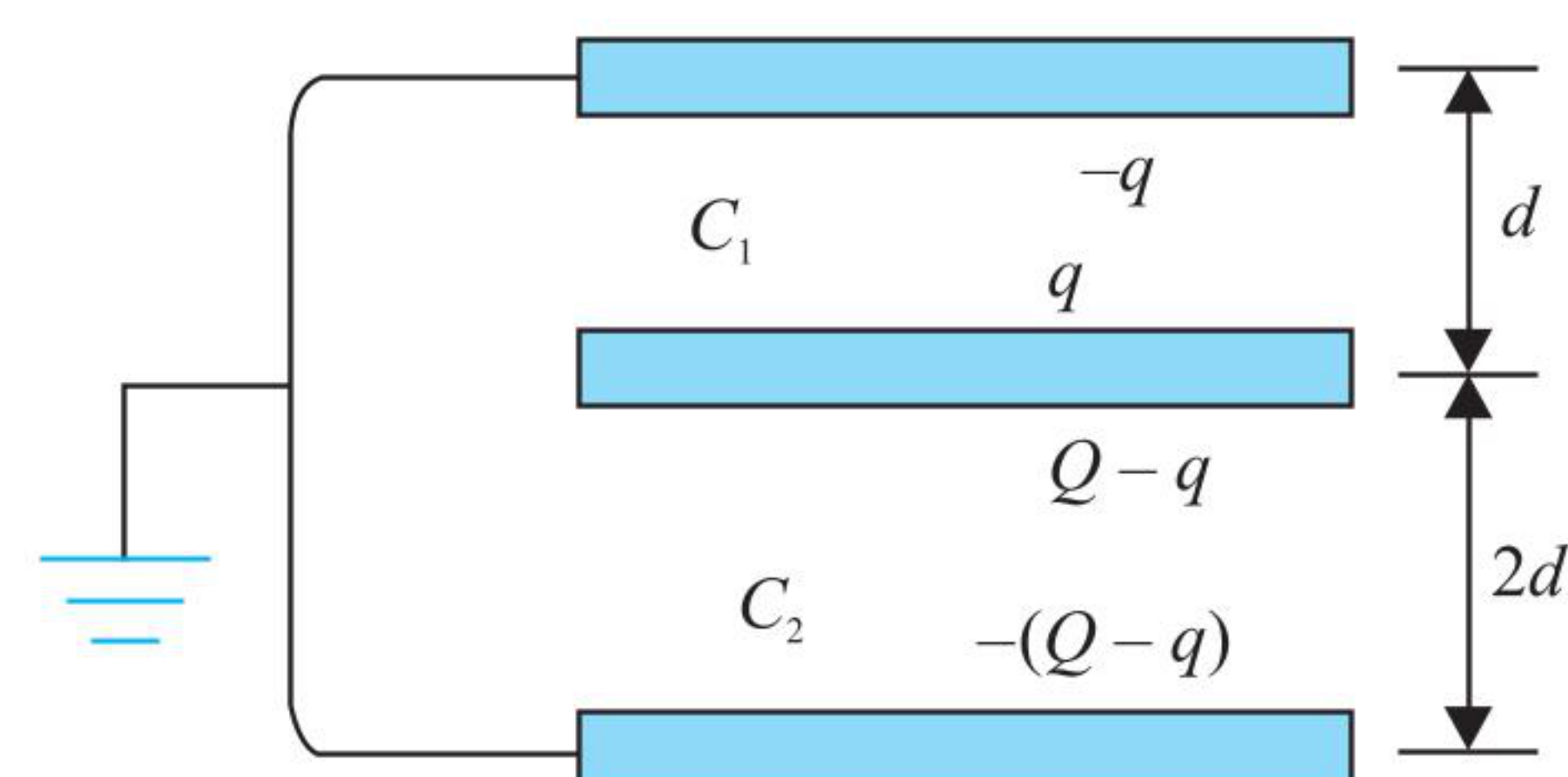
$$\text{And } q_3 + q_4 = 60$$

$$q_3 = 20 \mu\text{C} \Rightarrow q_4 = 40 \mu\text{C}$$

$$\text{And } q_2 = -20 \mu\text{C} \Rightarrow q_5 = -40 \mu\text{C}$$

3. $C = \frac{\epsilon_0 A}{d} \Rightarrow C \propto \frac{1}{d}$

$$\frac{C_1}{C_2} = \frac{2d}{d} = 2$$



Potential of middle plate

$$V = \frac{q}{C_1} = \frac{Q-q}{C_2}$$

$$\text{or } \frac{q}{Q-q} = \frac{C_1}{C_2} = 2 \quad \text{or } q = \frac{2Q}{3}$$

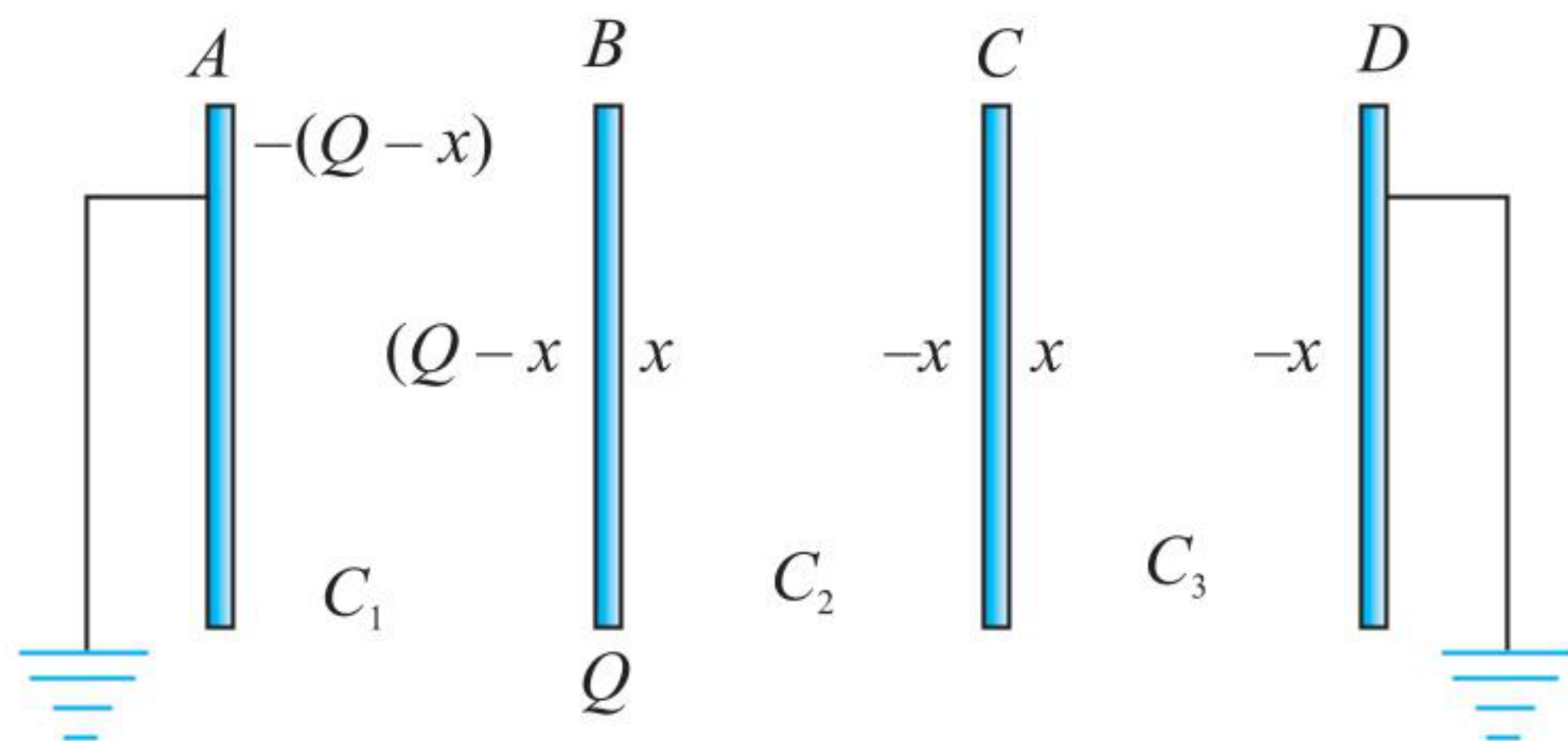
So the induced charge on upper plate is $-2Q/3$ and on lower plate is $-Q/3$.

4. Moving from plate A to D

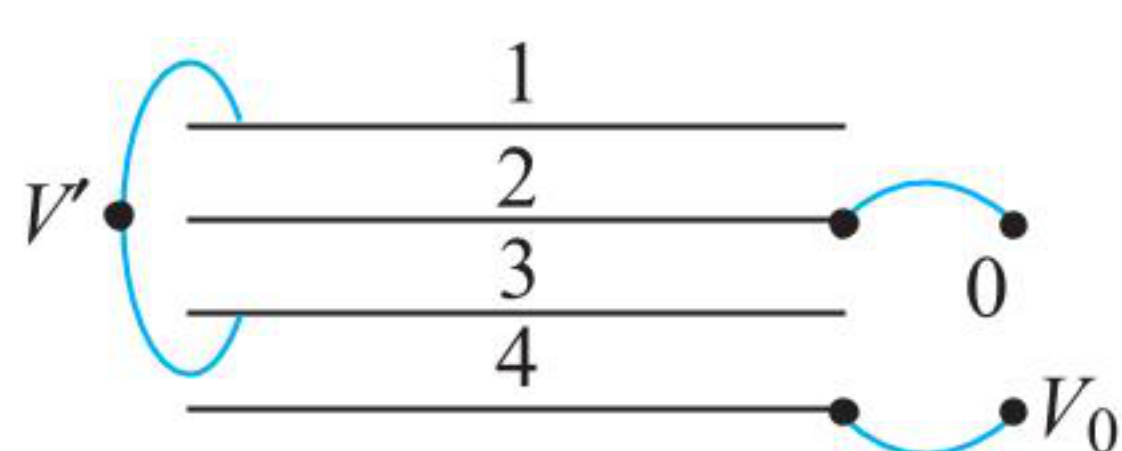
$$0 + \frac{(Q-x)}{C} - \frac{x}{C} - \frac{x}{C} = 0 \quad \text{or} \quad x = \frac{Q}{3}$$

 Hence, potential difference between B and C

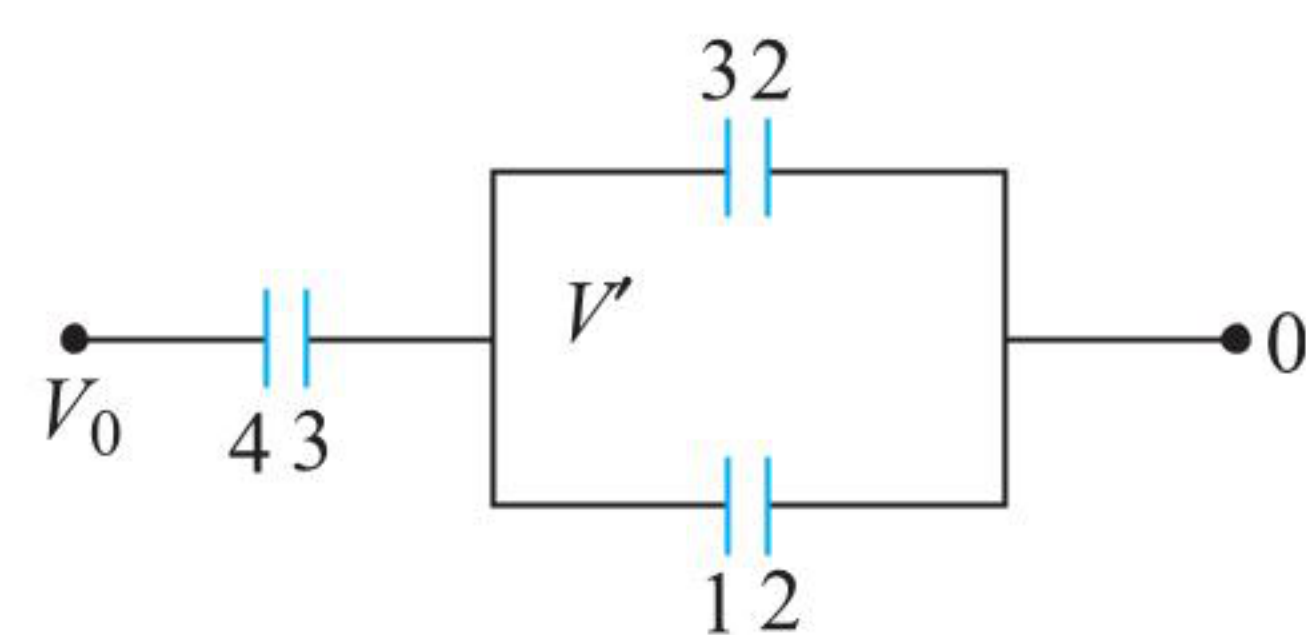
$$V_{BC} = \frac{x}{C} = \frac{Q}{3C} = \frac{Qd}{3\epsilon_0 S}$$



5. (a)



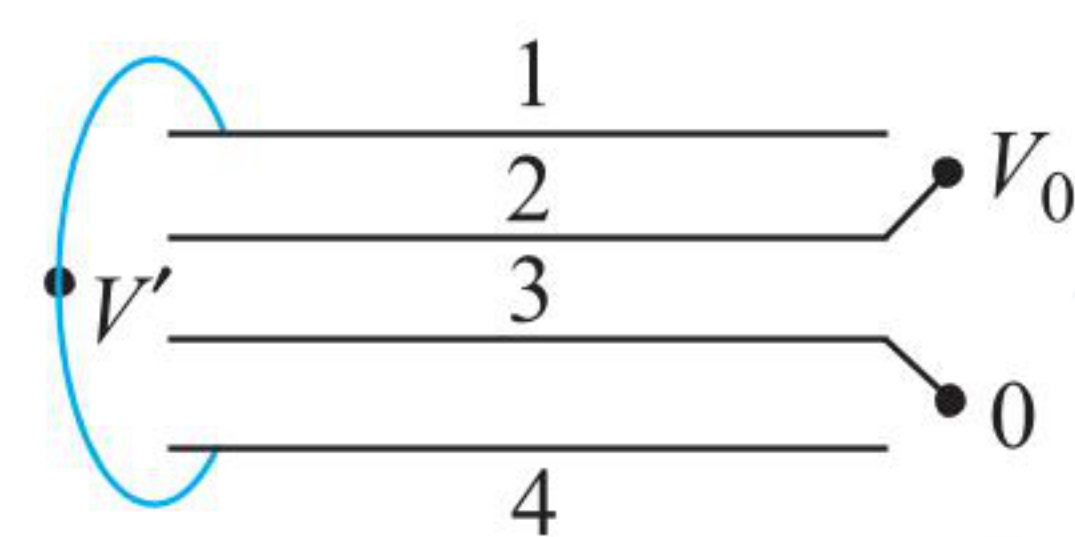
Equivalent circuit



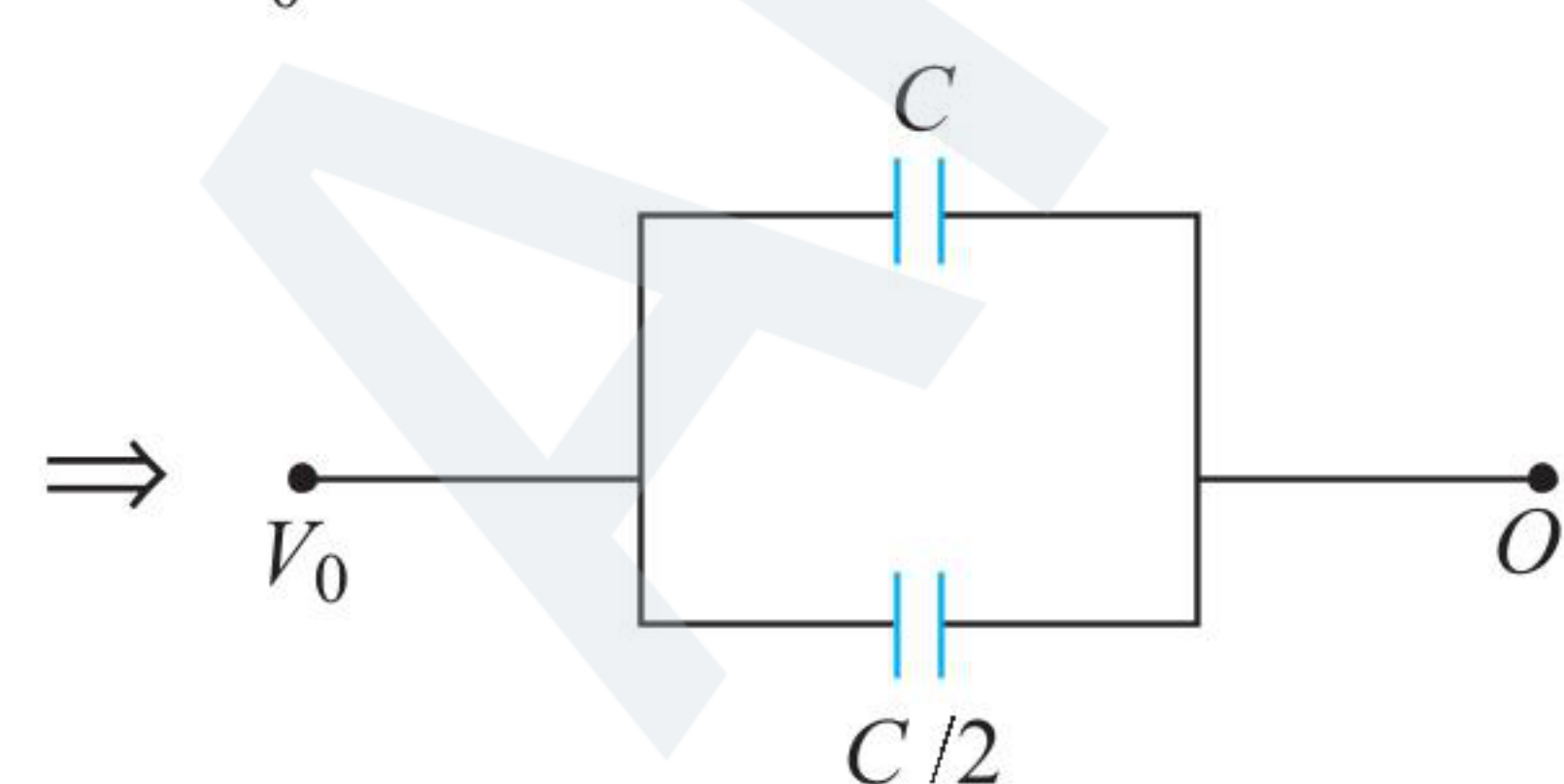
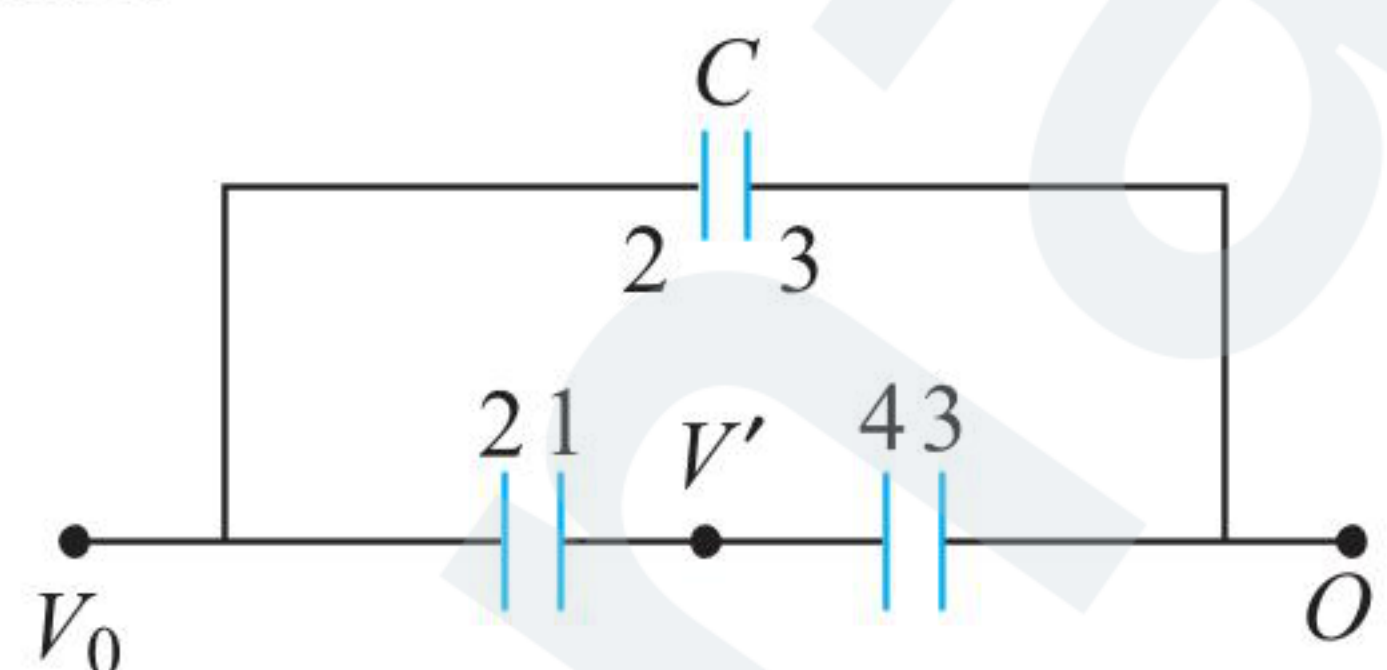
$$\Rightarrow V_0 = \frac{C}{2} + \frac{2C}{2} = \frac{3C}{2}$$

$$C_{eq} = \frac{2C}{3} = \frac{2\epsilon_0 A}{3d}$$

(b)

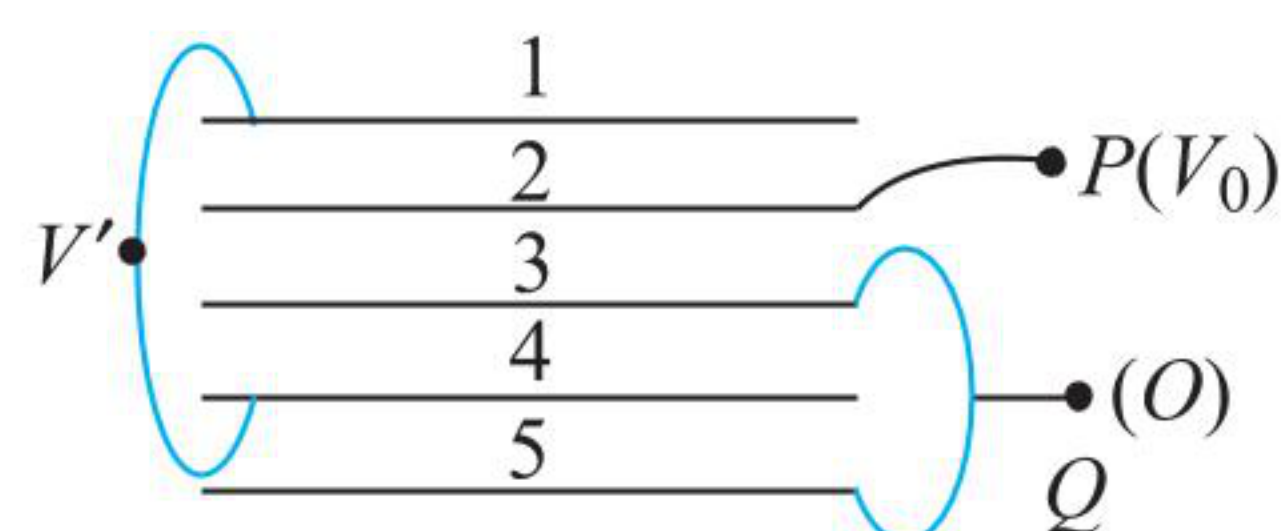


Equivalent circuit

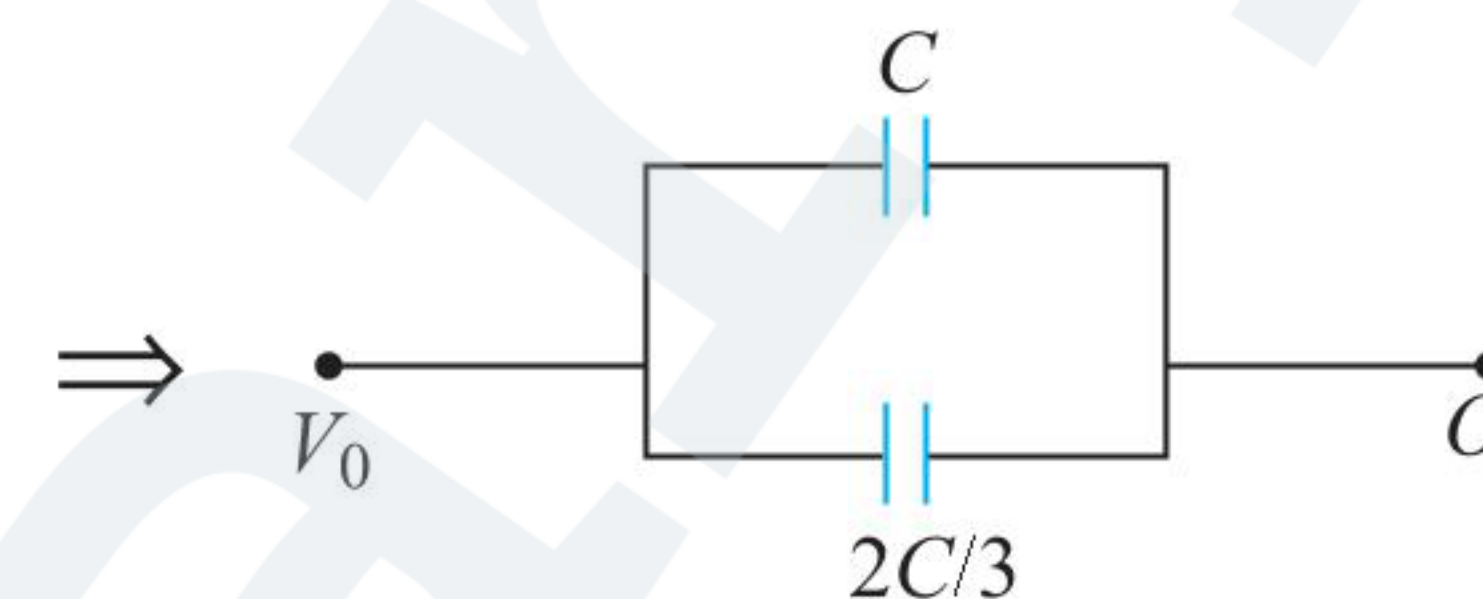
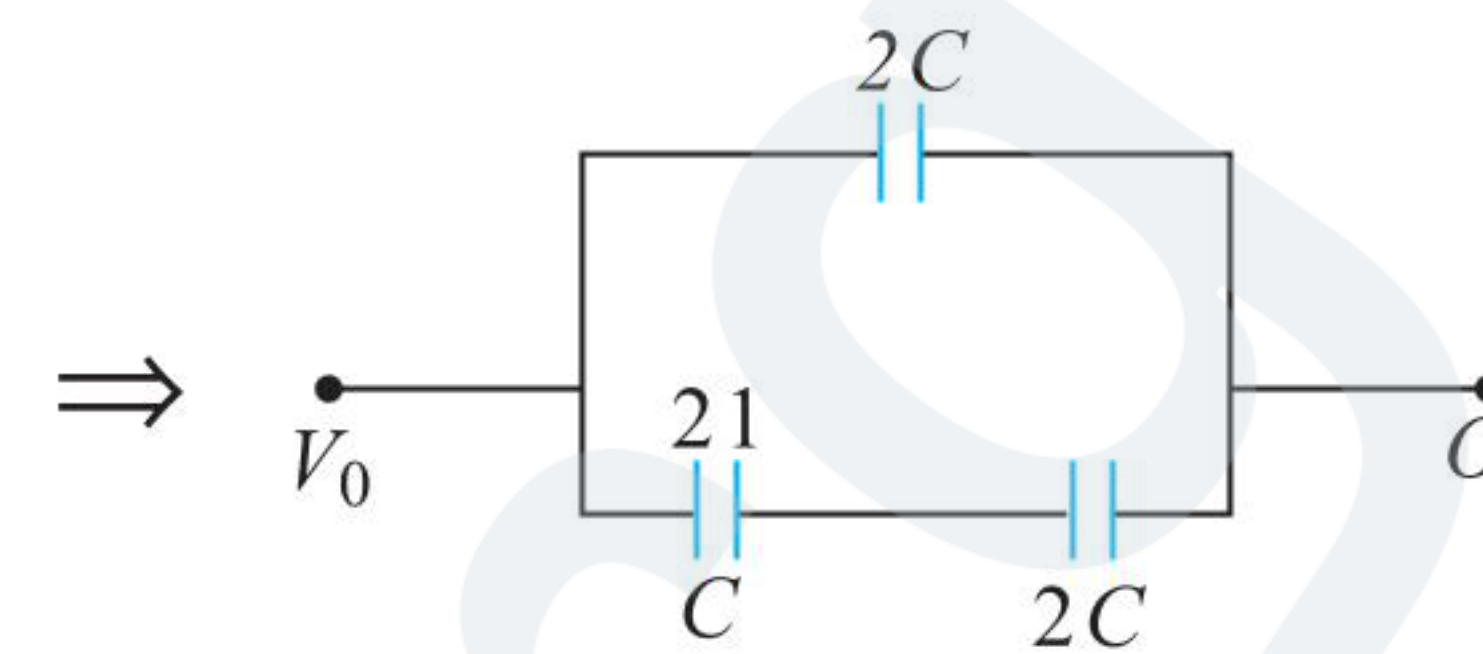
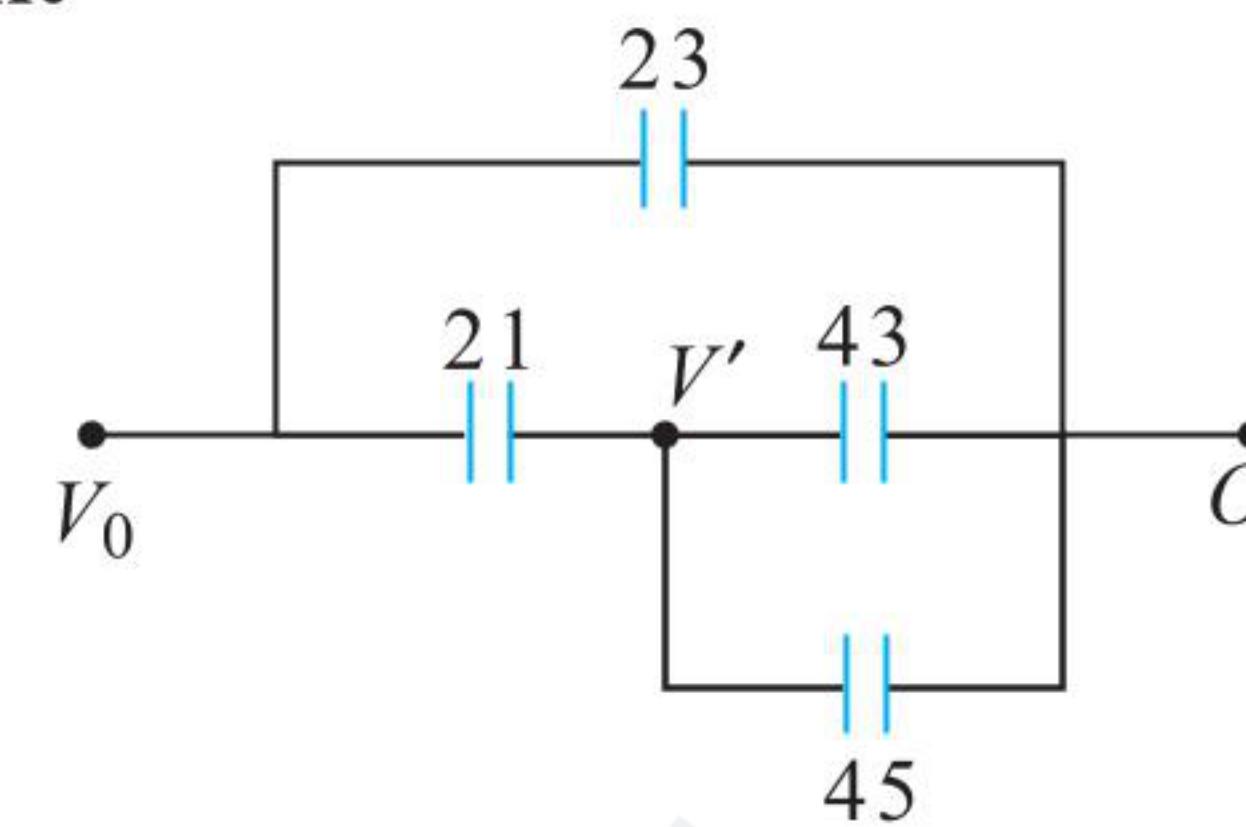


$$C_{eq} = \frac{3}{2}C = \frac{3\epsilon_0 A}{2d}$$

(c)

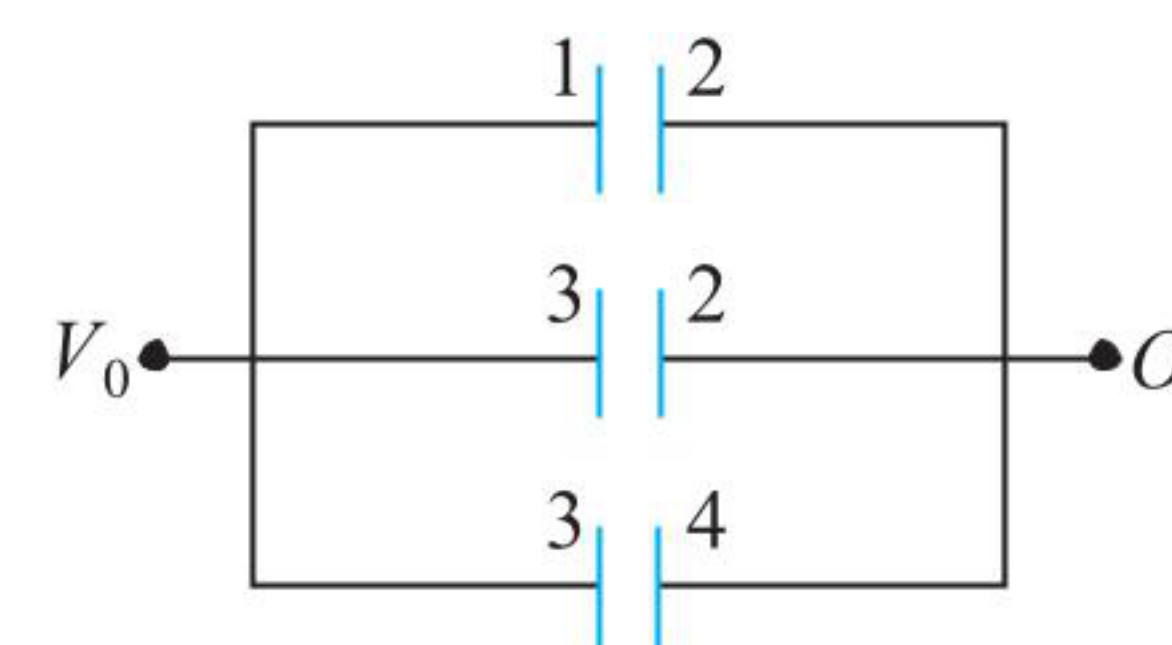
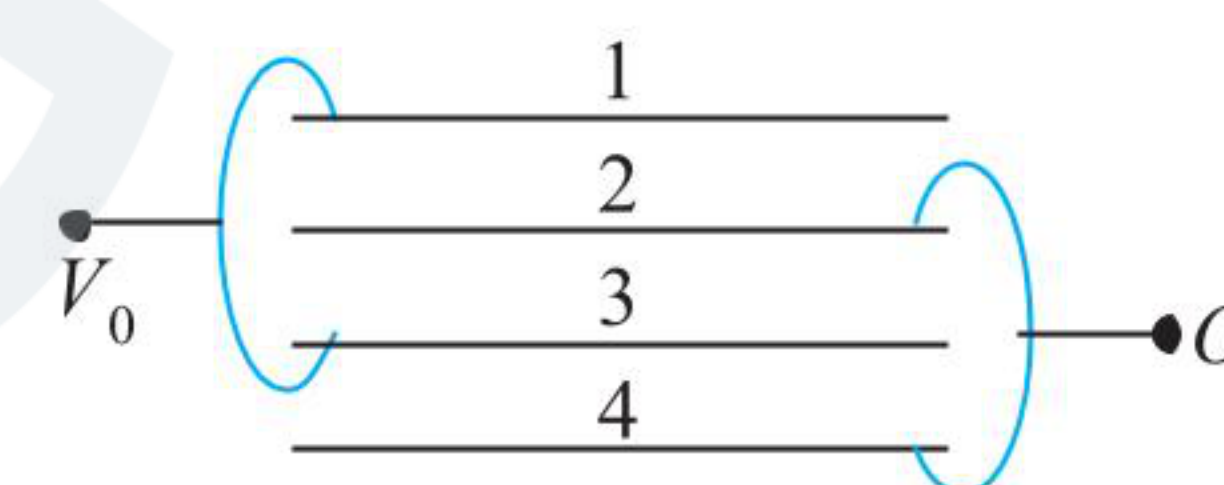


Equivalent circuit



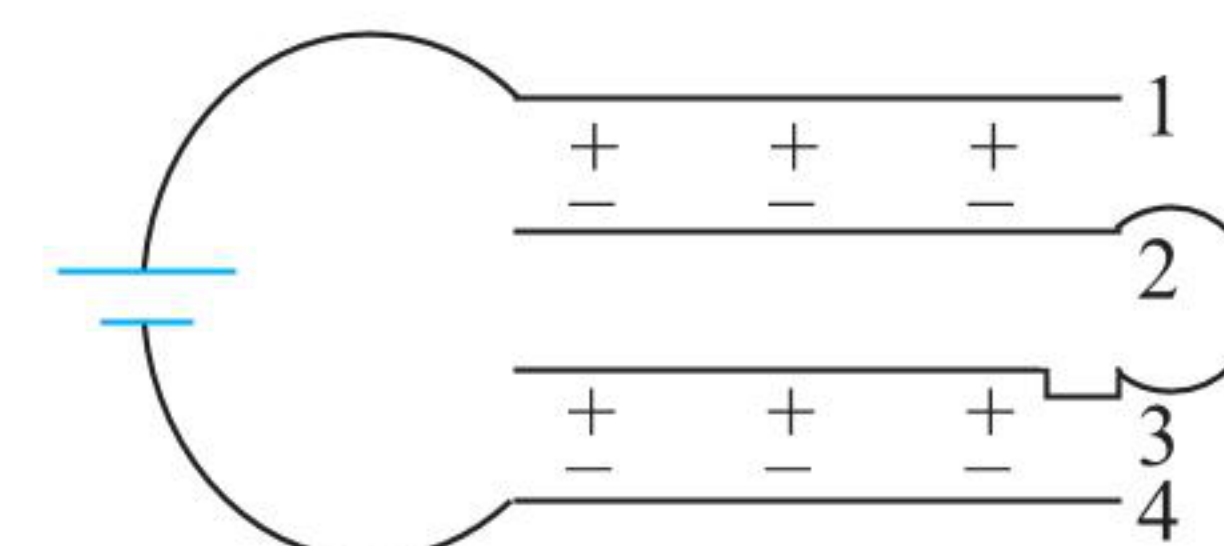
$$C_{eq} = \frac{5C}{3} = \frac{5\epsilon_0 A}{3d}$$

(d)

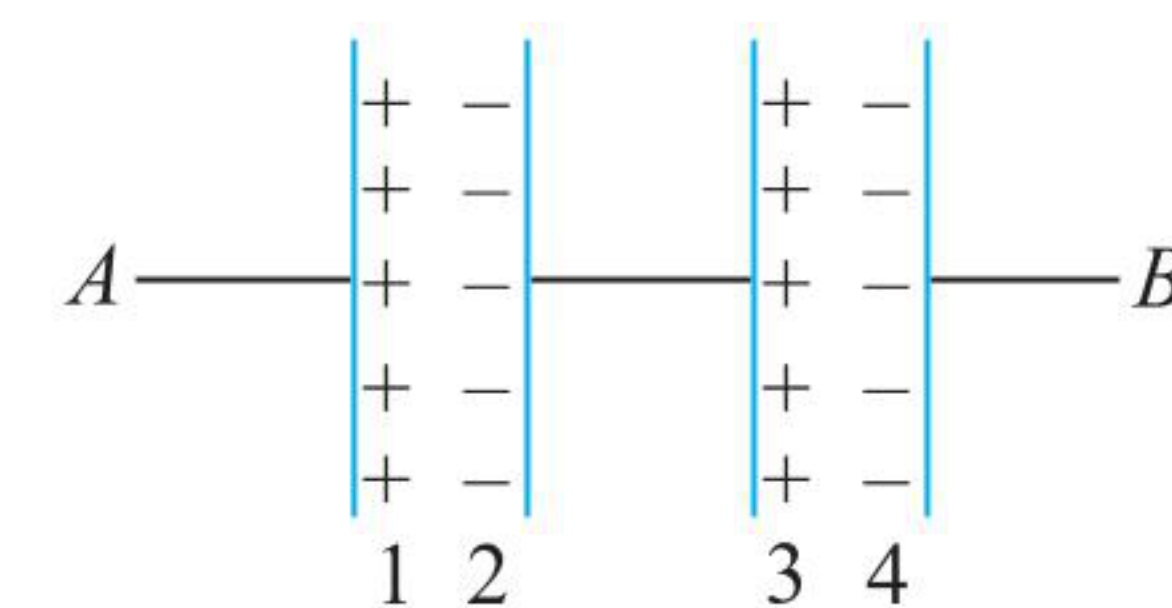


$$C_{eq} = 3C = \frac{3\epsilon_0 A}{d}$$

6.

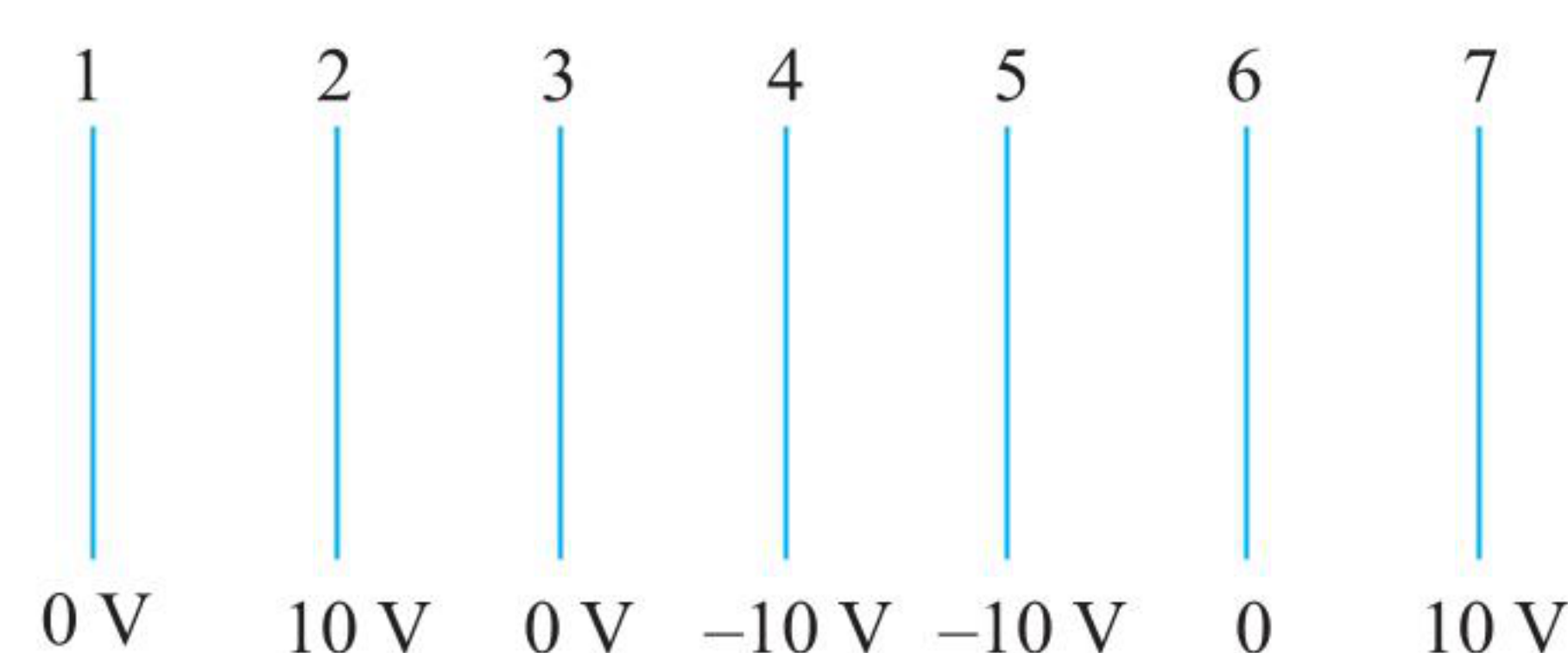


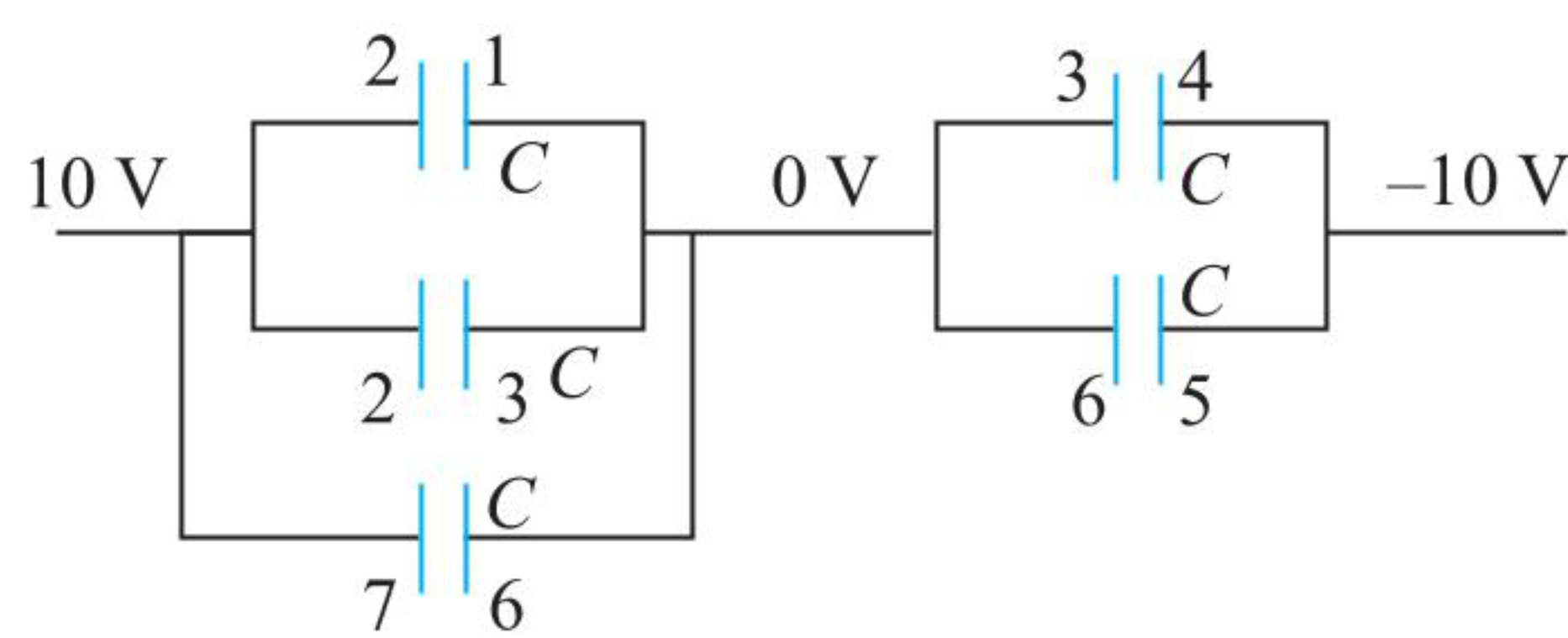
Plates can be rearranged as shown in figure.

 Plates 1 and 2 and plates 3 and 4 form two capacitors, which are in series between A and B . Plates 2 and 3 do not form any capacitor as they are at the same potential.


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{\epsilon_0 A}{d}\right) \times \left(\frac{\epsilon_0 A}{3d}\right)}{\frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{3d}} = \frac{\epsilon_0 A}{4d}$$

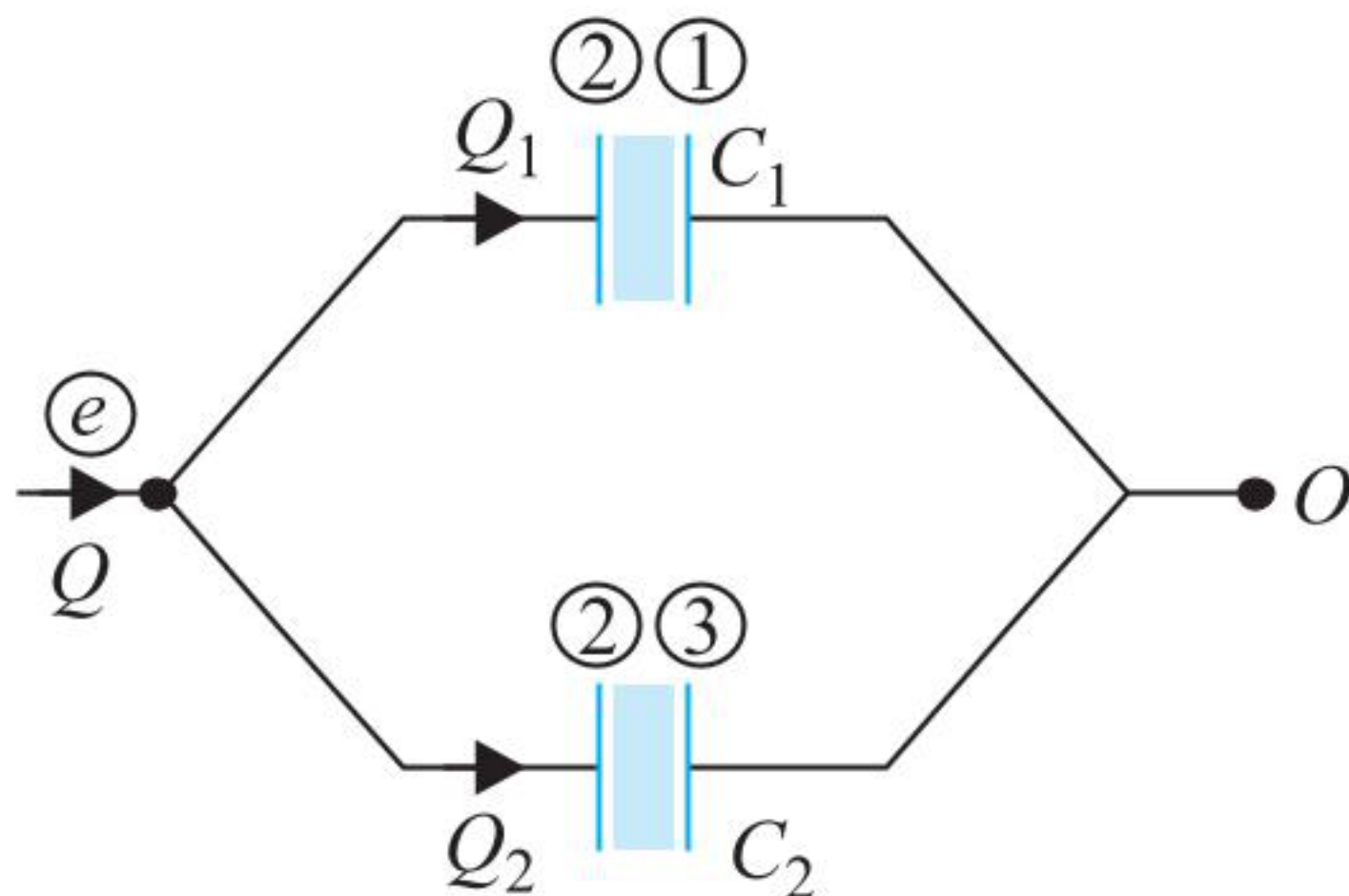
7.





$$C_{eq} = \frac{3C \times 2C}{3C + 2C} = \frac{6C}{5} = \frac{6\epsilon_0 A}{5l}$$

8. Equivalent circuit is two capacitors in parallel



$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{2\epsilon_0 A}{d} \quad \dots(i)$$

$$C_2 = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{3\epsilon_0 A}{2d} \quad \dots(ii)$$

C_1 and C_2 are in parallel.

$$C_{eq} = C_1 + C_2 = \frac{7\epsilon_0 A}{2d}$$

Charge drawn through the battery is

$$Q = C_{eq} \epsilon = \frac{7\epsilon_0 A \epsilon}{2d}$$

$$\text{Charge in } C_1 \text{ is } Q_1 = C_1 \epsilon = \frac{2\epsilon_0 A \epsilon}{d}$$

$$\text{And charge in } C_2 \text{ is } Q_2 = C_2 \epsilon = \frac{3\epsilon_0 A \epsilon}{2d}$$

The charge on outer surfaces of plates will be zero.

$$\text{Hence, charge on plate (1) is } q_1 = -\frac{2\epsilon_0 A \epsilon}{d}$$

$$\text{Charge on plate (2) is } q_2 = \frac{2\epsilon_0 A \epsilon}{d} + \frac{3\epsilon_0 A \epsilon}{2d} = \frac{7\epsilon_0 A \epsilon}{2d}$$

$$\text{Charge on plate (3) is } q_3 = \frac{-3\epsilon_0 A \epsilon}{2d}$$

$$\text{Energy stored in system is } U = \frac{1}{2} C_{eq} \epsilon^2 = \frac{7\epsilon_0 A \epsilon^2}{4d}$$

9. The charge appearing on outer most surface should be zero,

$$q_1 = q_6 = 0$$

and $V_{AB} = V$ with $V_A > V_B$

$$|q_2| = |q_3| = CV = \left(\frac{\epsilon_0 A}{d} \right) V$$

$$q_2 = +\frac{\epsilon_0 AV}{d} \text{ and } q_3 = -\frac{\epsilon_0 AV}{d}$$

Similarly, $V_{BC} = V$ with $V_C > V_B$

$$|q_4| = |q_5| = CV = \left(\frac{\epsilon_0 A}{2d} \right) V$$

$$\text{With } q_5 = +\frac{\epsilon_0 AV}{2d} \text{ and } q_4 = -\frac{\epsilon_0 AV}{2d}$$

Exercises

Single Correct Answer Type

$$1. (4) C_{eq} = C + 2C + 3C + \dots + nC$$

$$= \frac{(n+1)n}{2} C$$

2. (1) As battery is disconnected, so charge will remain the same. It is given that final potential is the same. So final capacitance should be equal to initial capacitance, i.e.,

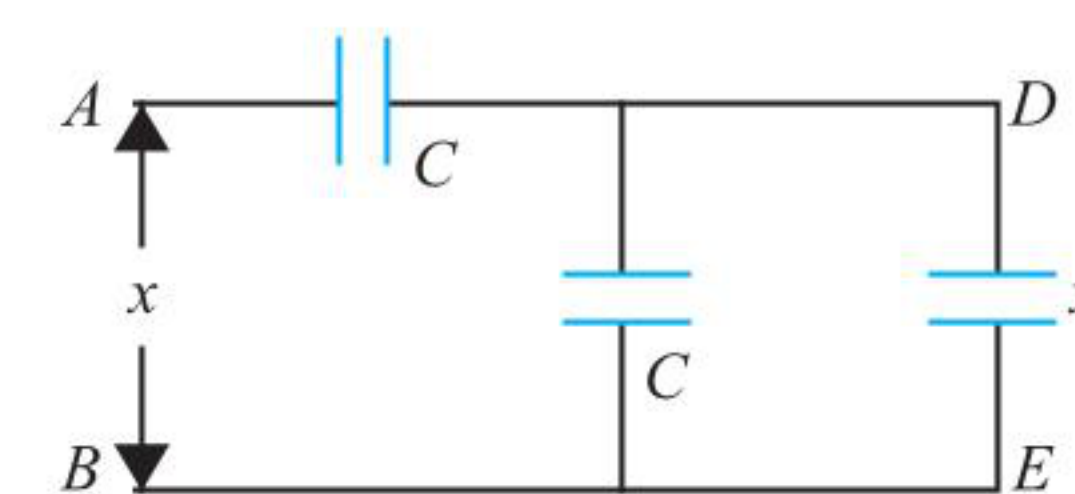
$$\frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{(1.6 + d) - t(1 - 1/K)}$$

$$\text{or } K = 5$$

3. (3) There are two capacitors in parallel.

$$U = \frac{1}{2} CV^2, C = \frac{2\epsilon_0 A}{d}$$

4. (2) Let capacitance between A and B be x. If we remove the first two capacitors, then capacitance across D and E will also be x. So the circuit can be redrawn as follows:



$$x = \frac{C(C+x)}{C+(C+x)}$$

$$\text{Solve to get } x = \frac{\sqrt{5}-1}{2} C$$

$$5. (4) C_{eq} = \frac{\epsilon_0}{\frac{d}{K_1} + \frac{d}{K_2} + \frac{d}{K_3}}$$

$$\text{Here, } K_1 = K_3 = 1, K_2 = \epsilon/\epsilon_0$$

$$6. (2) V - 0 = \frac{Q}{C} = \frac{Q(b-a)}{K4\pi\epsilon_0 ab}$$

$$\text{or } V = \frac{9 \times 10^9 \times 2.5 \times 10^{-6} (0.13 - 0.12)}{32 \times 0.13 \times 0.12} = 450 \text{ V}$$

$$7. (4) W = U_2 - U_1 = \frac{q^2}{2} \left[\frac{1}{C_2} - \frac{1}{C_1} \right]$$

$$C_1 = \frac{\epsilon_0 A}{d}, C_2 = \frac{C_1}{2} = \frac{\epsilon_0 A}{2d}$$

$$q = C_1 V = \frac{\epsilon_0 AV}{d}$$

$$\text{Solve to get } W = \frac{1}{2} \frac{\epsilon_0 AV^2}{d}$$

Alternatively:

$$W = Fd = \frac{Q^2}{2A\epsilon_0} d = \frac{C_1^2 V^2}{2\epsilon_0 A} d = \frac{1}{2} \frac{\epsilon_0 AV^2}{d}$$

8. (4) $V = Ed$. As d increases, V also increases. Note that E remains the same.

$$9. (3) V_A - \frac{q}{C_1} - E - \frac{q}{C_2} = V_B \text{ or } V_A - V_B - E = q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$\text{or } q = -\frac{40}{3} \mu\text{C}$$

10. (2) Capacitors B and C are in parallel, then A is in series.

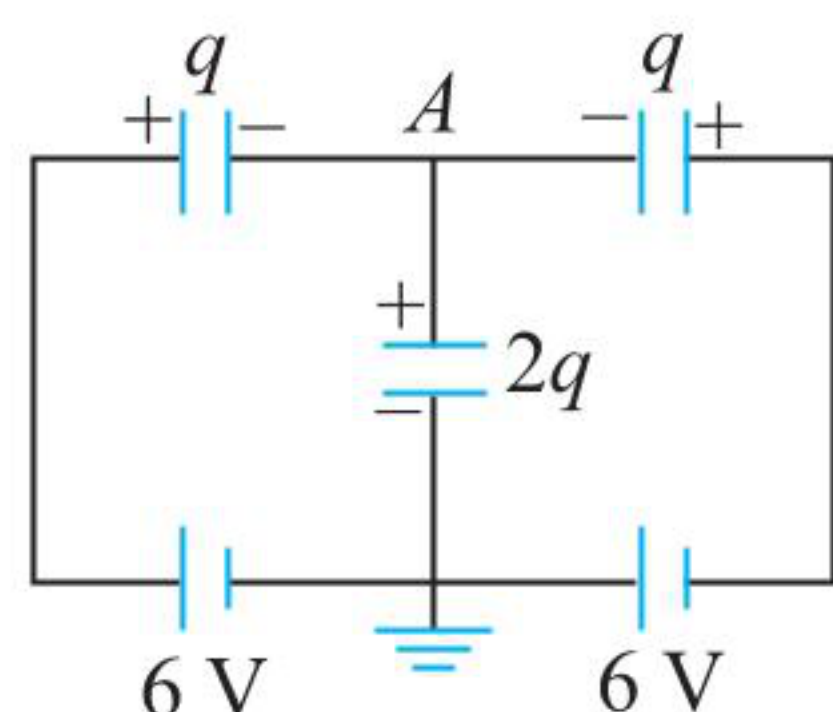
$$C_{\text{eq}} = \frac{2 \times (3+4)}{2 + (3+4)} = \frac{14}{9} \mu\text{F}$$

$$Q = C_{\text{eq}} V = \frac{14}{9} (7-6) = \frac{14}{9} \mu\text{C}$$

Q will be divided between B and C . So charge on B is

$$q = \frac{3 \times (14/9)}{3+4} = \frac{2}{3} \mu\text{C}$$

11. (1) $6 = \frac{q}{2} + \frac{2q}{4}$ or $q = 6 \mu\text{C}$



12. (3) Initial charge on C_1 is $Q_1 = C_1 V = 110 \mu\text{C}$

Let x charge flow through wires.

$$\frac{Q_1 - x}{C_1} = \frac{x}{C_{\text{eq}}}$$

$$\text{where } C_{\text{eq}} = \frac{C_2 \times C_3}{C_2 + C_3}.$$

Solve to get $x = 60 \mu\text{C}$.

13. (3) At junction A , Q_1 will be divided into Q_2 and Q_3 . Hence, $Q_1 = Q_2 + Q_3$. C_2 and C_3 are in parallel, so potential on them will be the same, i.e., $V_2 = V_3$.

V will be divided into V_1 and V_2 (or V_3).

Hence, $V = V_1 + V_2$ or $V = V_1 + V_3$

14. (1) $C_1 = C$, $V_1 = V_0$, $C_2 = KC$, $V_2 = 0$, and $V_{\text{common}} = V$

We know that

$$V_{\text{common}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$$\therefore V = \frac{CV_0 + KC \times 0}{C + KC}$$

$$\text{or } K = \frac{V_0}{V} - 1$$

15. (1) $V_1 = \frac{C \times 100}{3C + C} = 25 \text{ V}$, $V_2 = 100 - 25 = 75 \text{ V}$

16. (4) Start with C_3 and C_4 in parallel, then C_2 in series, then C_5 in parallel, then C_1 in series, and finally C_6 in parallel.

17. (3) $W = \frac{1}{2} C [(2V)^2 - V^2] = \frac{3}{2} CV^2$

$$W' = \frac{1}{2} C [(2V)^2 - (2V)^2] = 6CV^2$$

$$= \frac{6 \times 2 W}{3} = 4 W$$

18. (2) $A = 90 \text{ cm}^2$, $d = 2 \text{ mm}$

$$E = \frac{V}{d}, V = 400 \text{ V}$$

$$u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \left(\frac{V}{d} \right)^2$$

$$= \frac{1}{2} (8.85 \times 10^{-12}) \left(\frac{400}{2 \times 10^{-3}} \right)^2 = 0.177 \text{ Jm}^{-3}$$

19. (1) 22 V get will divide into series combination of C_3 and C_4 .

20. (3) $V_1 - V_2$ will be divided between C_1 and C_2 in series. Potential difference across C_1 is

$$V_1 - V_2 = \frac{C_2 (V_1 - V_2)}{C_1 + C_2}$$

$$\text{or } V_D = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

21. (3) $U = \frac{1}{2} CV^2$

Potential of each capacitor now is $V/2$.

$$U' = \frac{1}{2} C \left(\frac{V}{2} \right)^2 = \frac{U}{4}$$

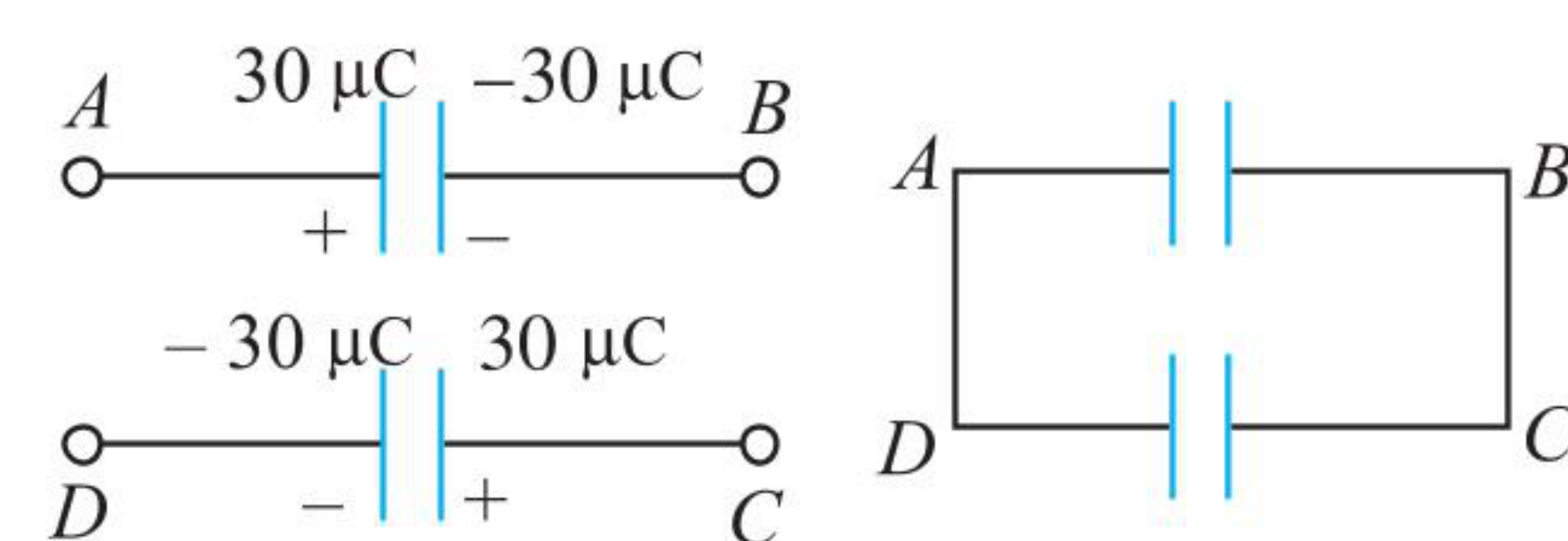
22. (1) Given $\frac{\epsilon_0 A}{d} = 10$

$$C' = \frac{\epsilon_0 A/2}{d} + \frac{4\epsilon_0 (A/2)}{d} = 10 \left[\frac{1}{2} + 2 \right] = 25 \mu\text{F}$$

23. (2) The whole amount of energy stored in capacitor will convert into heat.

$$\text{Heat} = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2 = 0.01 \text{ J}$$

24. (4) $V = \frac{Q_1 + Q_2}{C_1 + C_2} = 0$, where $Q_1 = 30 \mu\text{C}$, $Q_2 = -30 \mu\text{C}$



Final potential difference is zero. Final energy is zero. Charge flow is $30 \mu\text{C}$ from A to d .

25. (2) Potential of larger capacitor after the first charging is

$$V_1 = \frac{C_0 V_0}{C + C_0}$$

After second charging, potential is

$$V_2 = \frac{C_0 V_1}{C + C_0} = \left(\frac{C_0}{C + C_0} \right)^2 V_0$$

After n^{th} charging, potential is

$$V_n = \left(\frac{C_0}{C + C_0} \right)^n V_0$$

But $V_n = V$

$$\text{So } C = C_0 \left[\left(\frac{V_0}{V} \right)^{1/n} - 1 \right]$$

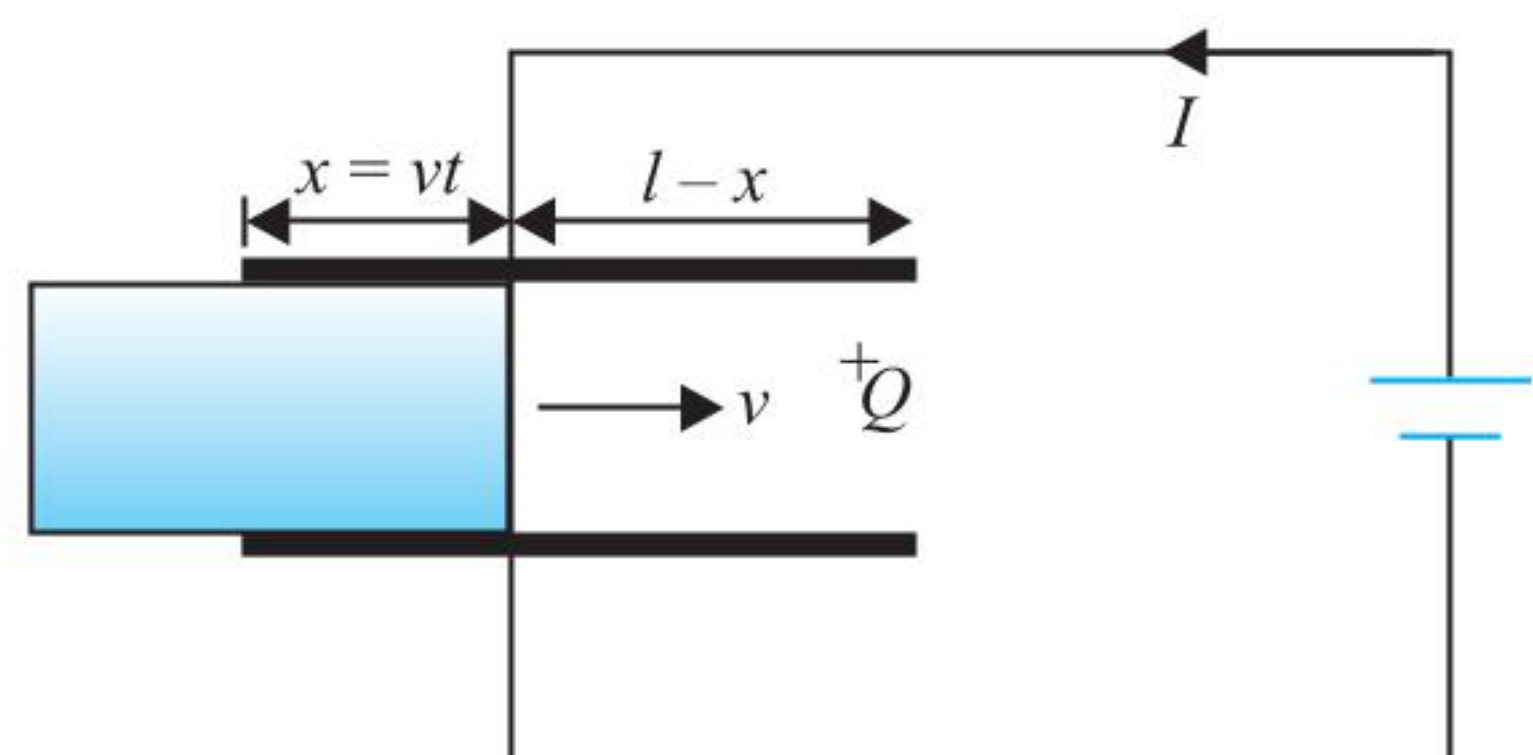
26. (3) $0 - \frac{Q}{C} + 10 - \frac{Q}{2C} = 0$

$$Q = \frac{20 C}{3} = \frac{20 \times 6}{3} = 40 \mu\text{C}$$

$$27. (2) C = \frac{K\epsilon_0 vtb}{d} + \frac{\epsilon_0(l-vt)b}{d}; q = CV$$

$$I = \frac{dq}{dt} = V \frac{dC}{dt} = V \left[\frac{K\epsilon_0 bv}{d} - \frac{\epsilon_0 bv}{d} \right] = \text{constant}$$

So till the dielectric is fully inserted, a constant current will flow. When the dielectric is fully inside, C will be constant, so $I = 0$. And when dielectric starts coming out, again a constant current will flow but in opposite direction.



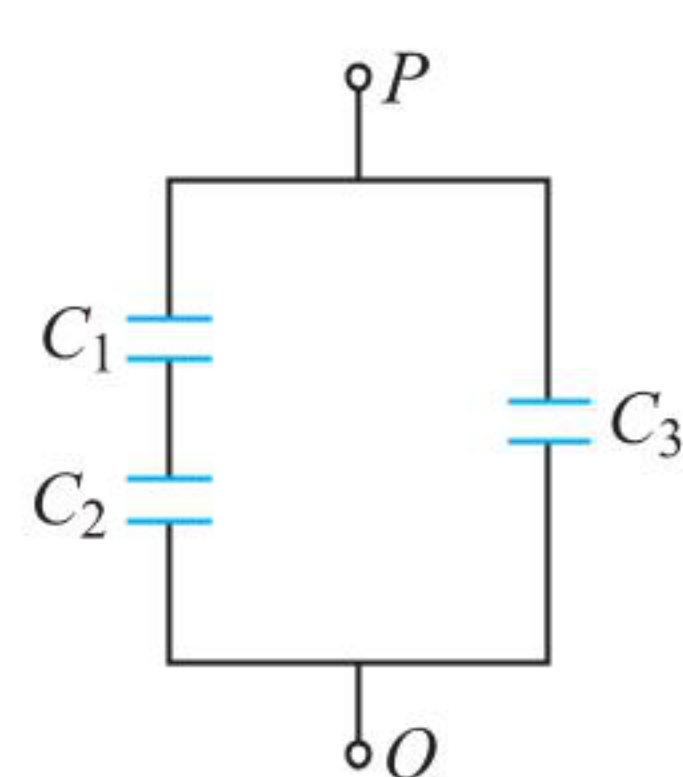
$$28. (2) 2000 \times 0.04 = \frac{1}{2} \times 40 \times 10^{-6} V^2$$

$$\text{or } V^2 = 4 \times 10^6 \text{ or } V = 2000 \text{ V}$$

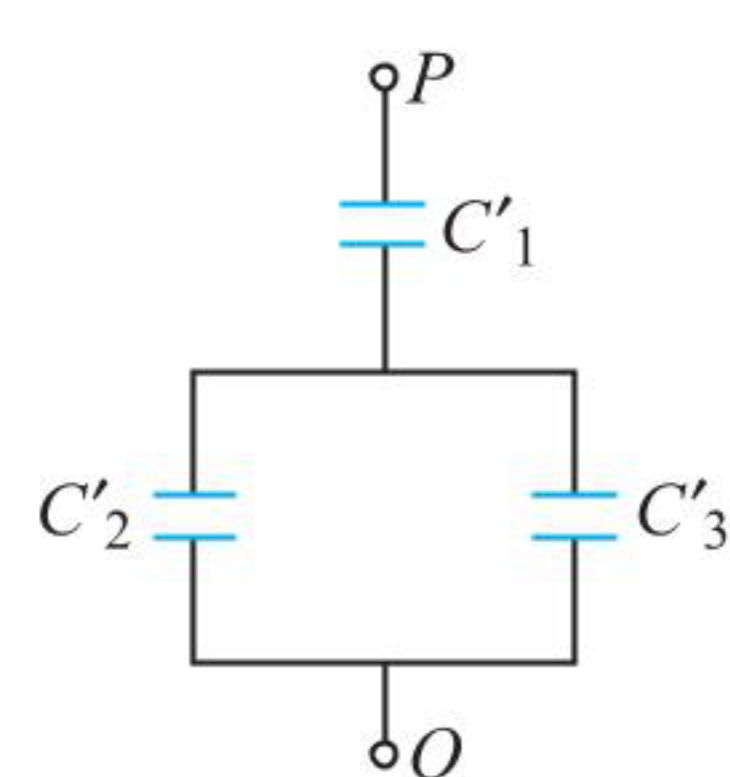
29. (1) We can make equivalent circuit of given system in two ways as in Fig. (a) and (b). But Fig. (b) is wrong and Fig. (a) is correct.

$$\text{Here } C_1 = \frac{\epsilon_0 A / 2}{t / 2} = \frac{\epsilon_0 A}{t}$$

$$C_2 = \frac{K\epsilon_0 A / 2}{t / 2} = \frac{K\epsilon_0 A}{t} \text{ and } C_3 = \frac{\epsilon_0 A / 2}{t} = \frac{\epsilon_0 A}{2t}$$



(a) (Right)



(b) (Wrong)

$$\text{Now, } C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$= \frac{\epsilon_0 A}{t} \left[\frac{3K+1}{2(K+1)} \right]$$

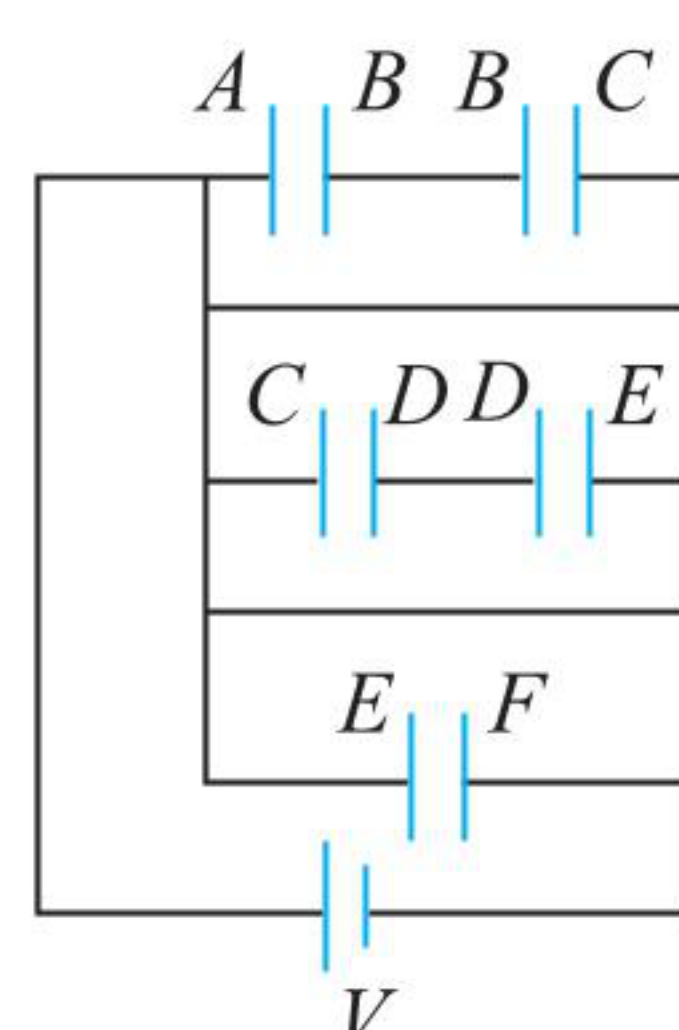
30. (3) After closing S_1 , charge on C_1 is $q = 6 \times 20 = 120 \mu\text{C}$. Now, S_1 is opened. On closing S_2 , charge q will be distributed between C_1 and C_2 according to their capacitances. So charge on C_2 is

$$q_2 = \frac{C_2 q}{C_1 + C_2} = \frac{3 \times 120}{3 + 6} = 40 \mu\text{C}$$

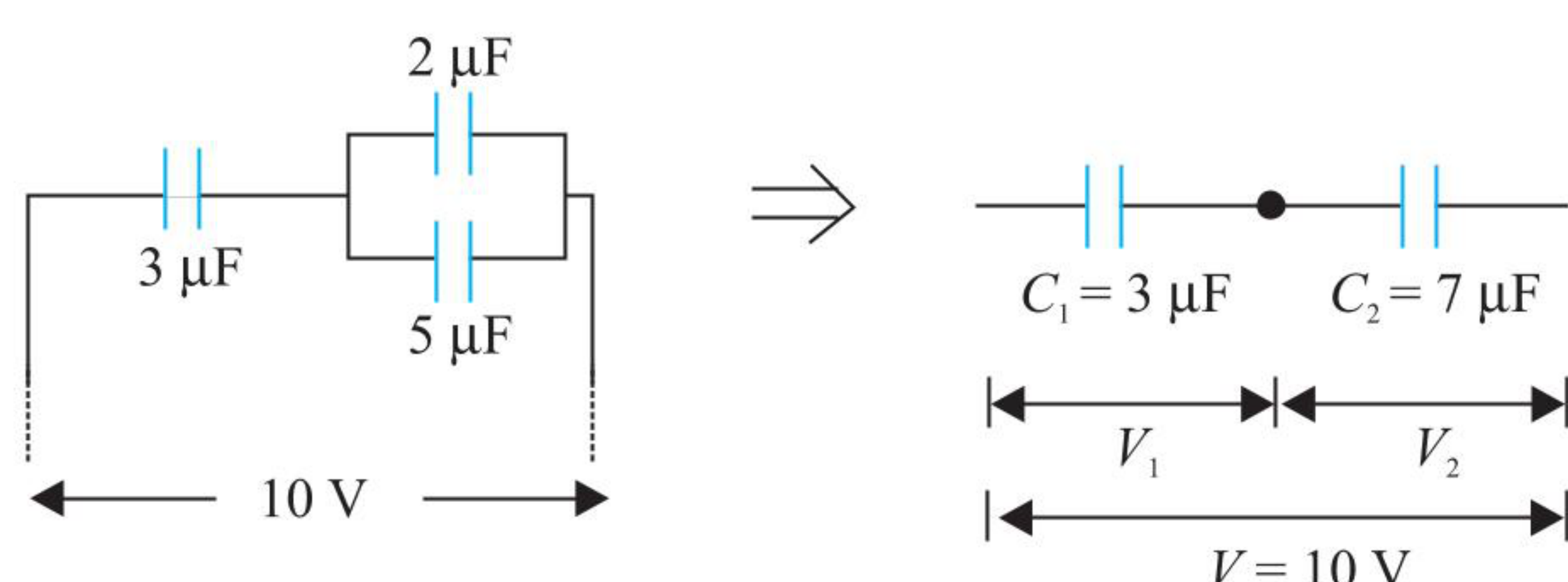
31. (3) $V_A = V_C$ and $V_C = V_E$

$$\text{Net energy stored is } U = \frac{1}{2} CV^2$$

$$C = \frac{\epsilon_0 A}{d} \quad (\text{For } EF)$$



32. (3) Potential difference across upper branch is 6 V.



$$V_2 = V \left(\frac{C_1}{C_1 + C_2} \right) = 10 \left[\frac{3}{3 + 7} \right] = 3 \text{ V}$$

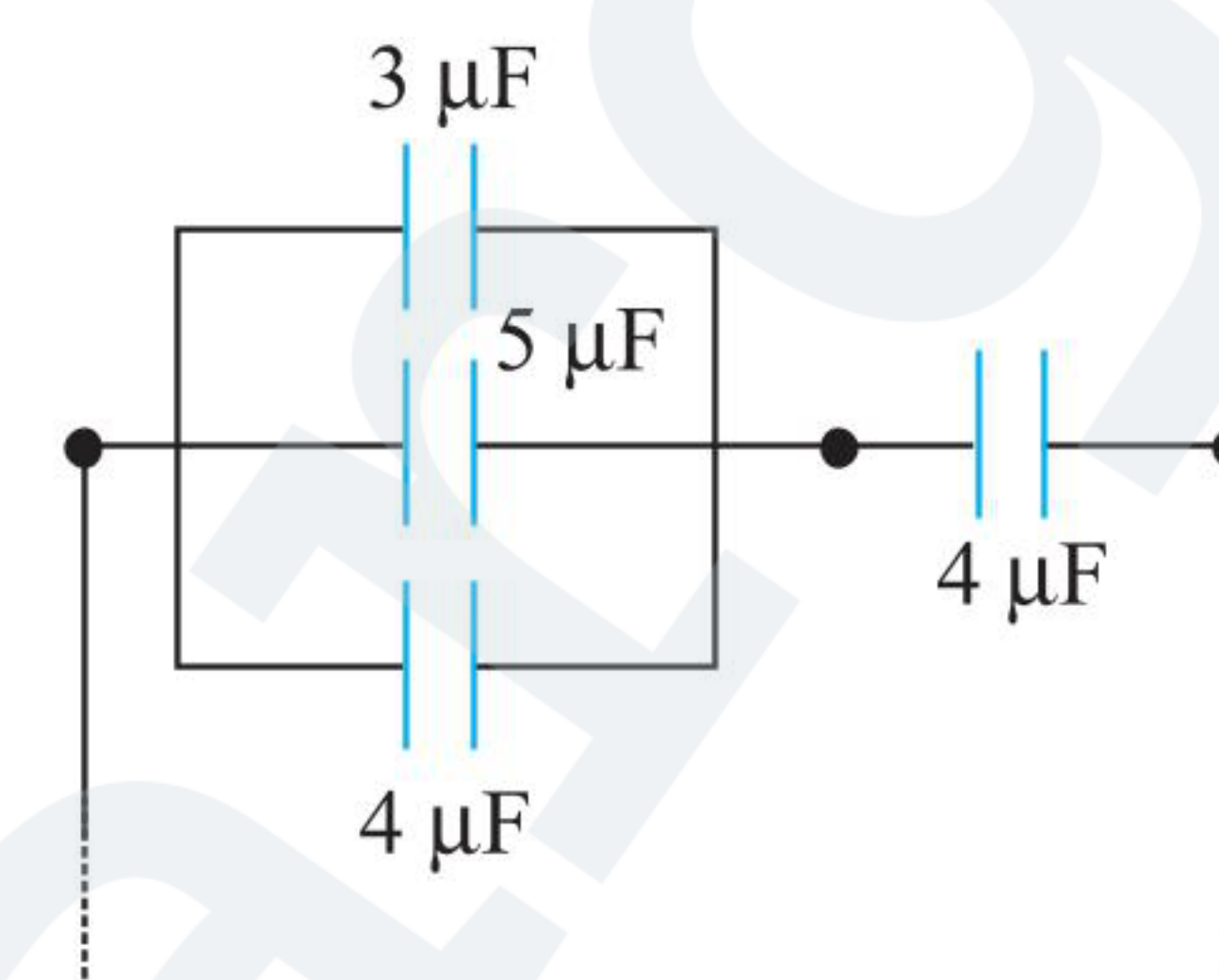
Hence, potential difference across $5 \mu\text{F}$ capacitor is 3 V.

Thus, charge in $5 \mu\text{F}$ capacitor is

$$Q_{5\mu\text{F}} = 5 \times 3 = 15 \mu\text{C}$$

33. (1) Potential difference across 5F capacitor is $120/5 = 24 \text{ V}$.

Hence, potential difference across all three capacitors connected in parallel is $V_1 = 24 \text{ V}$.

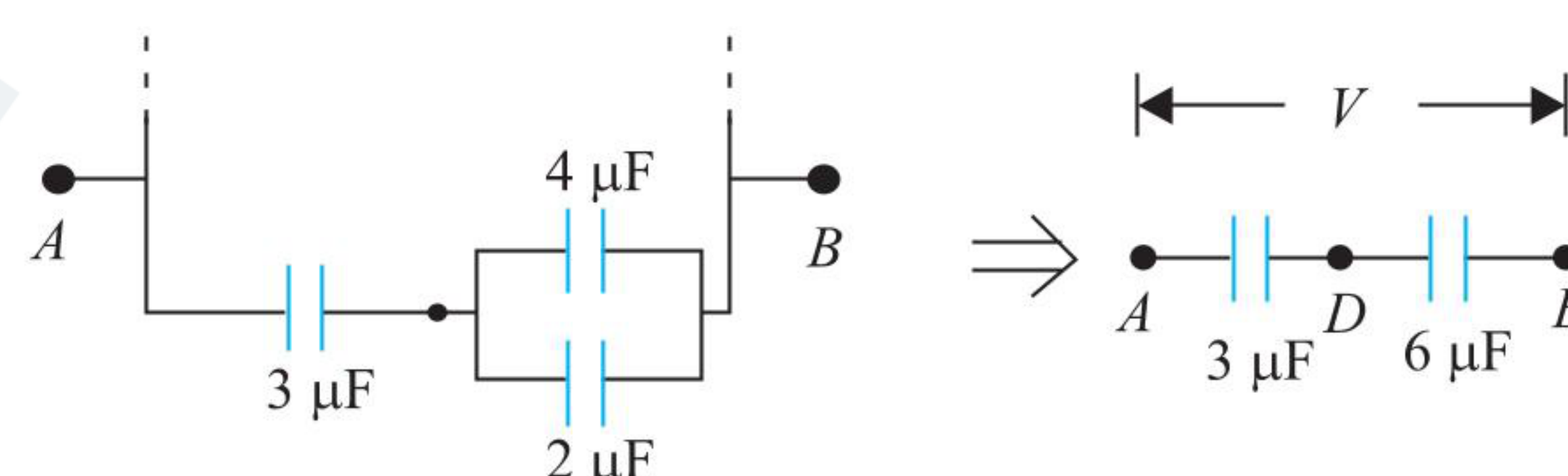


$$V_1 = V \left(\frac{C_2}{C_1 + C_2} \right)$$

$$\text{or } V = V_1 \left(\frac{C_1 + C_2}{C_2} \right)$$

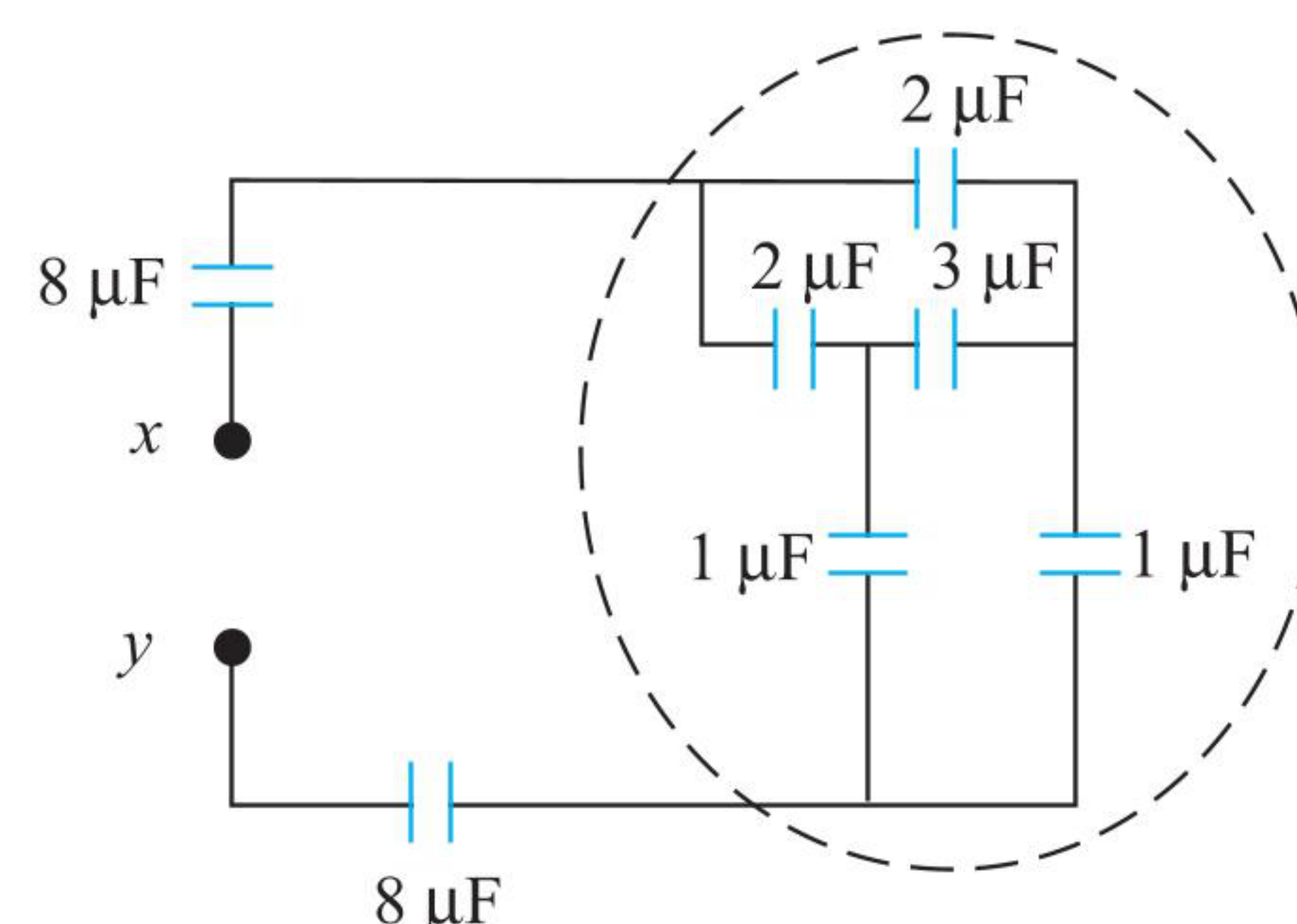
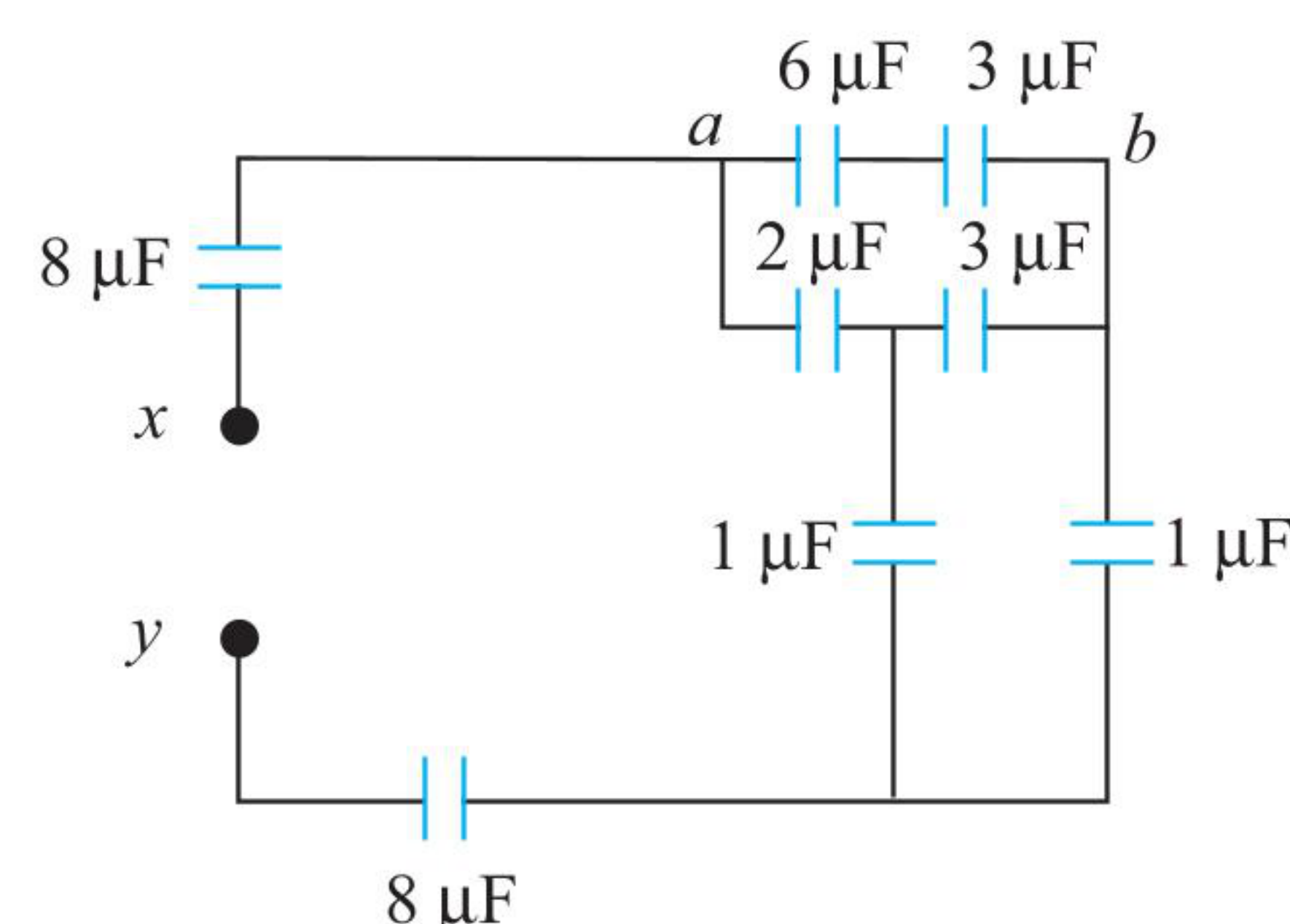
$$= 24 \left[\frac{12 + 4}{4} \right] = 96 \text{ V}$$

$$\therefore V_{ab} = V = 96 \text{ V}$$

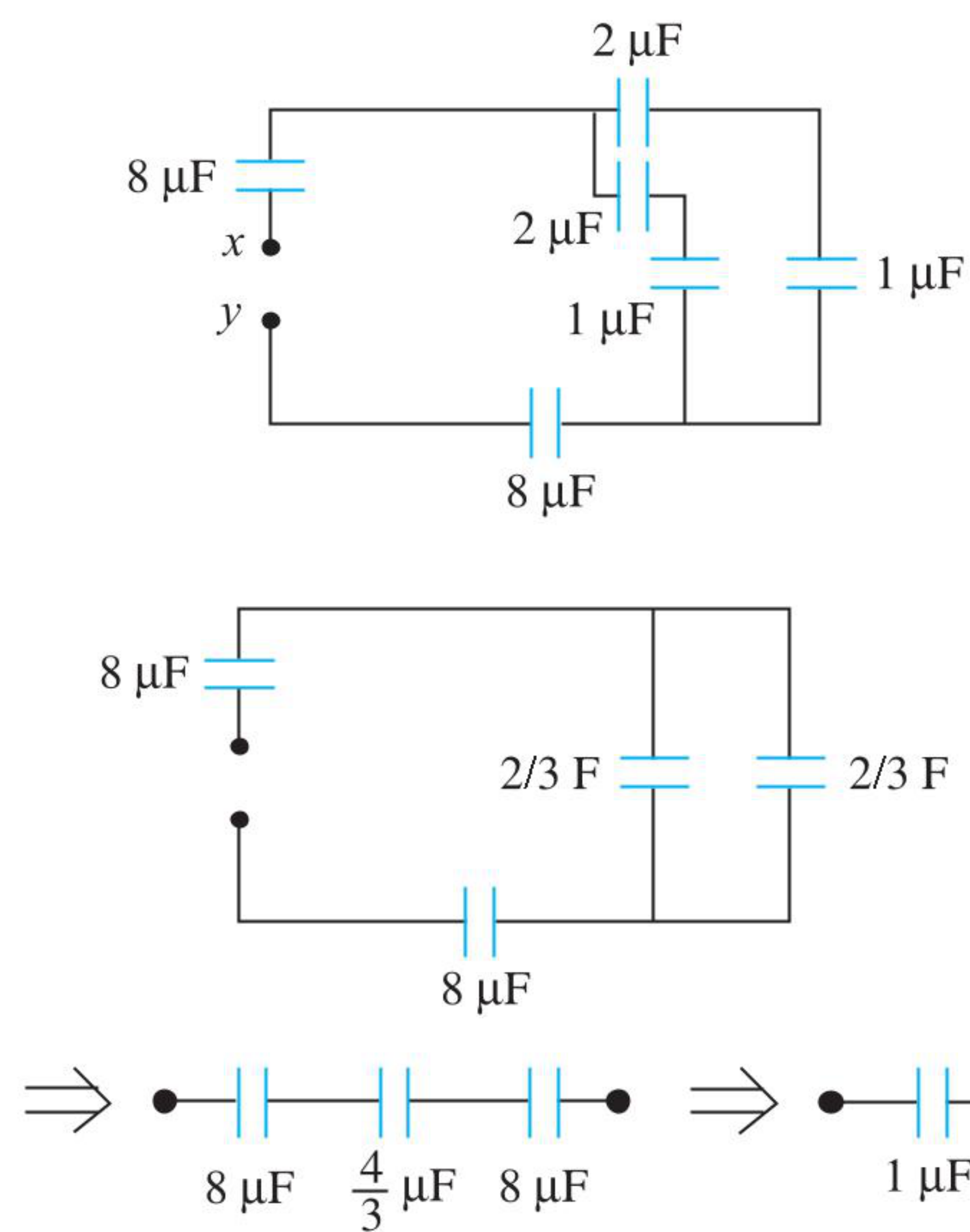


$$V_{AD} = V \left(\frac{6}{30 + 6} \right) = 96 \left[\frac{6}{30 + 6} \right] = 16 \text{ V}$$

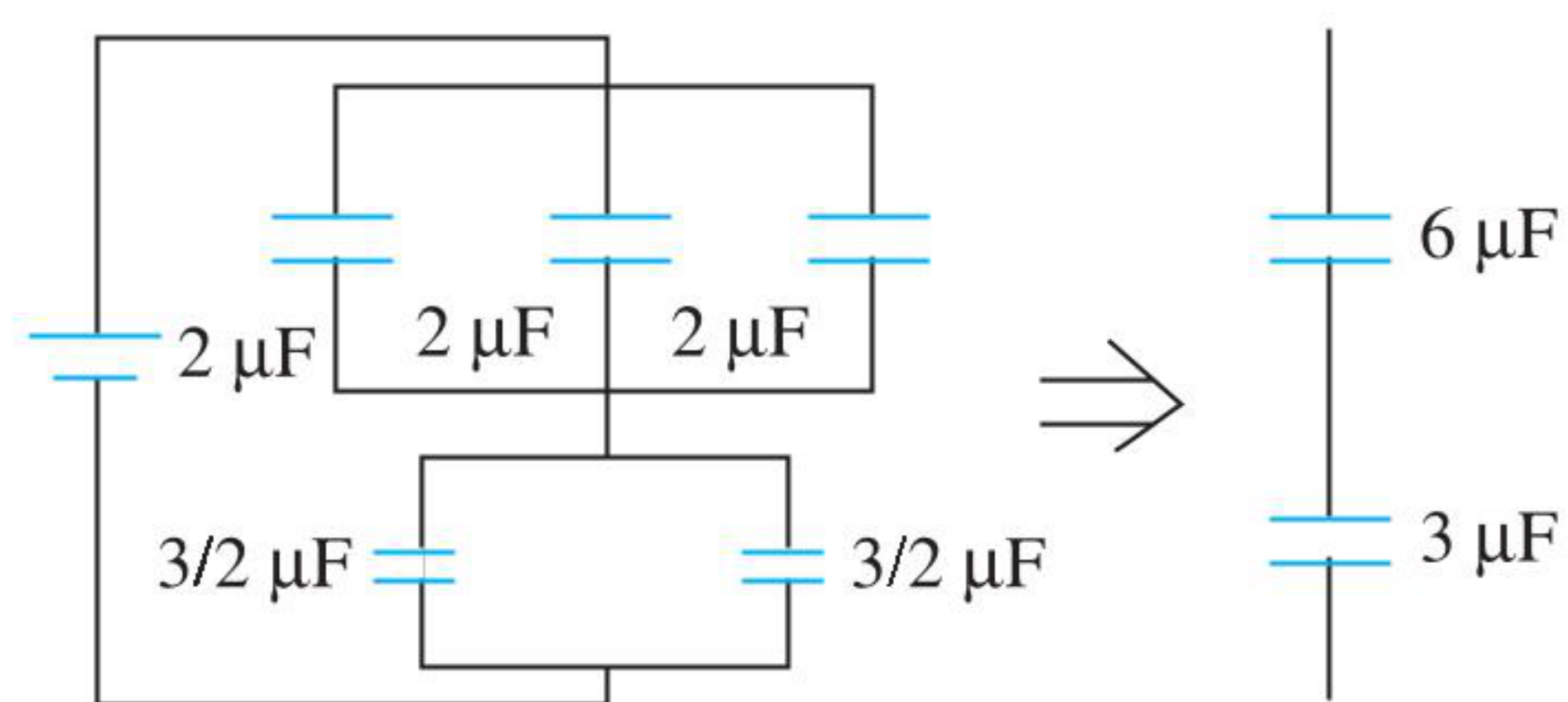
34. (2)



Dotted circuit makes balanced Wheatstone bridge. The circuit can be simplified as follows:



35. (1) The circuit can be simplified as follows:



Hence $C_{eq} = 2 \mu\text{F}$. Thus, charge supplied by battery is $Q = 2 \times 20 = 40 \mu\text{C}$.

Charge on required capacitor is $20 \mu\text{C}$

36. (4) Figure (a) can be simplified as follows:

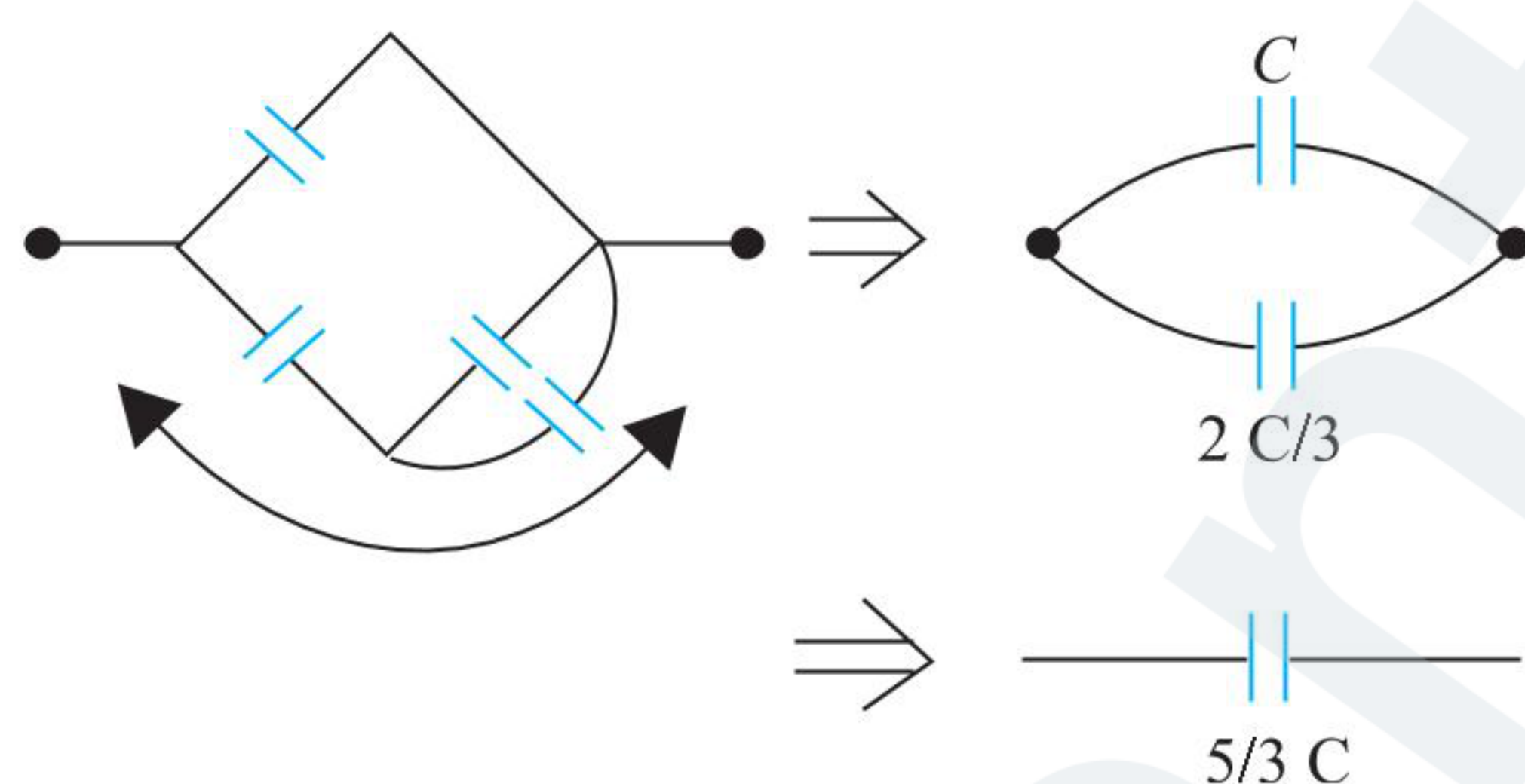


Figure (b) can be simplified as follows:

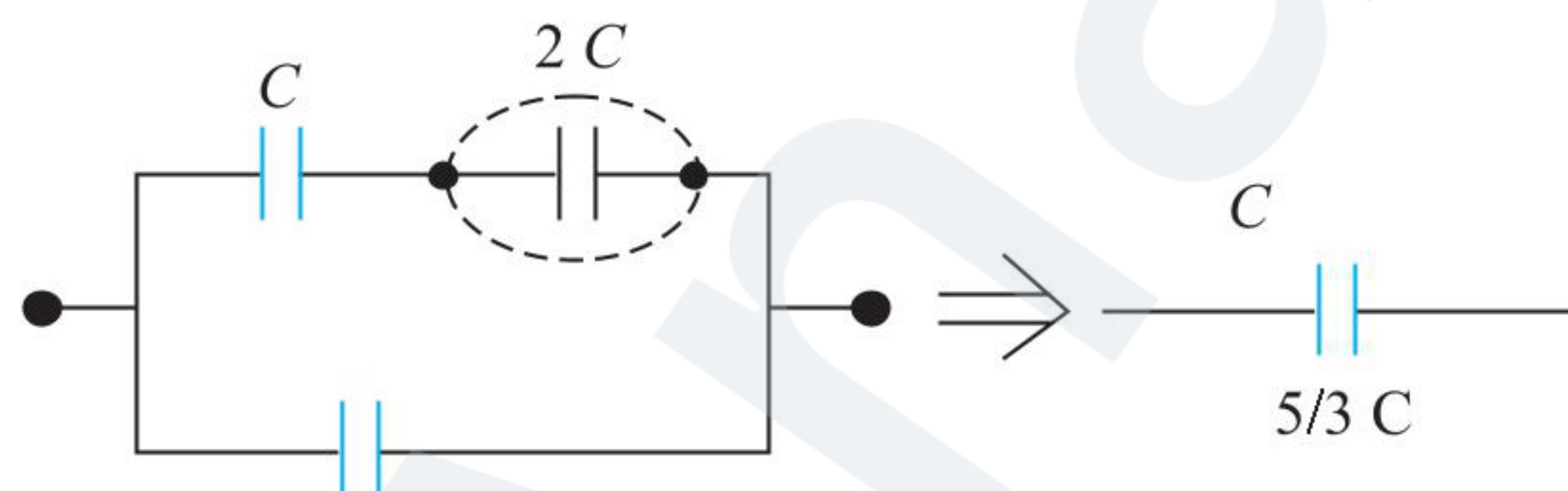
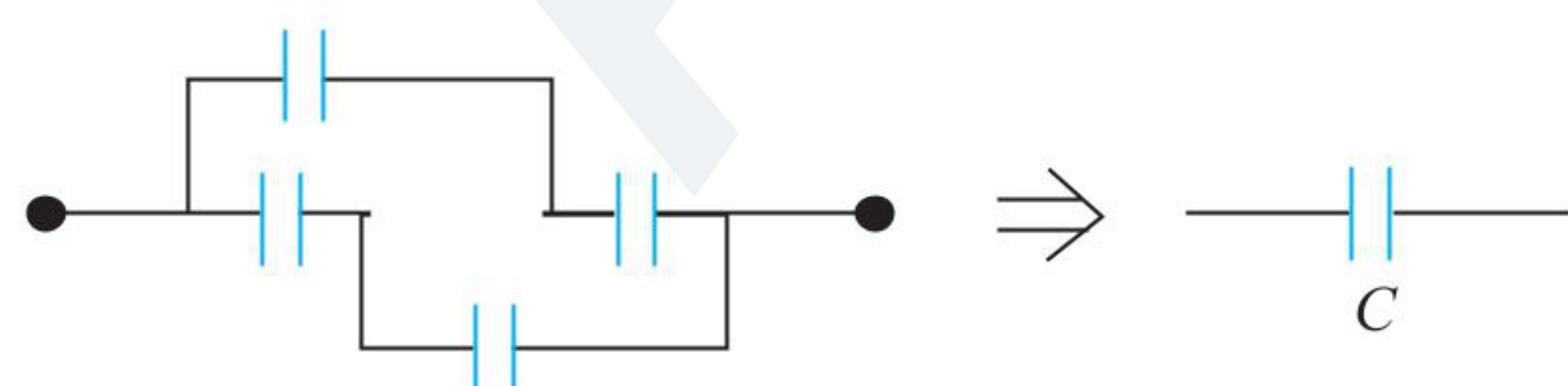


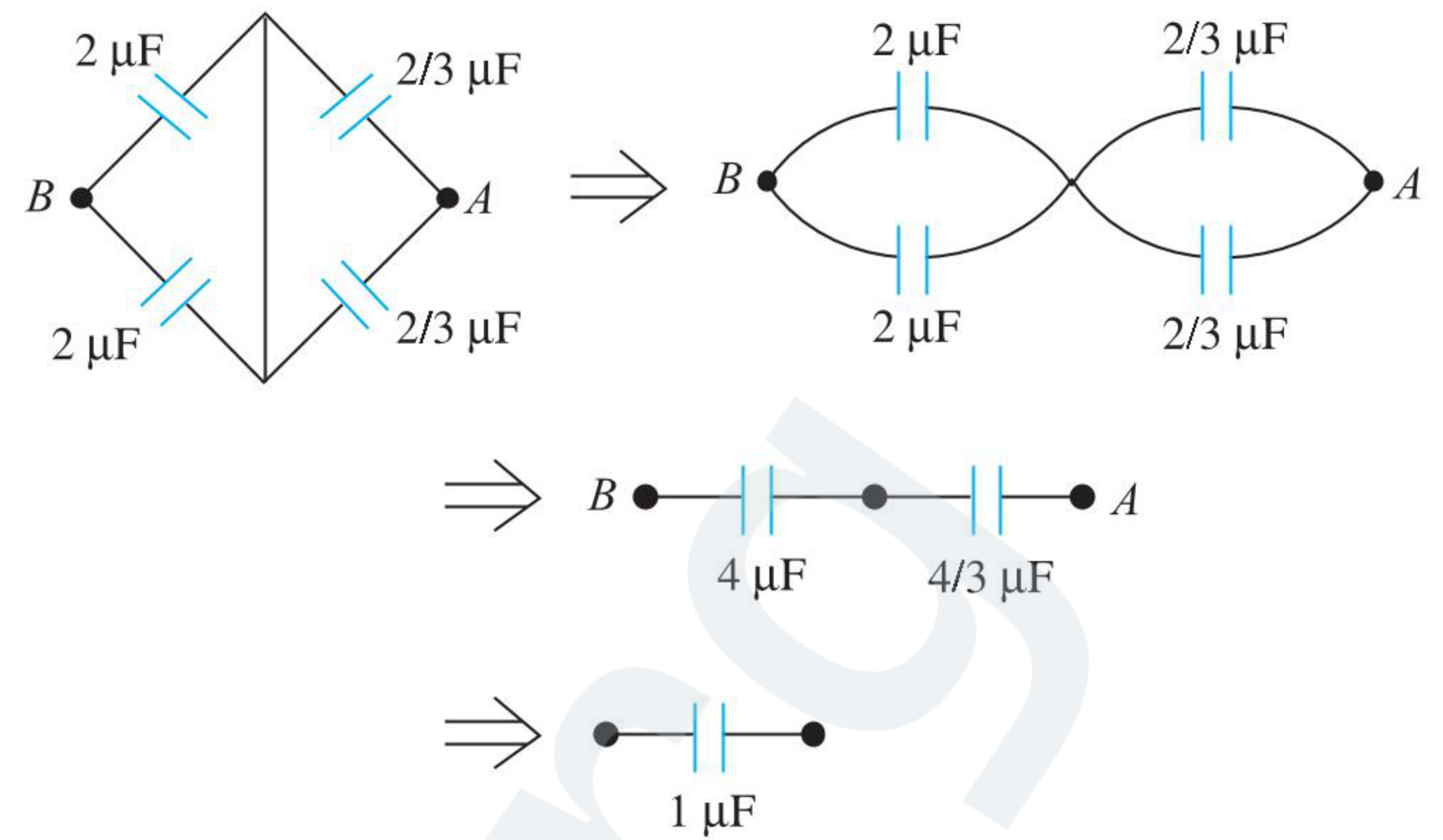
Figure (c) is balanced Wheatstone bridge, which can be simplified as follows



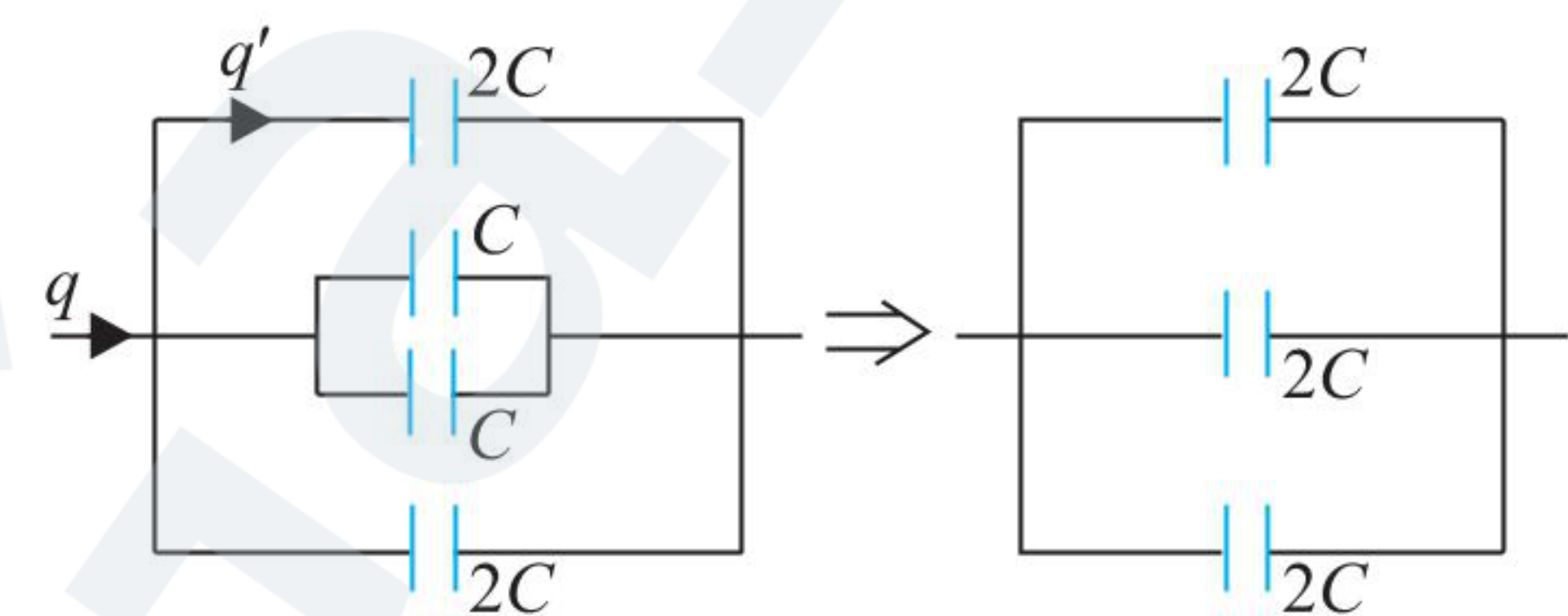
Hence ratio is

$$\frac{5}{3}C : \frac{5}{3}C : C = 5 : 5 : 3$$

37. (3) The redrawn circuit is as follows:



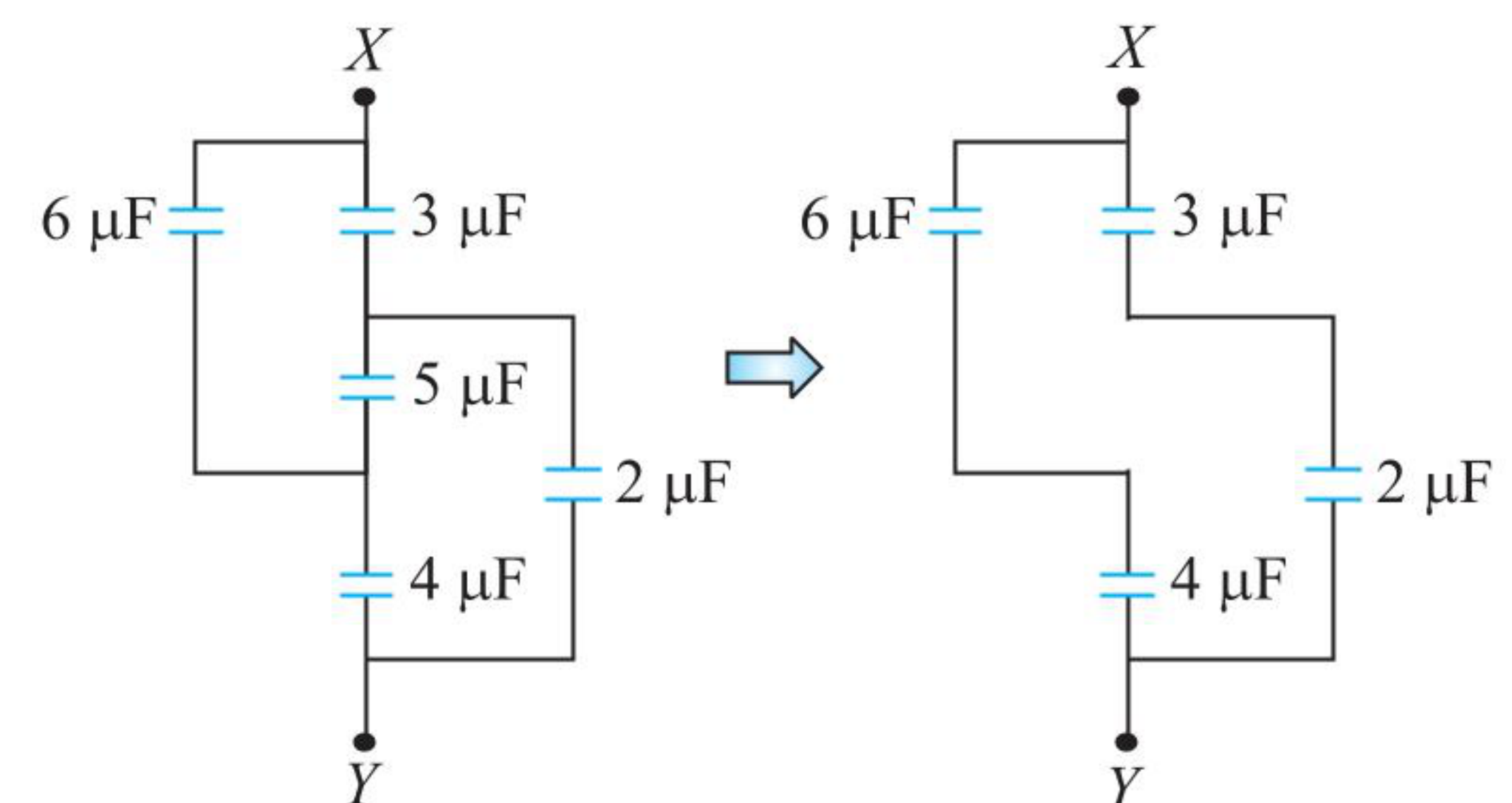
38. (2) Equivalent circuit may be drawn as follows:



Here $C_{eq} = 6C = 6 \mu\text{F}$.

Charge supplied by battery is $q = 6 \times 6 = 36 \mu\text{C}$. Hence, charge on each branch will be $12 \mu\text{C}$, and so charge on required capacitor is $6 \mu\text{C}$.

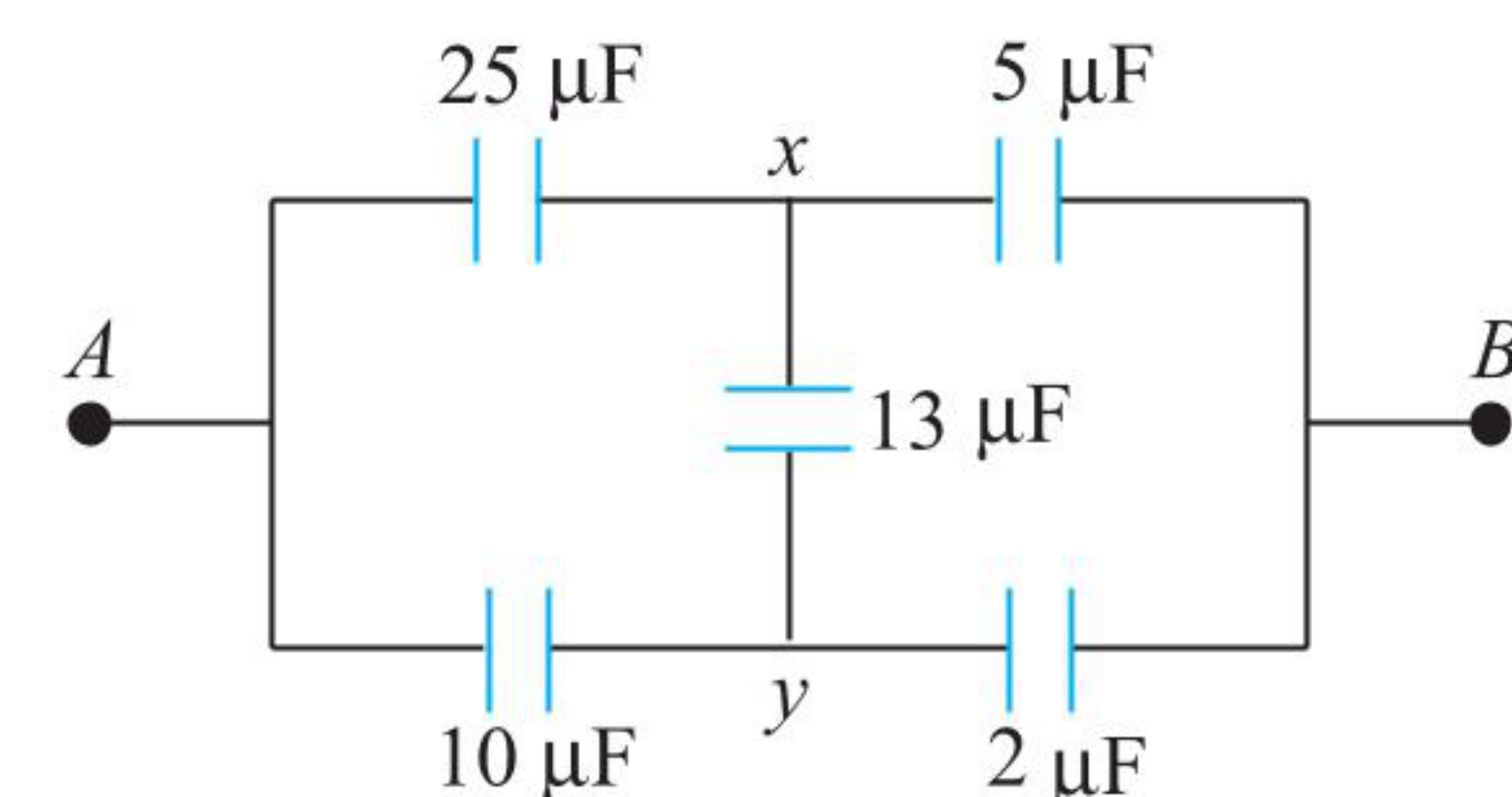
39. (3) Equivalent circuit may be drawn as follows:



As the structure is balanced Wheatstone bridge, so $5 \mu\text{F}$ capacitor will draw no charge from battery and may be removed. Now the circuit is a simple circuit.

$$C_{eq} = \frac{4 \times 6}{(6 + 4)} + \frac{3 \times 2}{(3 + 2)} = \frac{18}{5} \mu\text{F}$$

40. (1) Circuit can be redrawn as follows:



Now, $V_x = V_y$. Hence

$$\frac{25 \mu\text{F}}{5 \mu\text{F}} = \frac{7 \mu\text{F}}{2 \mu\text{F}}$$

$$C_{eq} = \left(\frac{5 \times 25}{5 + 25} \right) + \left(\frac{10 \times 2}{10 + 2} \right) = \frac{35}{6} = \frac{35}{6} \mu\text{F}$$

41. (2) Initially, when the switch is closed on position 1, the capacitor C is connected in series with batteries E_1 and E_2 . From KVL, we have

$$\frac{Q_i}{C} - E_2 + E_1 = 0$$

or $Q_i = (E_2 - E_1)C$... (i)

Depending upon the sign of $(E_2 - E_1)$, charge Q_i on the left plate may be positive (if $E_2 > E_1$), or negative (if $E_2 < E_1$); charge on right plate would be equal and opposite. When the switch is moved to position 2, the left plate (earlier having charge $+Q_i$) will now have charge

$$Q_f = -E_1 C$$
 ... (ii)

The net charge flow through the circuit is

$$\Delta Q = Q_f - Q_i = [-E_1 - (E_2 - E_1)]C = -E_2 C$$

We can say that a net positive charge equal to $E_2 C$ is pulled by the battery of emf E_1 from the left plate of the capacitor, which flows through battery E_1 and is transferred to the right plate of the capacitor. Work done by battery E_1 in the process of charge transfer is

$$\Delta W = E_1 E_2 C$$
 ... (iii)

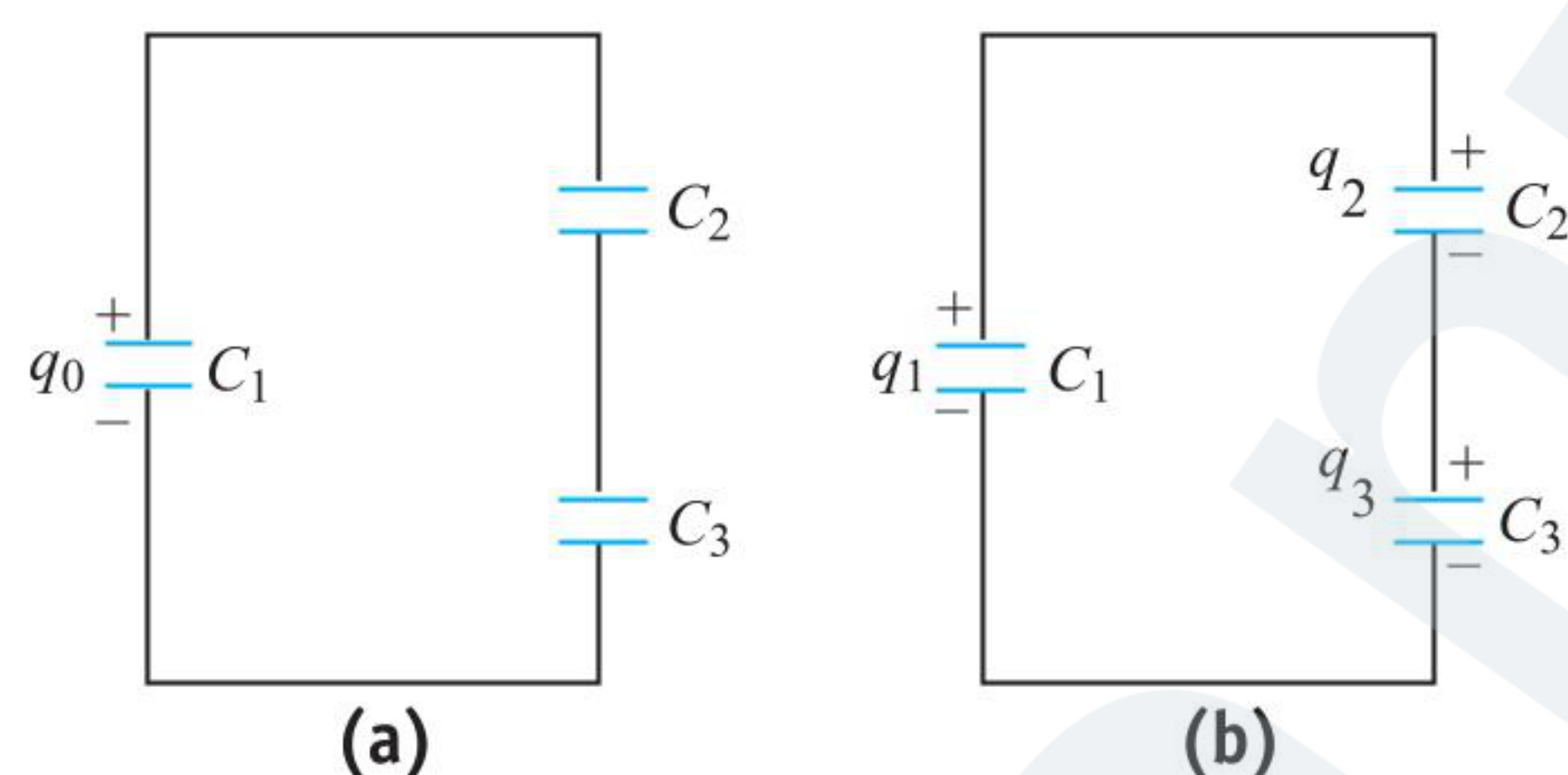
A part of this work changes the energy of the capacitor.

$$\begin{aligned} \Delta W_c &= \frac{Q_f^2}{2C} - \frac{Q_i^2}{2C} = \frac{1}{2} E_1^2 C - \frac{1}{2} (E_2 - E_1)^2 C \\ &= \frac{1}{2} (2E_1 E_2 - E_2^2) C \end{aligned}$$

and the remaining part is lost as Joule heat:

$$H = \Delta W - \Delta W_c = \frac{1}{2} E_2^2 C$$

42. (4) Initial situation after the reconnection is shown in Fig. (a) and the final situation in Fig. (b). The charge transferred by C_1 is $q_0 - q_1$ and the capacitors C_2 and C_3 are in series, so $q_2 = q_3$. Other way to solve the questions is to equalize the potentials across C_1 and series combination of C_2 and C_3 .

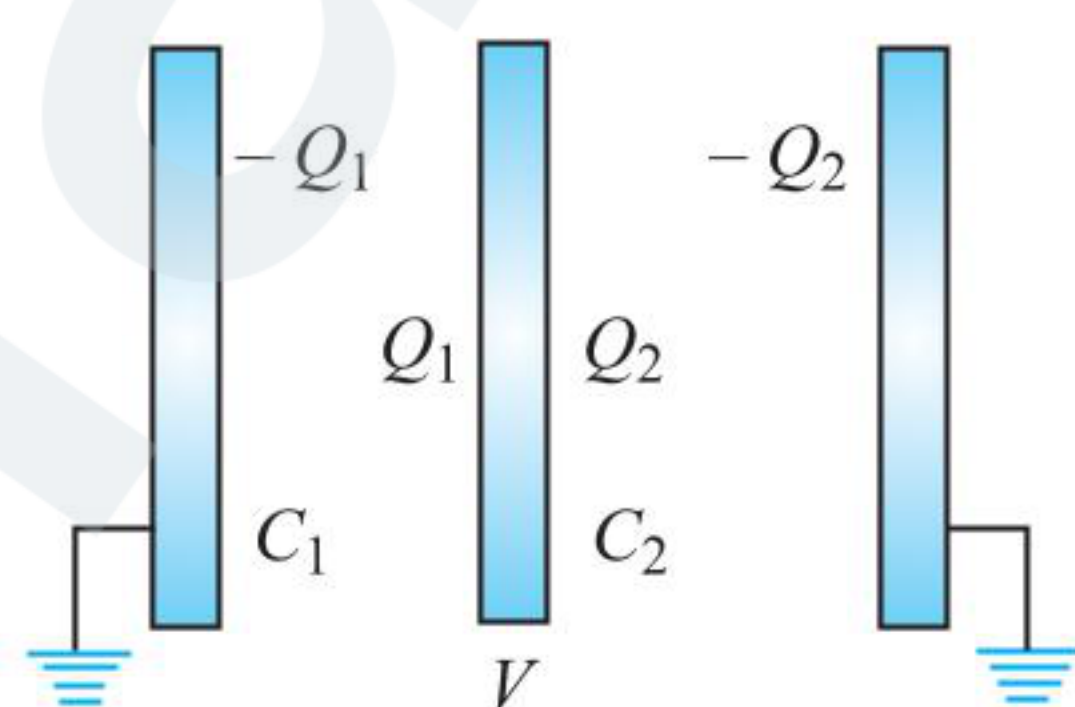


43. (4) $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$$Q_1 + Q_2 = Q$$

where $C_1 = \epsilon_0 A/a$ and $C_2 = \epsilon_0 A/b$. Solve to find

$$Q_2 = \frac{Qa}{a+b}$$



44. (2) $q_i = CV_i = 16 \times 5 = 80 \mu\text{C}$, $q_f = CV_f = 16 \times 10 = 160 \mu\text{C}$

Given $\frac{dq}{dt} = 40t \Rightarrow \int_{q_i}^{q_f} dq = \int_0^t 40t dt$

or $q_f - q_i = \frac{40t^2}{2}$

or $160 - 80 = 20t^2$ or $T = 2 \text{ s}$

45. (3) p and q are in parallel and then r in series.

46. (3) The charge on capacitor before dielectric is

$$q_1 = CV = 50 \times 10^{-9} \text{ C}$$

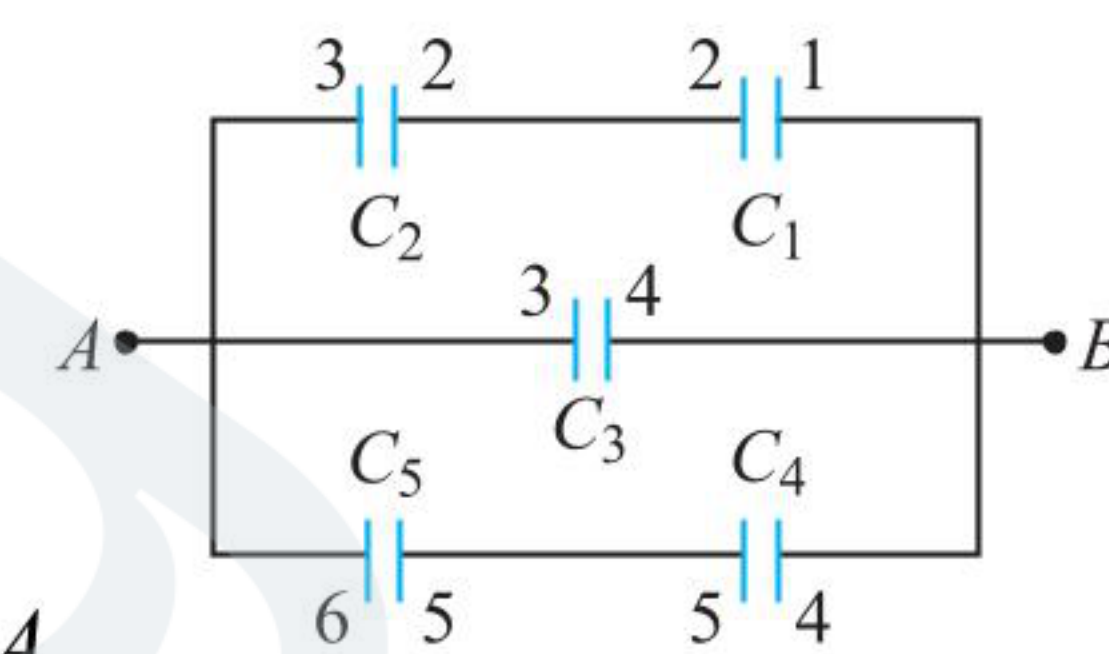
Final charge on capacitor after dielectric is

$$\begin{aligned} q_2 &= (KC)V = 5 \times 50 \times 10^{-9} \\ &= 250 \times 10^{-9} \text{ C} \end{aligned}$$

Charge flow from battery, $\Delta q = q_2 - q_1 = 200 \text{ nC}$

47. (1) Given plates can be rearranged as shown:

$$\begin{aligned} C_1 &= \frac{\epsilon_0 A}{d}; C_2 = \frac{\epsilon_0 A}{2d} \\ C_3 &= \frac{\epsilon_0 A}{3d}; C_4 = \frac{\epsilon_0 A}{2d}; C_5 = \frac{\epsilon_0 A}{d} \end{aligned}$$



C_1 and C_2 are in series and its effective capacity is

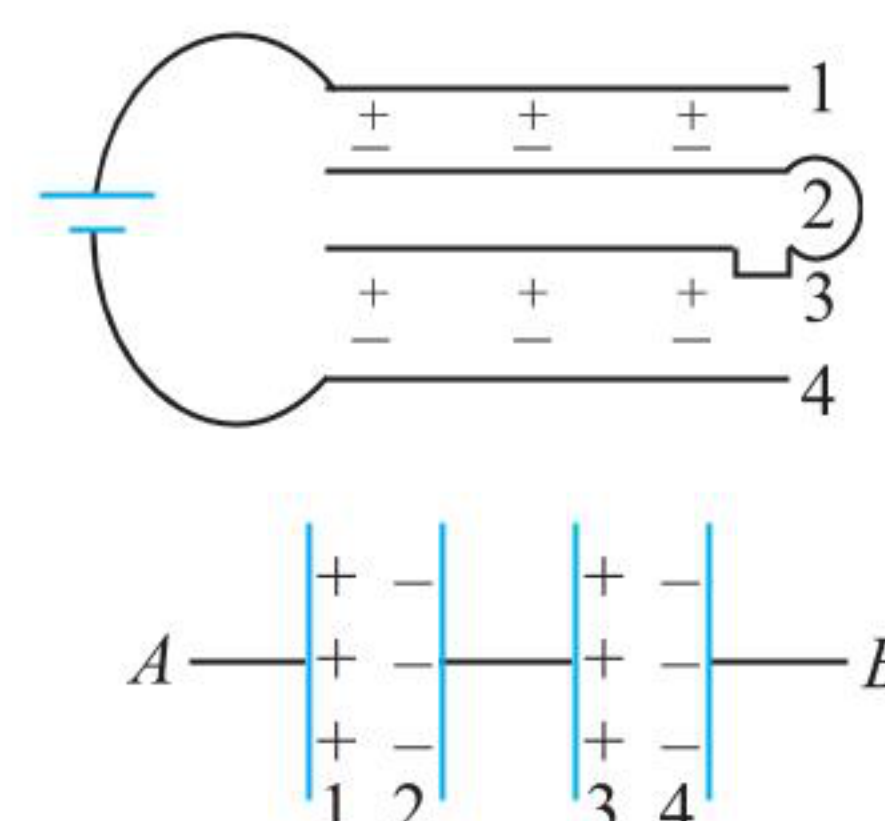
$$\frac{\frac{\epsilon_0 A}{d} \times \frac{\epsilon_0 A}{2d}}{\frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{2d}} = \frac{\epsilon_0 A}{3d}$$

Effective capacitance of C_4 and $C_5 = \frac{\epsilon_0 A}{3d}$

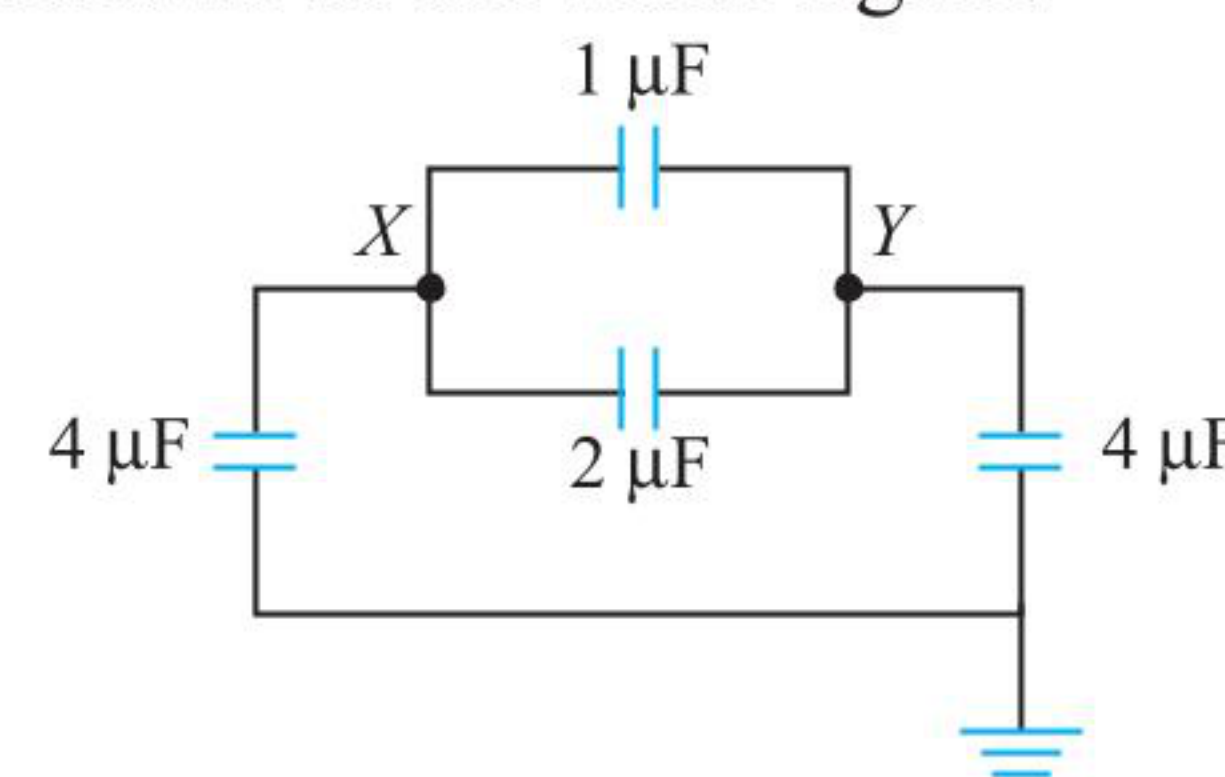
$$C_{AB} = \frac{\epsilon_0 A}{3d} + \frac{\epsilon_0 A}{3d} + \frac{\epsilon_0 A}{3d} = \frac{\epsilon_0 A}{d}$$

48. (2) Plates can be rearranged as shown in figure. Plates 1 and 2 and plates 3 and 4 form two capacitors which are in series between A and B . Plates 2 and 3 do not form any capacitor as they are at same potential.

$$\begin{aligned} \text{So, } C_{eq} &= \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{\epsilon_0 A}{d}\right) \times \left(\frac{\epsilon_0 A}{3d}\right)}{\frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{3d}} \\ &= \frac{\epsilon_0 A}{4d} \end{aligned}$$



49. (4) Circuit is redrawn as shown in figure.



In the circuit, capacitors $4 \mu\text{F}$ and $4 \mu\text{F}$ are connected in series. Combination of this is in parallel with $1 \mu\text{F}$ and $2 \mu\text{F}$ capacitors.

50. (1) The potential of point B is $+10 \text{ V}$, therefore no potential difference exists across $12 \mu\text{F}$ capacitor, hence $q_{12} = 0$. Charge on the $4 \mu\text{F}$ capacitor is $q_4 = (4)(10) = 40 \mu\text{C}$.

51. (1) Electrical force is balanced by the weight of the mass

$$mg = qE, \text{ where } E = \text{electric field}$$

or $mg = q \frac{V}{d}$ (where d = separation at the plates)

$$\frac{\epsilon_0 A}{d} = C \Rightarrow \frac{1}{d} = \frac{C}{\epsilon_0 A}$$

or $V = \frac{2mg\epsilon_0 A}{qC}$

$$= \frac{10^{-6} \times 10 \times 2.2 \times 10^{-12} \times 10^{-2}}{10^{-5} \times 4 \times 10^{-8}}$$

$$= \frac{10^{-6} \times 9.8 \times 8.88 \times 10^{-12} \times 100 \times 10^{-4}}{10^{-8} \times 0.04 \times 10^{-6}}$$

$$= 43 \text{ mV}$$

52. (4) Initial total energy = $\frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$

When the switch is open, dielectric is introduced. Then capacitance, $C = KC = 3C$.
Energy stored in C is

$$\frac{1}{2}3CV^2 = \frac{3}{2}CV^2$$

Since the switch is open, charge will be the same in B so energy

$$\text{is } B = \frac{1}{2} \frac{C}{3} V^2.$$

$$\text{So total final energy} = \frac{3}{2}CV^2 + \frac{1}{6}CV^2$$

$$= \frac{9CV^2 + 1CV^2}{6} = \frac{10}{6}CV^2$$

$$\text{So required ratio} = \frac{CV^2}{\frac{10}{6}CV^2} = \frac{3}{5} = 3 : 5$$

53. (2) Initially at switch position Q , by conservation of charge, the voltage V across the two capacitors is thus given by

$$q = q_1 + q_2 = C_1V + C_2V = (C_1 + C_2)V$$

$$\text{or } 18 = (3 + 6)V$$

$$\text{or } V = (18/9) = 2 \text{ V}$$

54. (1) $\frac{q_1}{1} = \frac{q_2}{2} = \frac{q_3}{3} = K$

$$q_1 + q_2 + q_3 = 6$$

Solving, we get

$$K = 1$$

$$\text{or } q_3 = 3 \mu\text{C}$$

55. (4) Charge on positive plate of A is C_1V_1 .

Charge on negative plate of B is $-C_2V_2$. When d plate of capacitor C is connected with the plate of A , then the total charge of (d, a) plate system will be C_1V_1 (conservation of charge).

Similarly on (c, s) plates, total charge will be $-C_2V_2$.

The total charge on (b, g) plate system will be $+C_2V_2 - C_1V_1$.

56. (3) Circuit is redrawn as shown in figure. Equivalent capacity of combination is

$$C_{\text{eq}} = \frac{3 \times 6}{9} = 2 \mu\text{F}$$

Hence, charge

$$q = C_{\text{eq}}V = 2 \mu\text{F} \times 30 \text{ V} = 60 \mu\text{C}$$

57. (4) The capacitance of capacitor C is $\epsilon_0 A/x$. Charge on capacitor

$$q = CV = \frac{\epsilon_0 AV}{x}; U = \frac{1}{2}qV = \frac{1}{2} \frac{\epsilon_0 AV^2}{x}$$

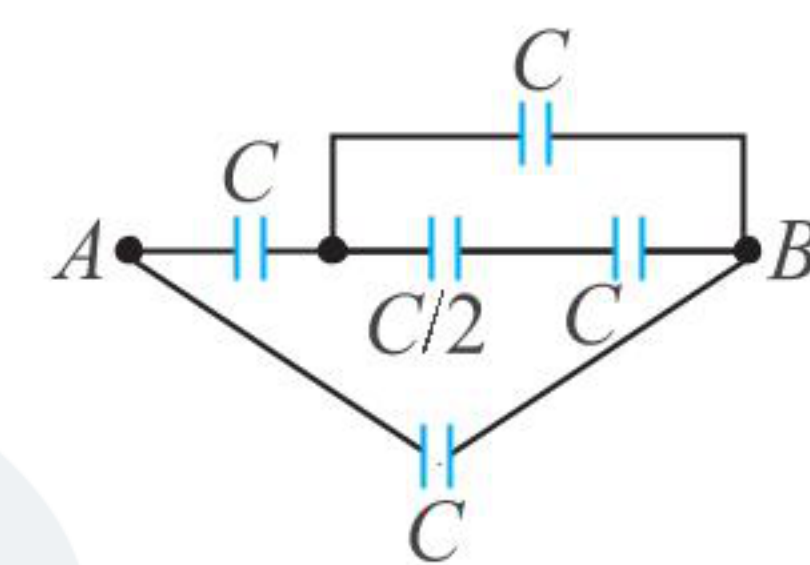
$$\text{So, } \frac{dU}{dt} = \frac{1}{2} \epsilon_0 AV^2 \left(-\frac{1}{x^2} \right) \frac{dx}{dt}$$

$$= \frac{1}{2} \epsilon_0 AV^2 v \left(-\frac{1}{x^2} \right)$$

From the earlier expression,

$$\frac{dU}{dt} \propto \frac{1}{x^2}$$

58. (2) $C_{\text{eq}} = \frac{\frac{4C}{3} \times C}{\frac{7C}{3}} + C = \frac{11C}{7}$



59. (1) The total charge on each plate will have to be same. They should also carry opposite charges.

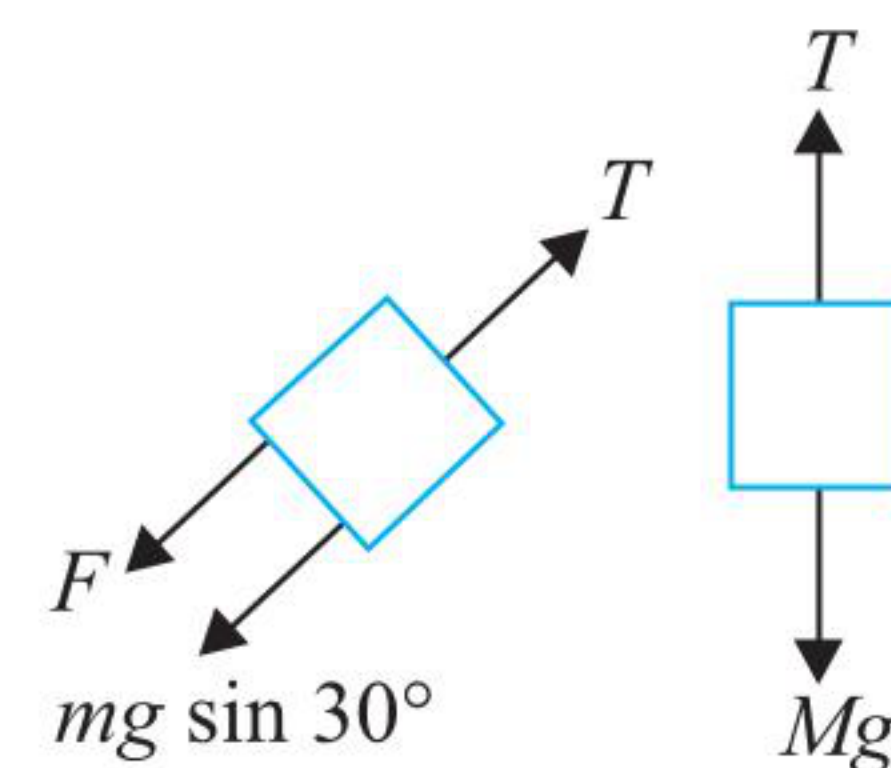
60. (1) For equilibrium, we have

$$T = Mg$$

$$\text{And } T = F + \sin 30^\circ$$

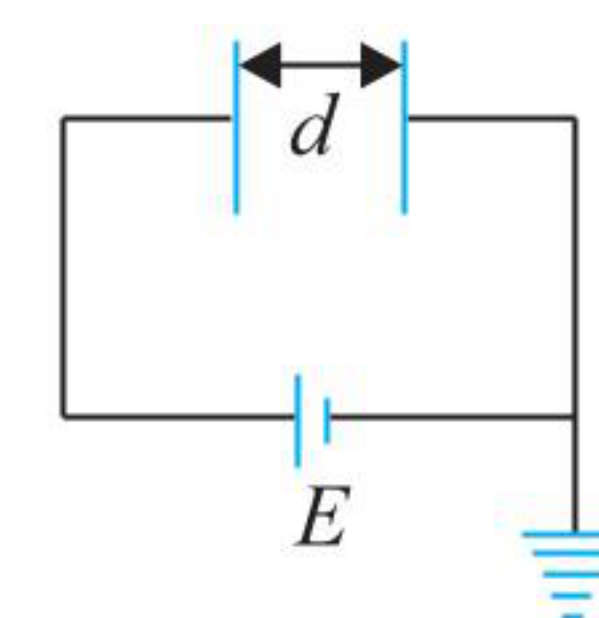
$$\text{or } Mg = F + \frac{mg}{2}$$

$$\text{or } m = \frac{m}{2} + \frac{F}{g} = \frac{m}{2} + \frac{E^2 \epsilon_0 A (k-1)}{2lgd}$$



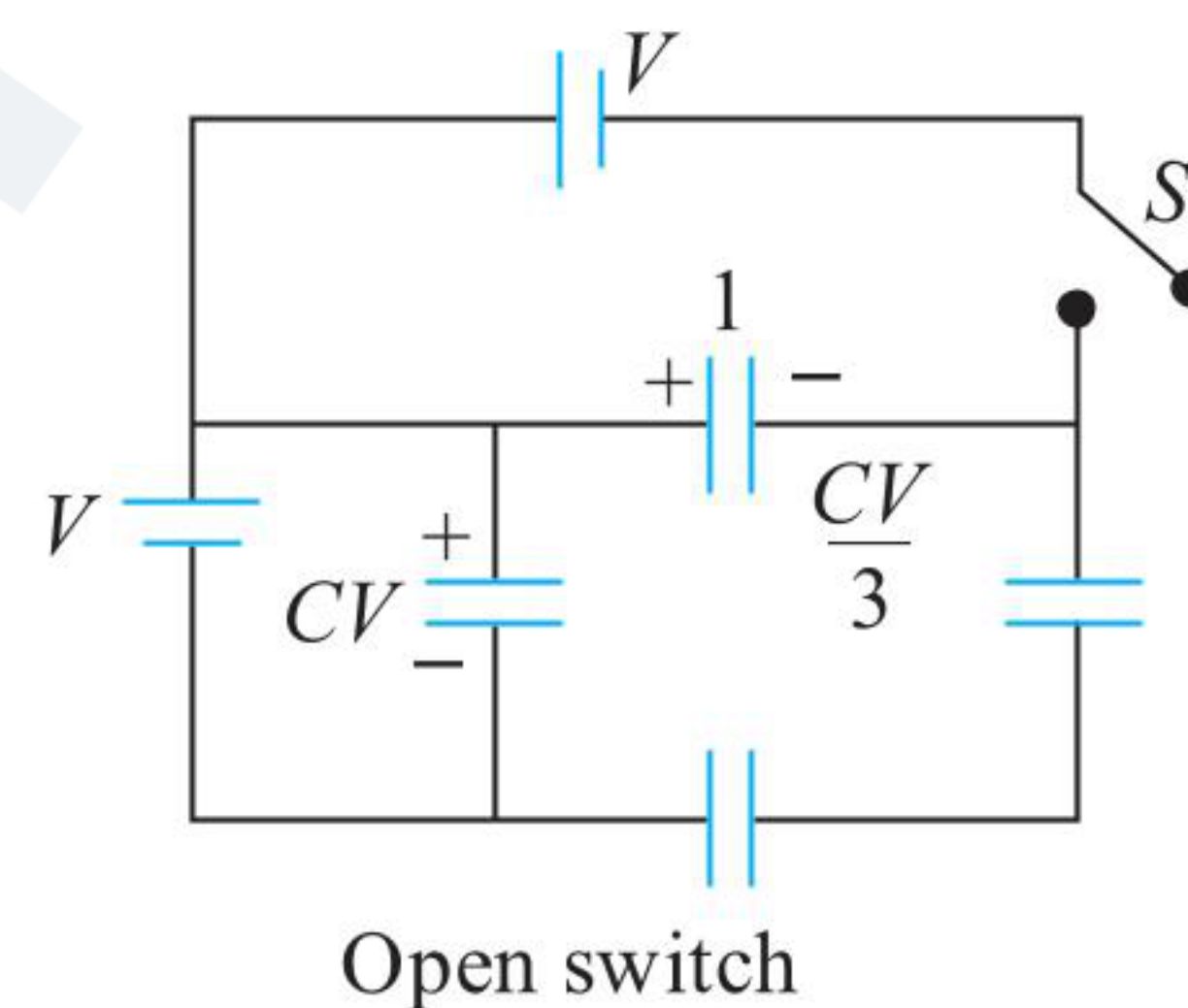
61. (2) The circuit can be redrawn as shown in figure. Charge on capacitor is

$$Q = CE = \frac{\epsilon_0 AE}{d}$$



Options (1), (3), and (4) are incorrect.

62. (4) Charge flowing from A to B :



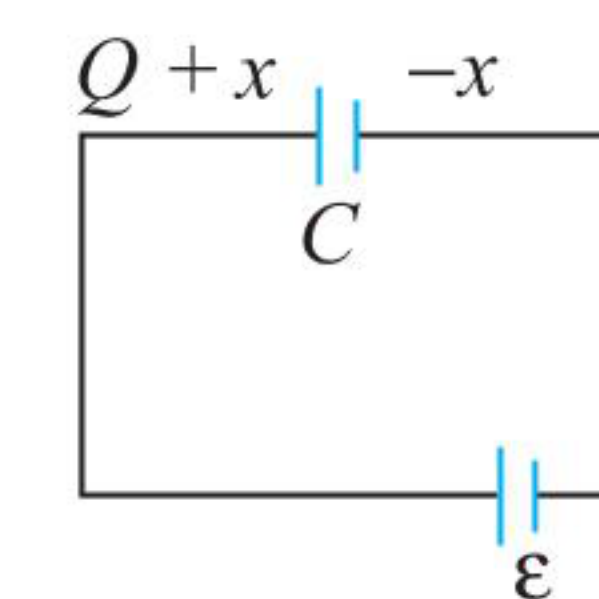
x = change in charge on capacitor 1

$$= CV - \frac{CV}{3} = \frac{2CV}{3}$$

63. (2) Electric field between the capacitor plates:

$$E = \frac{Q+x}{2A\epsilon_0} + \frac{x}{2A\epsilon_0} = \frac{1}{2A\epsilon_0} [Q+2x]$$

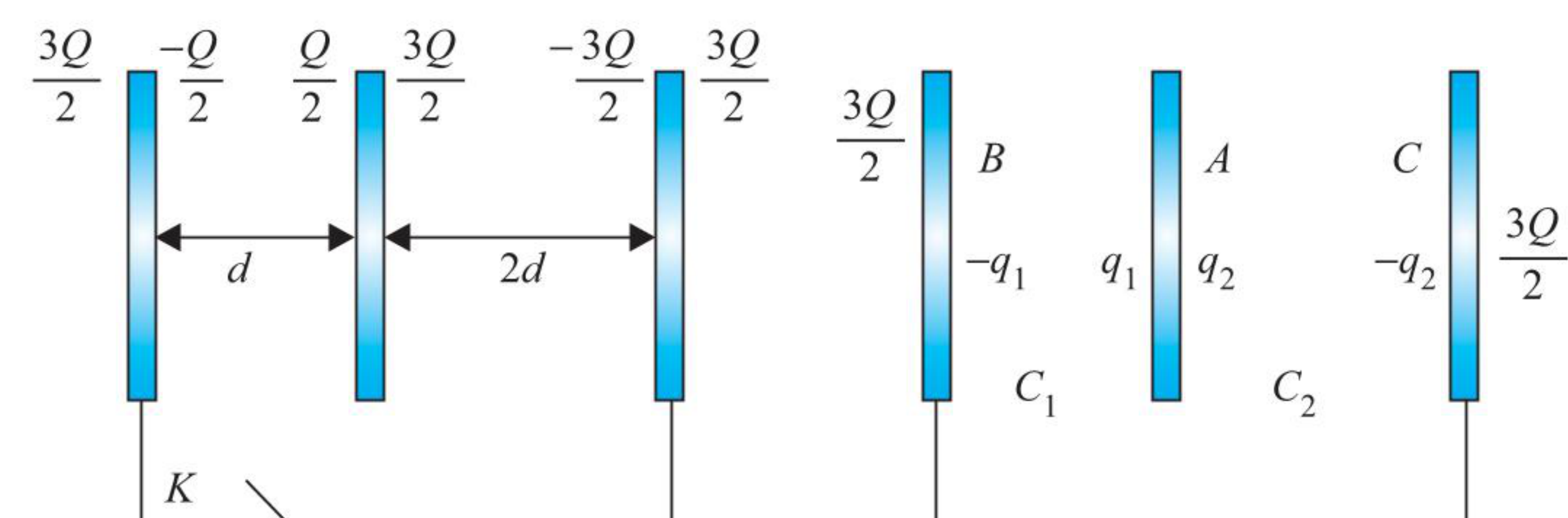
$$\text{Potential difference} = Ed = \frac{d}{2A\epsilon_0} [Q+2x] = \epsilon$$



$$\epsilon = \frac{Q+2x}{2C} - x = \frac{Q}{2} - C\epsilon$$

64. (1) This is the case of a balanced Wheatstone bridge. The middle five capacitors will have no charge and will be useless.

65. (1) Before closing the switch After closing the switch



$$q_1 + q_2 = 2Q$$

$$V_{AB} = V_{AC} \text{ or } \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

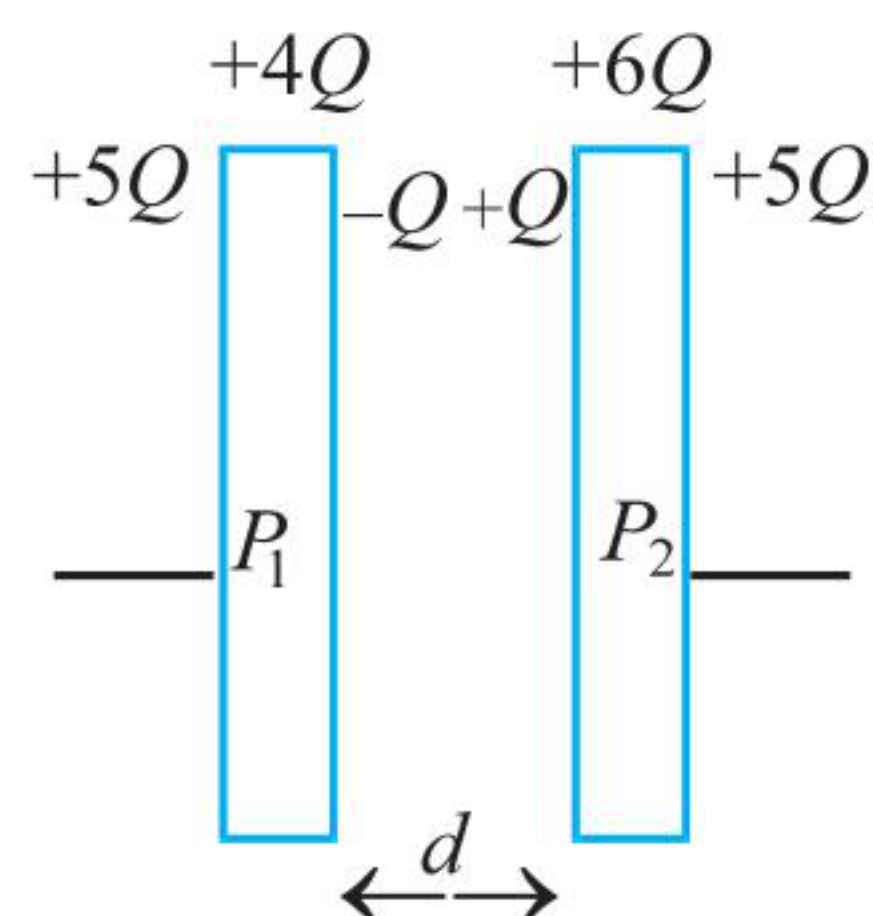
$$\frac{q_1}{q_2} = \frac{C_1}{C_2} = 2 \text{ or } q_1 = 2q_2$$

$$\text{Solving, we get } q_1 = \frac{4Q}{3}, q_2 = \frac{2Q}{3}$$

Charge flown through K is

$$-\frac{Q}{2} - (-q_1) = -\frac{Q}{2} + \frac{4Q}{3} = 5Q/6$$

66. (2) Potential difference between the plates is

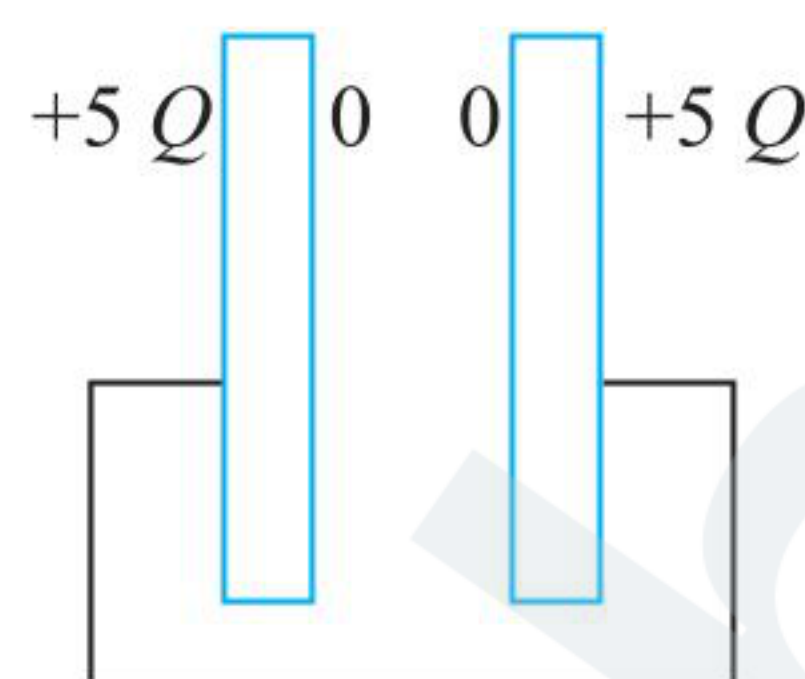


$$V_{P_1} - V_{P_2} = \frac{-Q}{(\epsilon_0 A/d)} = \frac{-Qd}{\epsilon_0 A}$$

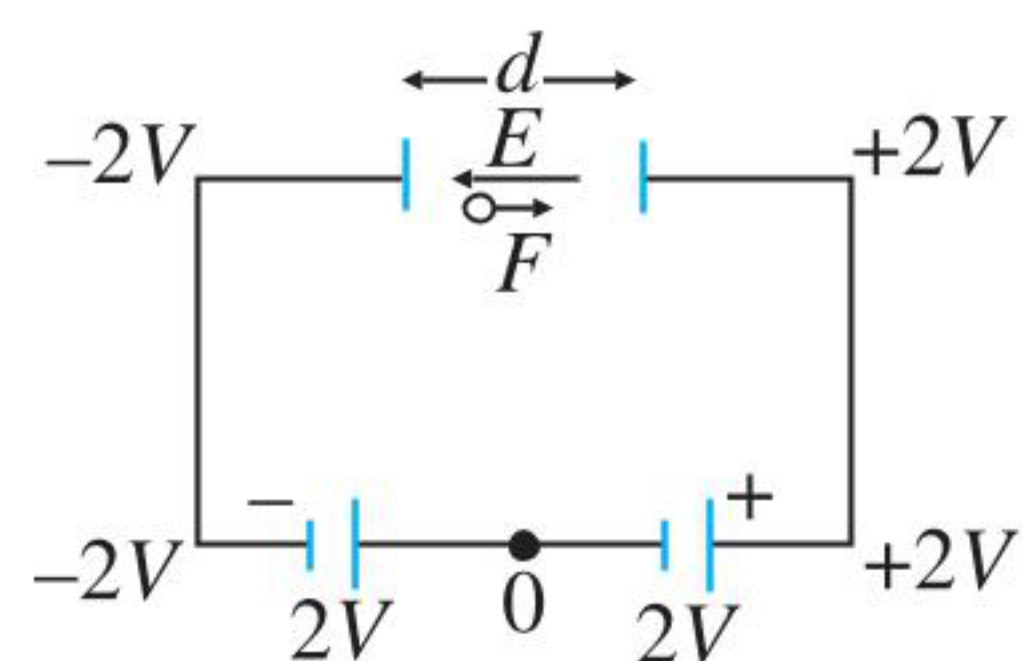
67. (4) Final charges are as shown. Energy stored in the capacitor will be zero finally.

$$\text{Heat produced} = \Delta H = U_i - U_f$$

$$= \frac{1}{2} \frac{Q^2}{\epsilon_0 A/d} - 0$$



68. (3) Circuit (3) represents these equipotential lines



69. (3) After connection the charge of capacitor $Q' = CV = C \frac{2Q_0}{C} = 2Q_0$

The final charge in inner plate of A will be $2Q_0$. Hence charge supplied by battery

$$\Delta Q = \left(2Q_0 - \left(-\frac{Q_0}{2} \right) \right) = \frac{5Q_0}{2}$$

Hence work done by battery

$$W = (\Delta Q)V = \frac{5Q_0}{2} \times \frac{2Q_0}{C} = \frac{5Q_0^2}{C}$$

70. (1) Let q_1 charge flows through S_1 and q_2 through S_2 . For capacitor of capacitance $2C$,

$$Q_0 + q_1 = 2CV = Q_0 \text{ or } q_1 = 0$$

For capacitor of capacitance C ,

$$\frac{Q_0}{3} + q_2 = CV = \frac{Q_0}{2} \text{ or } q_2 = \frac{Q_0}{6}$$

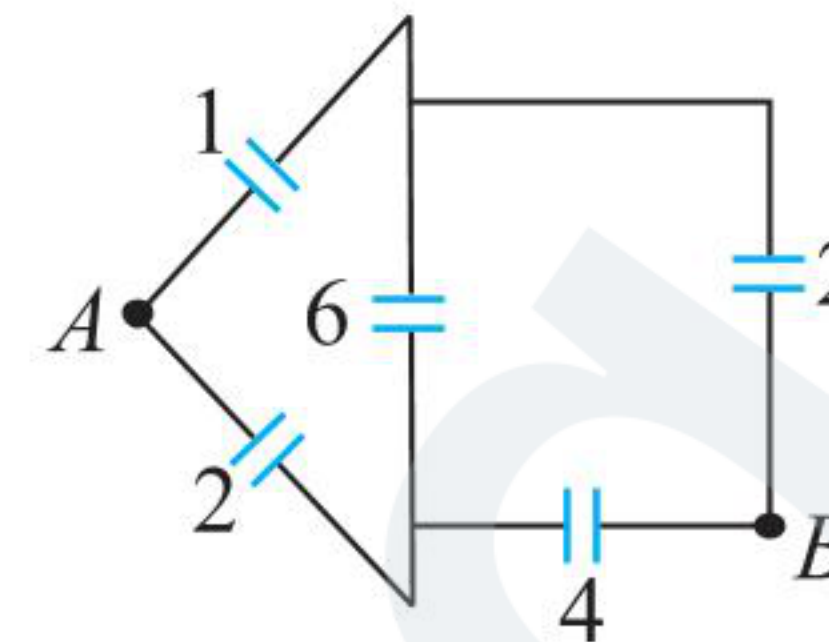
71. (2) Potential drops across C_1 and C_2 are 6 V and 4 V, respectively. Since they are in series, same charge flows through them and

$$\frac{C_1}{C_2} = \frac{V_2}{V_1} = \frac{2}{3}$$

72. (1) Dielectric constant on relative permittivity

$$K = \frac{E_{\text{ext}}}{E_{\text{net}}} = \frac{5}{3}$$

73. (1) The figure can be redrawn as follows:

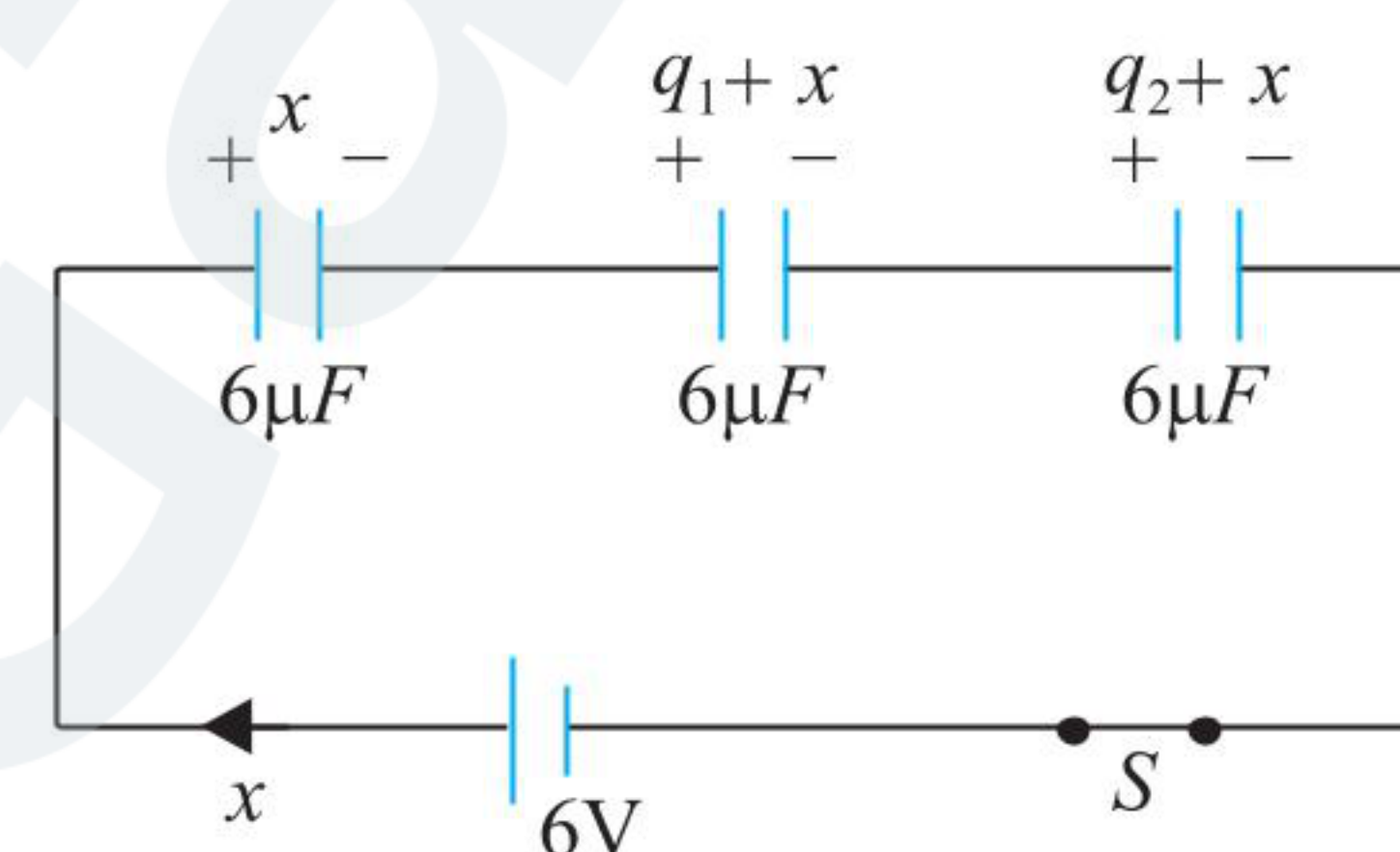


This is a balanced Wheatstone bridge. $6 \mu\text{F}$ is useless.

Find $C_{eq} = 2 \mu\text{F}$

74. (2) Let charge x flow through circuit on closing switch. Then final charge on each capacitor is as shown.

$$\frac{x}{6} + \frac{q_1 + x}{6} + \frac{q_2 + x}{6} = 6$$



$$\text{or } 3x + q_1 + q_2 = 36 \text{ or } x = 6 \mu\text{C}$$

75. (3) Case I: $C_{eq} = 2C, U_1 = \frac{1}{2} 2CE^2 = CE^2$

$$\text{Case II: } C_{eq} = \frac{8C}{3}, U_2 = \frac{1}{2} \frac{8C}{3} E^2 = \frac{4}{3} CE^2$$

Case III: The upper $2C$ will be short-circuited.

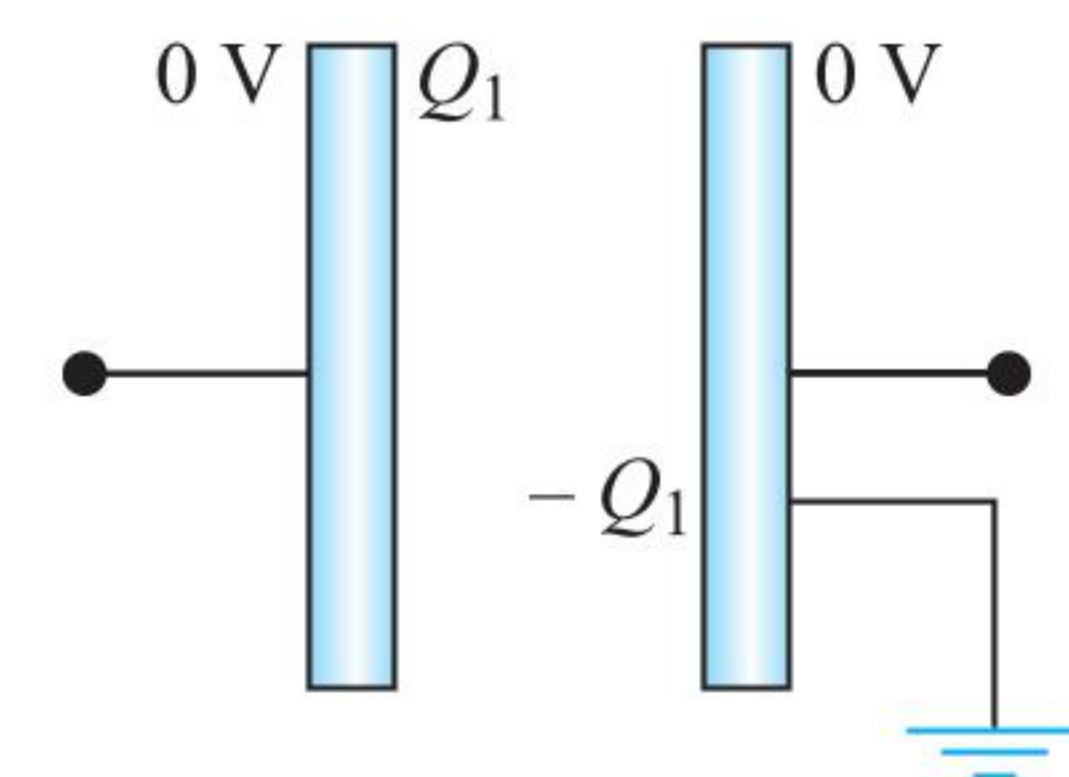
$$U_3 = \frac{1}{2} CE^2 + \frac{1}{2} 2CE^2 = \frac{3}{2} CE^2$$

$$U_1 : U_2 : U_3 = 6 : 8 : 9$$

Multiple Correct Answers Type

1. (2), (3), (4)

As the switch is closed, the charge on the outer surface of the second plate becomes zero. From the concept in electrostatics that electric field inside the bulk of the material of conductor is zero, we can find the charges on various faces. So it is clear that $Q_1 + Q_2$ charge goes from the second plate to the earth. Charge on capacitor is Q_1 and hence its potential is Q_1/C .



2. (2), (3), (4)

As the dielectric slab is pulled out, the equivalent capacity of the system decreases, and hence charge supplied by the battery decreases as potential of the system remains constant. It means charging of battery takes place and a positive charge flows from a to b . As the two capacitors are connected in series, so charge on both capacitors remains the same at all instants. From energy conservation law,

$$U_i + W_{\text{ext}} = U_f + \text{work done on battery} + \Delta H$$

As dielectric slab is attracted by the plates of capacitors, to pull it out, F has to perform some work, i.e., $W_{\text{ext}}(F) > 0$.

3. (1), (2), (4)

As electric field is conservative field hence $W_{\text{ele}} \Rightarrow -\Delta U = -W_{\text{ext}}$. Hence option (1) is correct. As $-\int \vec{F}_{\text{elec}} d\vec{x} \Rightarrow -W_{\text{ele}} \Rightarrow W_{\text{ext}}$. Hence option (2) is correct.

$$C = \frac{\epsilon_0 A}{d_1}, C' = \frac{\epsilon_0 A}{d_2}$$

$$\text{Extra charge flown} = Q' - Q = (C' - C)V$$

$$= \epsilon_0 A V \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

Work done by battery is

$$W_b = V \times \text{charge flown} = \epsilon_0 A V^2 \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

Change in potential energy of capacitor is

$$\Delta U = \frac{1}{2} (C' - C) V^2 = \frac{1}{2} \epsilon_0 A V^2 \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

It means $W_{\text{battery}} = 2\Delta U$. Hence option (4) is correct.

4. (1), (3), (4)

This system can be considered as two capacitors in parallel, with

$$C_1 = \frac{\epsilon_0 A}{2d} \text{ and } C_2 = \frac{K\epsilon_0 A}{2d}.$$

C_1 is due to upper half of two plates, while C_2 is due to lower half.

As potential difference across two capacitors is the same, charges would be different as capacitors are different.

5. (1), (2), (3)

$$\text{Effective capacitance is } C = \frac{4 \times 5}{9} = \frac{20}{9} \mu\text{F}$$

$$\text{Charge on } C_1 \text{ is } Q_1 = CV = \frac{20}{9} \times 9 = 20 \mu\text{C}$$

$$\text{Charge on } C_2 \text{ is } Q_2 = \left(\frac{C_2}{C_2 + C_3} \right) Q_1 = \frac{2}{3} \times 20 = 8 \mu\text{C}$$

$$\text{Charge on } C_3 \text{ is } Q_3 = \left(\frac{C_3}{C_2 + C_3} \right) Q_1 = \frac{3}{3} \times 20 = 12 \mu\text{C}$$

$$\text{Since, } \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$$

$$\text{and } Q_1 = Q_2 + Q_3 = Q_2 + \frac{C_3}{C_2} Q_2 = Q_2 \left(\frac{C_2 + C_3}{C_2} \right)$$

Therefore,

$$Q_2 = \left(\frac{C_2}{C_2 + C_3} \right) Q_1$$

$$Q_3 = \left(\frac{C_3}{C_2 + C_3} \right) Q_1$$

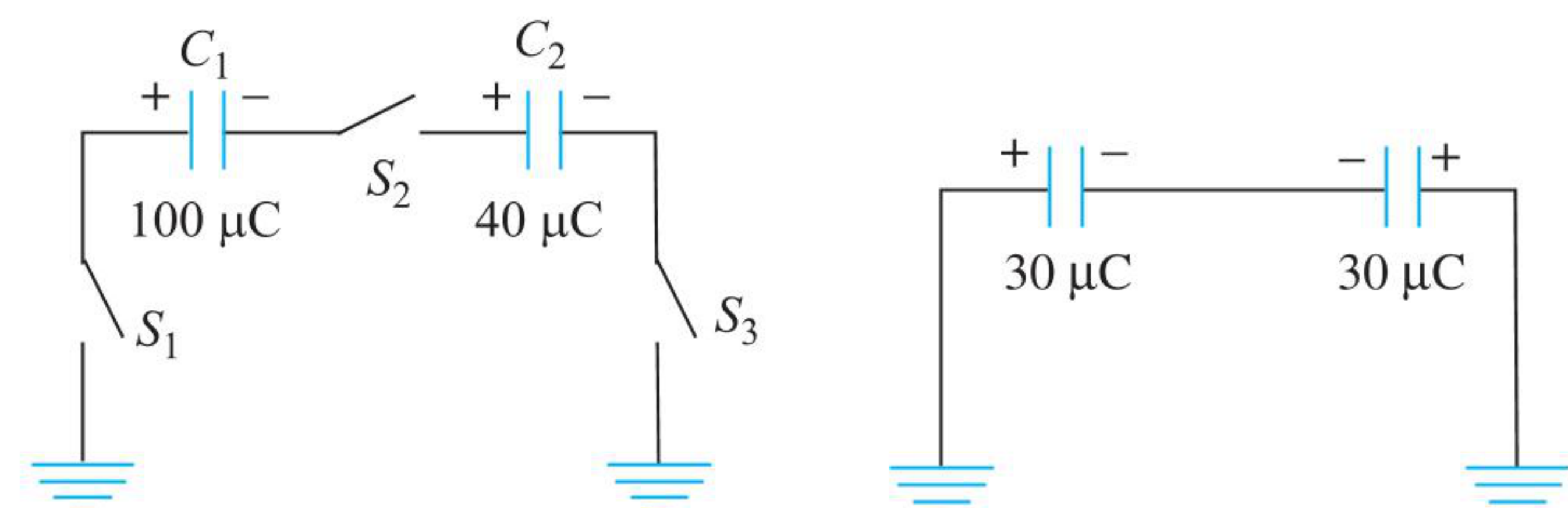
Thus, options (1), (2), and (3) are correct.

6. (1), (3)

When switches are open, charges on the capacitor are shown.

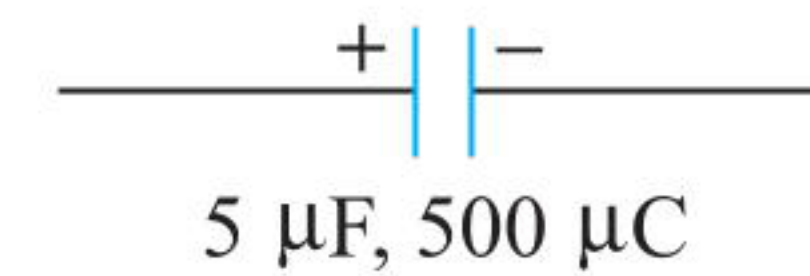
When switches are closed, capacitors become parallel; charges are shown on two capacitors. Comparing two diagrams, it is clear that $70 \mu\text{C}$ of charge will pass through switches S_1 and S_3 .

Hence, choices (1) and (3) are correct and choices (2) and (4) are wrong.



7. (2), (3)

Initial condition just after the connection of battery:



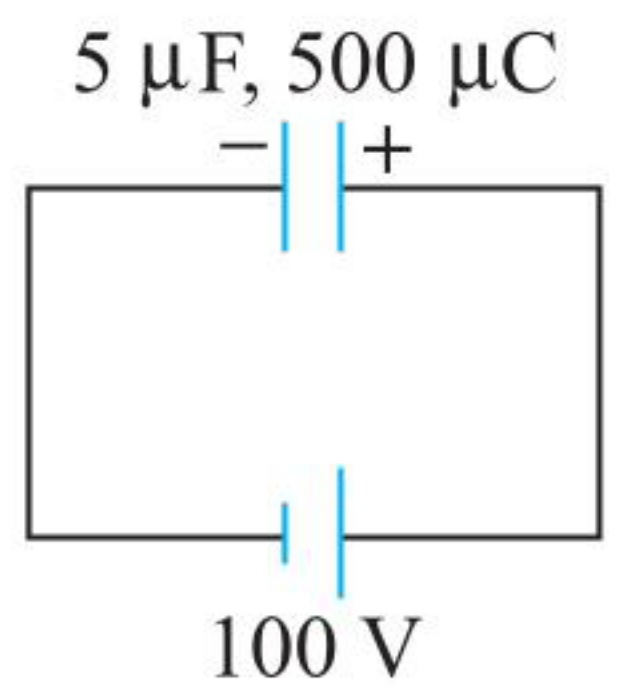
Condition after a long time:

It means battery supplies $1000 \mu\text{C}$ charge from its positive terminal and an equal and opposite charge enters from its negative terminal, i.e., charge flow through battery is $1000 \mu\text{C}$. Work done by the battery is

$$W_{\text{battery}} = 100 \text{ V} \times 10^3 \mu\text{C} = 0.1 \text{ J}$$

From energy conservation law, $U_i + W_{\text{battery}} = U_f + \Delta H$

$$U_i = U_f \text{ so } \Delta H = 0.1 \text{ J}$$



8. (1), (3), (4)

$$8 - \frac{q_1}{2} - 2 - \frac{q_3}{2} = 3 \quad \text{or} \quad 6 - 3 = \frac{q_1}{2} + \frac{q_3}{2}$$

$$\text{or } q_1 + q_3 = 6 \quad \dots(i)$$

$$8 - \frac{q_1}{2} + \frac{q_2}{2} = 2 \quad \text{or} \quad \frac{q_1}{2} - \frac{q_2}{2} = 6$$

$$\text{or } q_1 - q_2 = 12 \quad \dots(ii)$$

$$\text{Also, } q_1 + q_2 = q_3 \quad \dots(iii)$$

Solving (i), (ii), and (iii), we get $q_1 = 6 \mu\text{C}$, $q_2 = -6 \mu\text{C}$, and $q_3 = 0$.

Hence, choices (1), (4), and (3) are correct. Choice (2) is wrong.

9. (1), (3), (4)

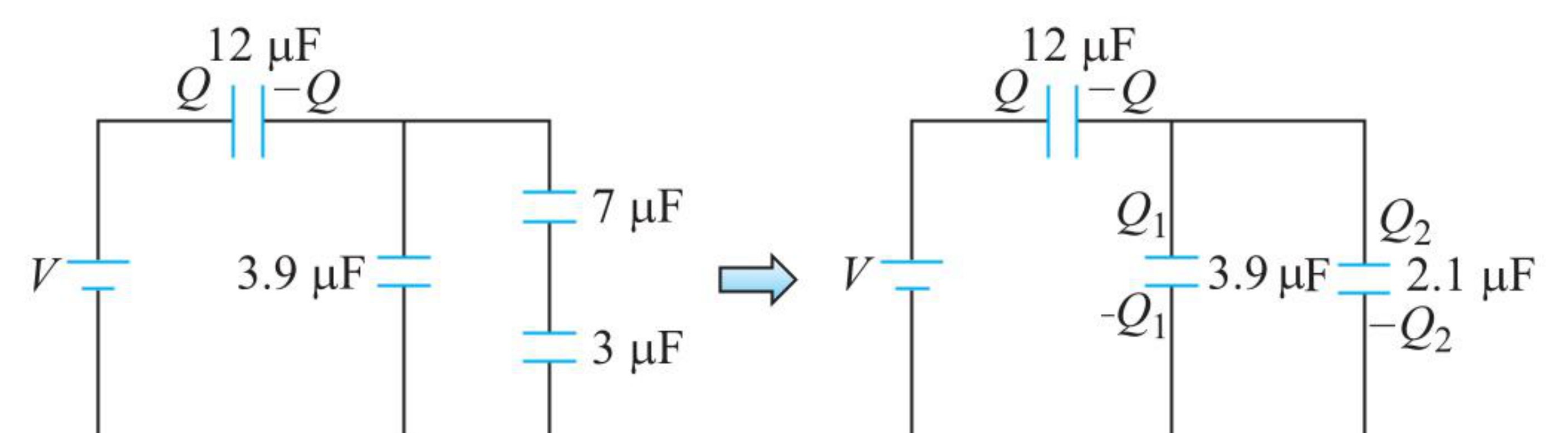
Capacitors $1 \mu\text{F}$, $2 \mu\text{F}$, and $3 \mu\text{F}$ are in parallel, and their total capacitance is $6 \mu\text{F}$. Thus, we have three capacitors in series, each of capacitance $6 \mu\text{F}$ across the 12 V power supply. So potential drop across each is $12/3 = 4 \text{ V}$. So options (3) is correct. Charge on $2 \mu\text{F}$ capacitor is $CV = 2 \times 4 = 8 \mu\text{C}$. So option (1) is correct.

Charge on $6 \mu\text{F}$ capacitor is $6 \times 10^{-6} \times 4 = 24 \mu\text{C}$. So option (2) is wrong.

Thus, correct options are (1), (3), and (4).

10. (1), (2), (4)

Equivalent capacitance of network is $C_{\text{eq}} = 4 \mu\text{F}$.



$$\text{Charge } Q = C_{\text{eq}} V = 4 \text{ V } \mu\text{C}$$

The charge on the $7 \mu\text{F}$ or $3 \mu\text{F}$ capacitor is

$$Q_2 = (7 \mu\text{F})(6 \text{ V}) = 42 \mu\text{C}$$

$$\frac{Q_2}{2.1 \mu\text{F}} = \frac{Q_1}{3.9 \mu\text{F}} \quad \text{or} \quad Q_1 = (42 \mu\text{C}) \frac{(3.9 \mu\text{F})}{(2.1 \mu\text{F})} = 78 \mu\text{C}$$

$$Q = Q_1 + Q_2 = (78 \mu\text{C} + 42 \mu\text{C}) = 120 \mu\text{C} = 4 \text{ V } \mu\text{C}$$

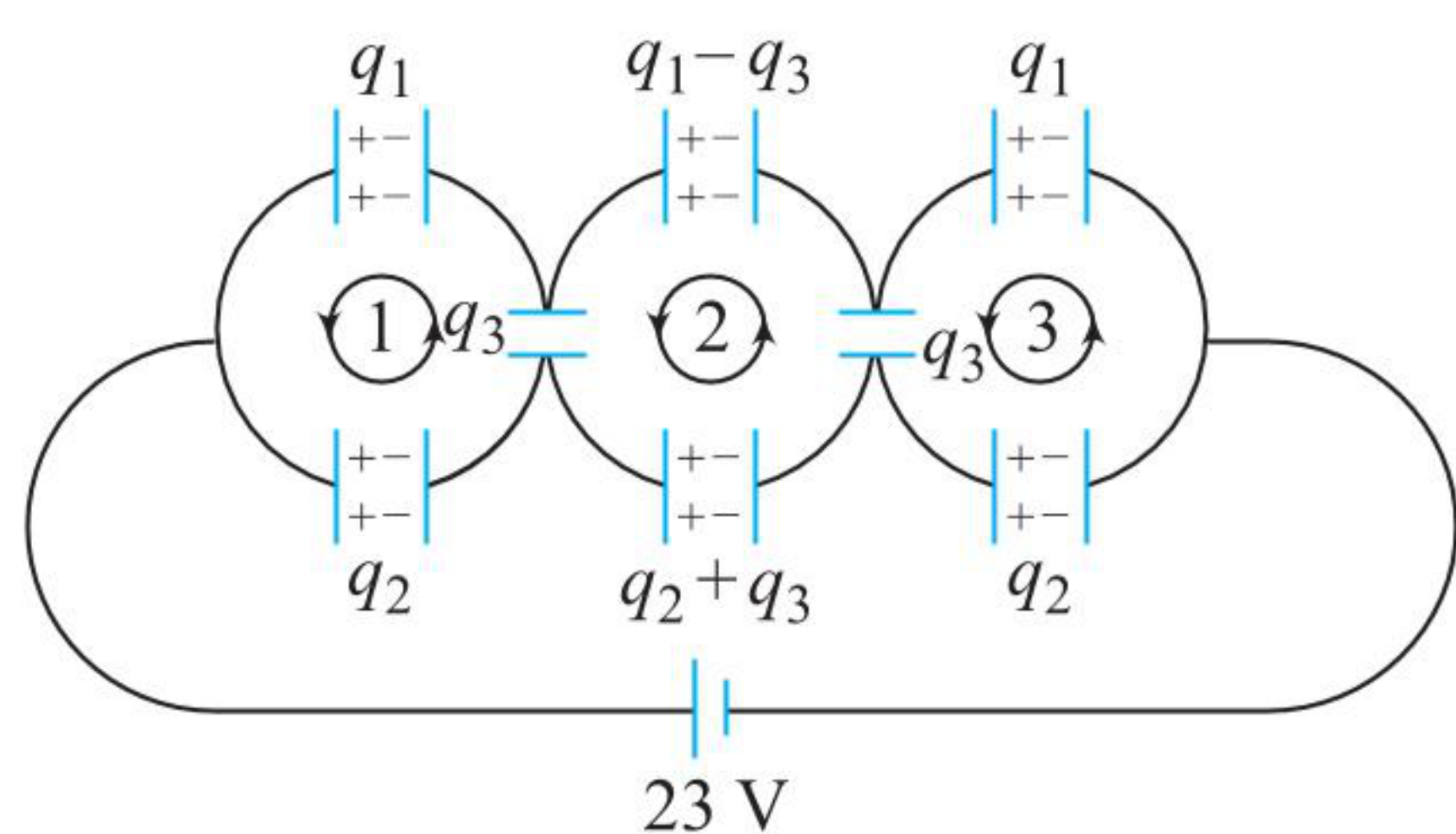
Emf of the battery is $V = 30 \text{ V}$

The potential drop across $12 \mu\text{F}$ capacitor is

$$\frac{Q}{12 \mu\text{F}} = \frac{120 \mu\text{C}}{12 \mu\text{F}} = 10 \text{ V}$$

11. (1), (2)

The distribution of charge is shown in the figure, in compliance with the point rule. Applying loop rules for loops A, B, and D, we get



$$-\frac{q_2}{5} + \frac{q_3}{0.75} + \frac{q_1}{15} = 0$$

$$\text{or } q_1 - 3q_2 + 20q_3 = 0 \quad \dots(i)$$

$$-\frac{q_2 + q_3}{15} - \frac{q_3}{0.75} + \frac{q_1 - q_3}{5} - \frac{q_3}{0.75} = 0$$

$$\text{or } 3q_1 - q_2 - 44q_3 = 0 \quad \dots(ii)$$

$$\text{or } 23 - \frac{q_2}{5} - \frac{q_2 + q_3}{15} - \frac{q_2}{5} = 0$$

$$\text{or } 345 = 7q_2 + q_3 \quad \dots(iii)$$

Solving for q_1, q_2, q_3 , we get

$$q_1 = \frac{19 \times 345}{92}, q_2 = \frac{13 \times 345}{92}, q_3 = \frac{345}{92}$$

$$\text{Potential difference between } A \text{ and } B \text{ is } \frac{q_3}{0.75} = \frac{345}{92} \times \frac{4}{3} = 5 \text{ V}$$

Potential difference between E and F is also 5 V but in the opposite direction.

12. (1), (4)

When capacitors are connected in parallel, initial capacitance is $C = 2\epsilon_0 A/d$. After the distance between the plates is changed, the capacitance becomes

$$C = \frac{\epsilon_0 A}{d+a} + \frac{\epsilon_0 A}{d-a} \\ = \frac{2\epsilon_0 A}{d - (a^2/d)}$$

which is greater than initial one. Hence, option (1) is correct and option (2) is incorrect. When capacitors are connected in series, then

$$\frac{1}{C} = \frac{2d}{\epsilon_0 A}$$

After the distance between the plates is changed,

$$\frac{1}{C} = \frac{d+a}{\epsilon_0 A} + \frac{d-a}{\epsilon_0 A} = \frac{2d}{\epsilon_0 A}$$

That is, the capacitance remains unchanged. Hence, option (4) is correct and option (3) is incorrect.

13. (1), (4)

Let q_1 and q_2 be the instantaneous charges on capacitors. Since they are in parallel, then

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \text{ and } q_1 + q_2 = Q$$

$$C_1 = \frac{\epsilon_0 A}{d_0 + vt}, C_2 = \frac{\epsilon_0 A}{d_0 - vt}$$

$$\text{So } \frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{d_0 - vt}{d_0 + vt} \text{ or } q_2 \left(\frac{d_0 - vt}{d_0 + vt} \right) - q_1 = 0$$

$$\text{So } q_2 = \frac{Q(d_0 + vt)}{2d_0} \text{ and } q_1 = \frac{Q(d_0 - vt)}{2d_0}$$

Hence, option (1) is correct and option (2) is incorrect.

$$i = \frac{-dq_1}{dt} \text{ or } \frac{dq_2}{dt} \text{ or } i = \frac{Qv}{2d_0}$$

which does not depend on time. So option (4) is correct and option (3) is incorrect.

14. (1), (3)

$$\text{Initial stored energy } U_i = \frac{1}{2} \frac{\epsilon_0 A V_1^2}{x}$$

$$\text{Final stored energy } U_f = \frac{1}{2} \frac{\epsilon_0 A V_0^2}{2(x+dx)}$$

$$\text{So } \Delta U = U_f - U_i = \frac{1}{2} \epsilon_0 A V_0^2 \left[\frac{1}{x+dx} - \frac{1}{x} \right]$$

$$= \frac{1}{2} \epsilon_0 A V_0^2 \left[\frac{x - x - dx}{x(x+dx)} \right]$$

$$= -\frac{1}{2} \frac{\epsilon_0 A V_0^2}{x^2} dx = -\frac{1}{2} \frac{\epsilon_0 A V_0^2}{x} \cdot \frac{dx}{x} = -\frac{U dx}{x}$$

So, option (1) is correct and option (2) is incorrect.

$$F = -\frac{dU}{dx} = -\frac{U}{x}$$

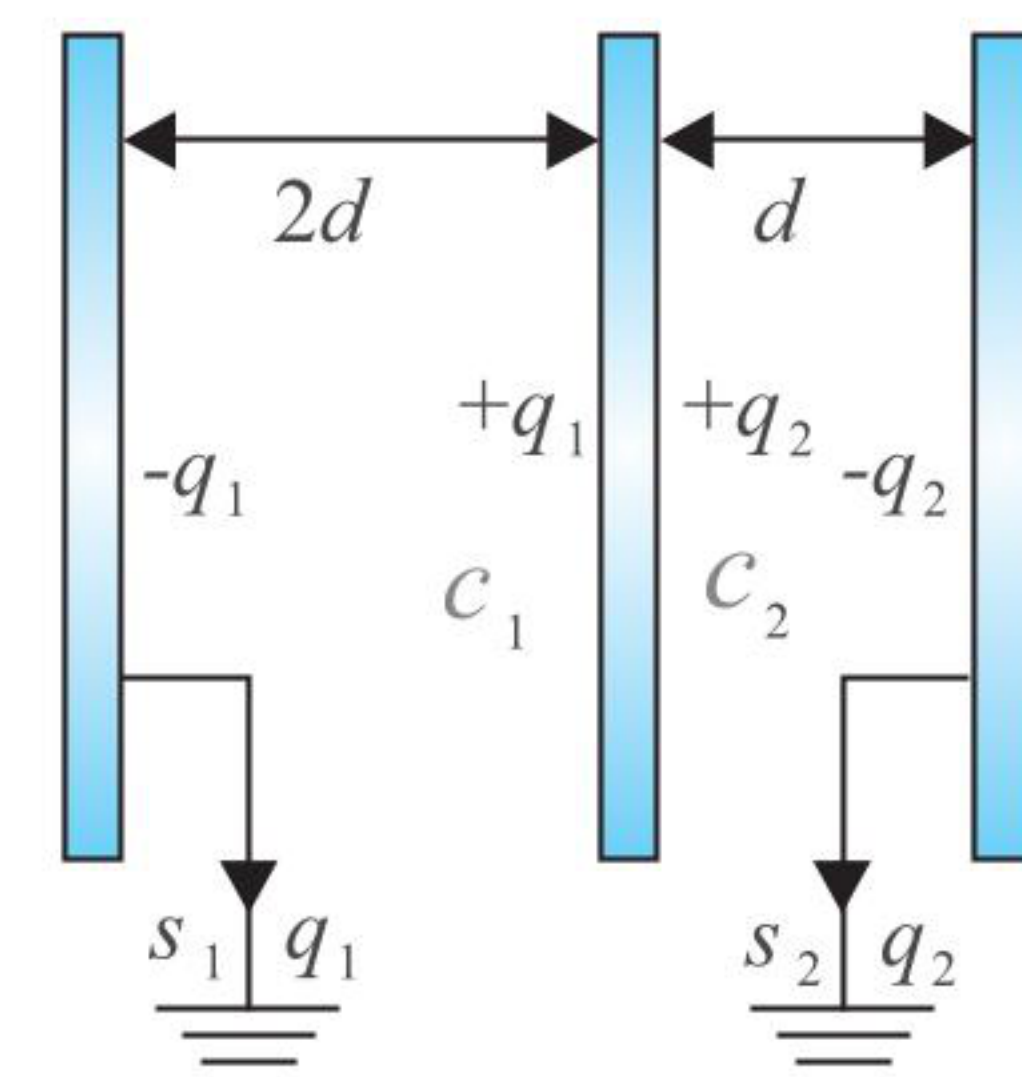
$$= \frac{1}{2} \frac{\epsilon_0 A}{x} \cdot \frac{V_0^2}{x} = -\frac{1}{2} \left(\frac{\epsilon_0 A}{x} V_0 \right) \frac{V_0}{x} = -\frac{1}{2} qE$$

So magnitude of attractive force is $1/2 qE$.

So, option (3) is correct and option (4) is incorrect.

15. (1), (2), (3)

When either A or C is earthed (but not both together), a parallel plate capacitor is formed with B , with $\pm Q$ charges on the inner surfaces. (The other plate, which is not earthed, plays no role.) Hence, charge of amount $+Q$ flows to the earth. When both are earthed together, A and C effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio 1 : 2, and hence share charge Q in the same ratio.



$$C_1 = \frac{\epsilon_0 A}{2d}, C_2 = \frac{\epsilon_0 A}{d} \frac{C_1}{C_2} = \frac{1}{2}$$

Now we can write

$$\frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{1}{2}$$

because potential difference of both the capacitors is same and $q_1 + q_2 = Q$. Solving these, we get $q_1 = Q/3, q_2 = 2Q/3$.

16. (2), (3)

$$C = \frac{Q}{V} \text{ Also, } C = \frac{\epsilon_0 A}{d}$$

As d increases, C will decrease. Therefore, V should increase, as Q remains the same.

$$E = \frac{V}{d} = \frac{Q}{A\epsilon_0} \text{ will be constant.}$$

Energy, $U = \frac{Q^2}{2C}$. As C decreases, U will increase.

17. (1), (4)

Suppose field in air gap is E , then in dielectric it is E/K .

For dielectric medium

$$dV = -\frac{E}{K} dx \text{ or } V = -\frac{E}{K} x$$

For air gap medium, $V = -Ex$.

18. (2), (4)

Plate 2 acquires a net positive charge and 3 acquires a net negative charge.

Due to induction, plate 1 acquires negative charge and plate 4 positive charge. 1 and 4 are equipotential (since they are joined).

Due to symmetry

$$V_{1-2} = V_{3-4}$$

$$\text{and } V_{2-3} = 2V_{2-1} = 2V_{4-3}$$

because charge capacitor 2-3 is double.

$$\text{Electric field} = \frac{\text{potential}}{d}$$

$$E_1 : E_2 : E_3 = 1 : 2 : 1 = \frac{1}{2} : 1 : \frac{1}{2}$$

and from plate 1 to 4

Potential first increases, then decreases, and again increases. Since we are going first in direction opposite of E and then in direction of E .

19. (1), (3), (4)

Here effective dielectric constant of conductor can be taken as infinity.

20. (2), (4)

As battery is disconnected, hence $Q = \text{constant}$, plates of capacitor are moved apart,

Hence C decreases $\left(C \propto \frac{1}{d}\right)$.

$$\text{Now, } V = \frac{Q}{C} \text{ and } U = \frac{Q^2}{2C}$$

$$V = \text{constant, } C = KC_0 \text{ i.e., } C > C_0$$

$$Q = CV, \text{ i.e., } Q > Q_0$$

$$U = \frac{1}{2} CV^2, \text{ i.e., } U < U_0$$

21. (1), (2), (3)

Being a conductor, each plate has the same potential at each point. But because $E = -(\Delta V/\Delta x)$, hence E is highest where the plates are closest to each other. Hence, surface charge density is also higher at the closer end.

22. (1), (3)

Capacities in different arms are

$$2F, 1F, \frac{1}{2}F, \frac{1}{4}F, \dots$$

$$C_{\text{eff}} = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots = 2 \left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right) = 4F$$

As PD across each row is same, thus the charge will be maximum across the capacitor in first row.

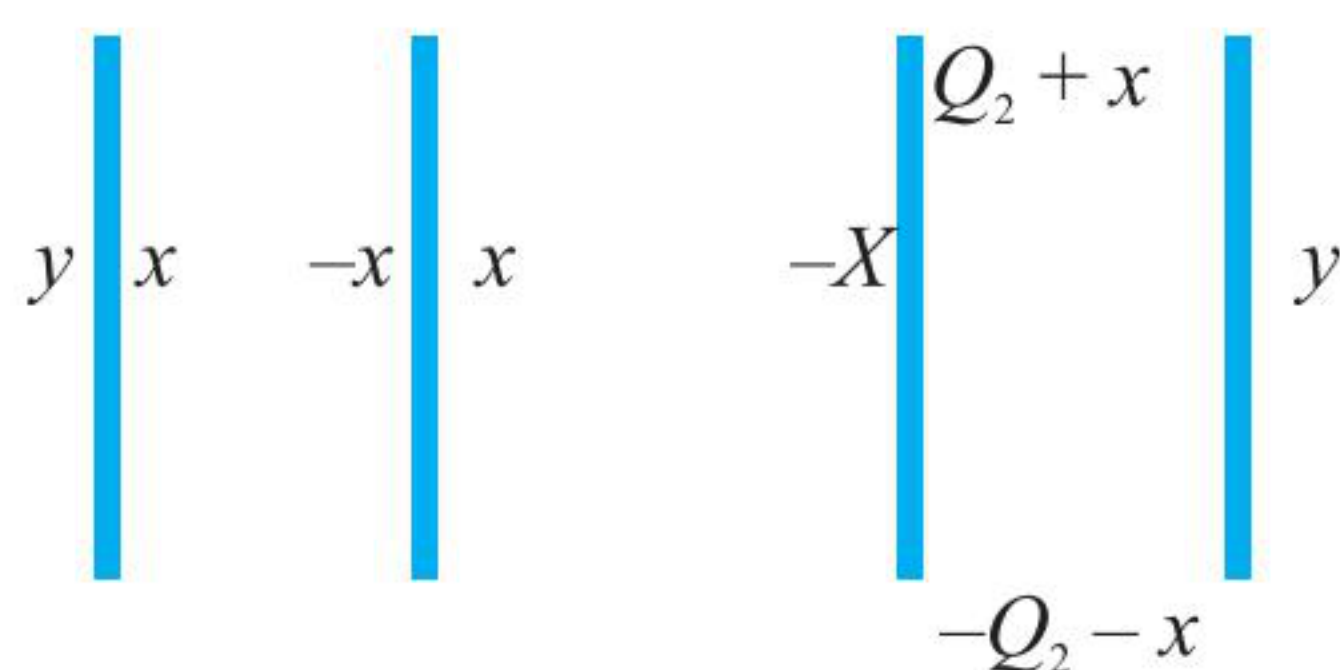
23. (1), (2)

$$xa + xb + (Q_2 + x)c = 0$$

$$x = \frac{-Q_2 c}{a + b + c}$$

$$Q_1 = y + x + y - Q_2 - x$$

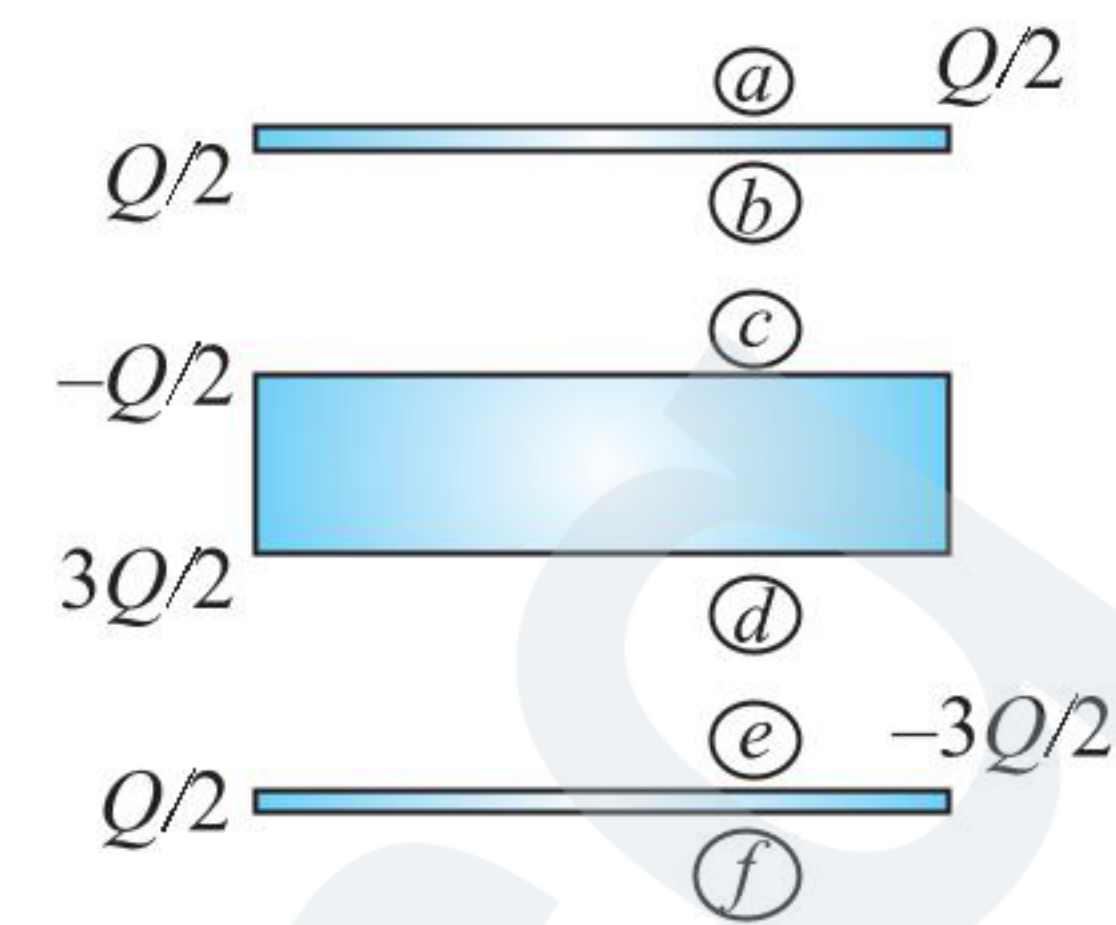
$$y = \frac{Q_1 + Q_2}{2}$$



$$V = \frac{Q_2 ca}{(a + b + c)S\epsilon_0}$$

24. (1), (2)

Initial charges on each surface

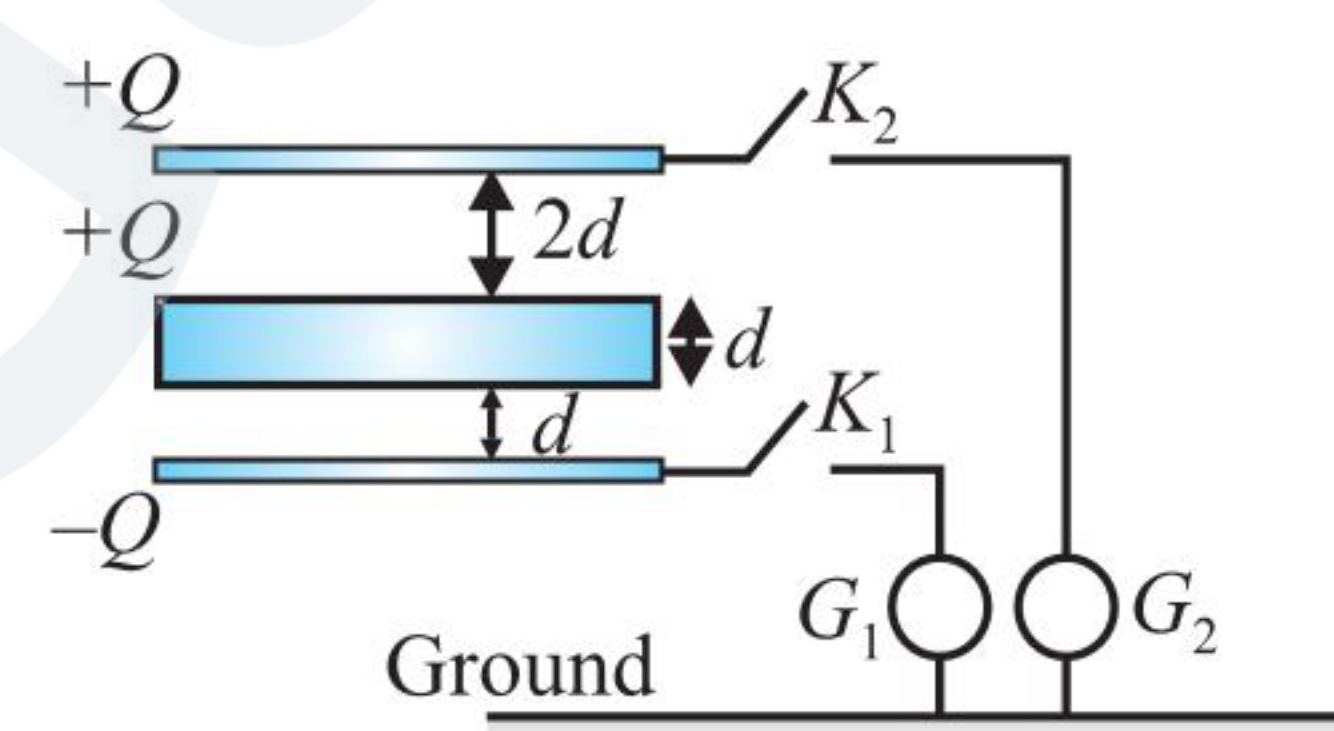


Charge on outer surfaces

$$q_a = q_b = \frac{Q + Q - Q}{2} = \frac{Q}{2}$$

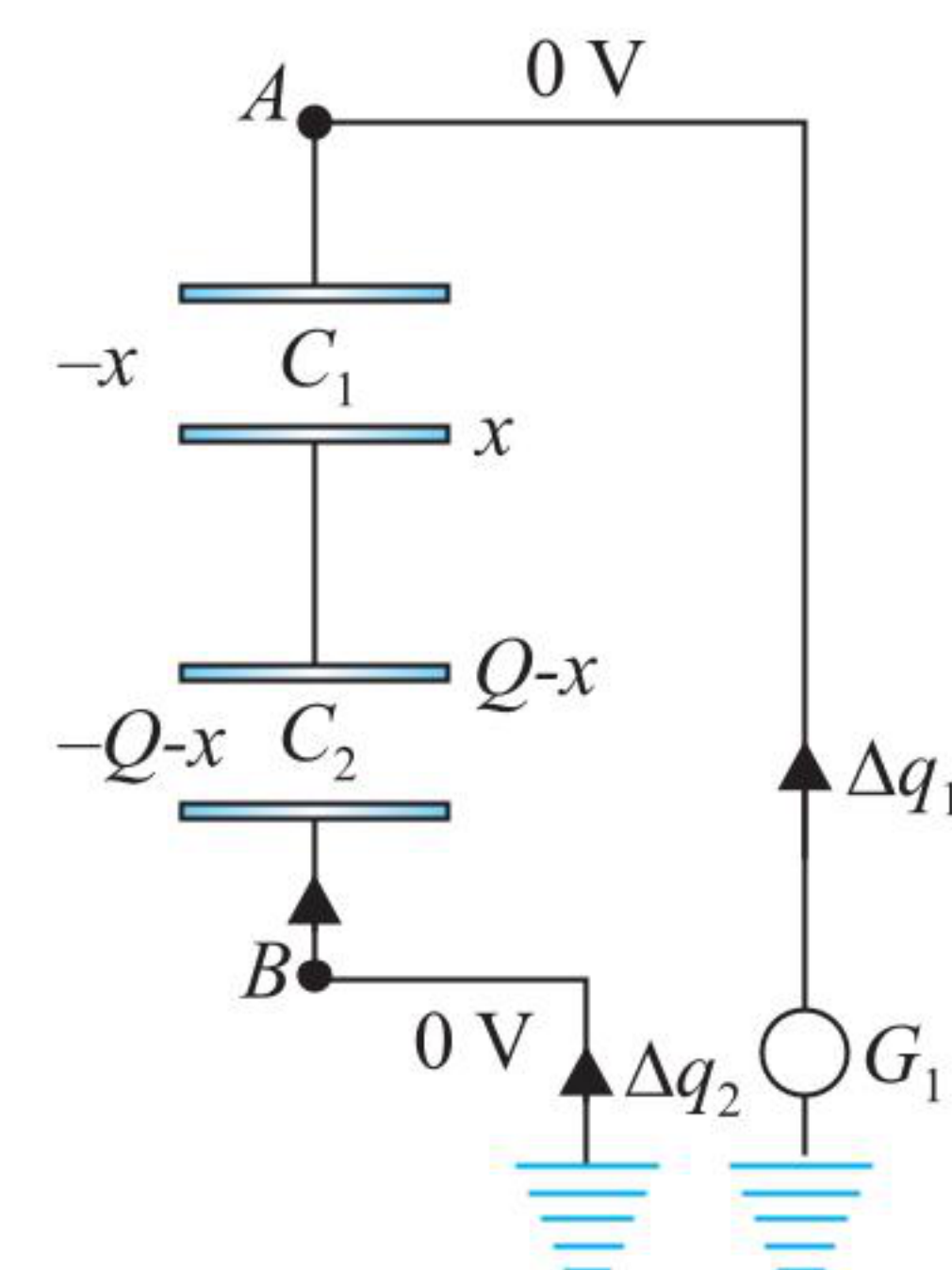
$$\text{and } q_b = \frac{Q - (Q - Q)}{2} = \frac{Q}{2}$$

$$C_1 = C = \frac{\epsilon_0 A}{2d} \text{ or } C_2 = 2C = \frac{\epsilon_0 A}{d}$$

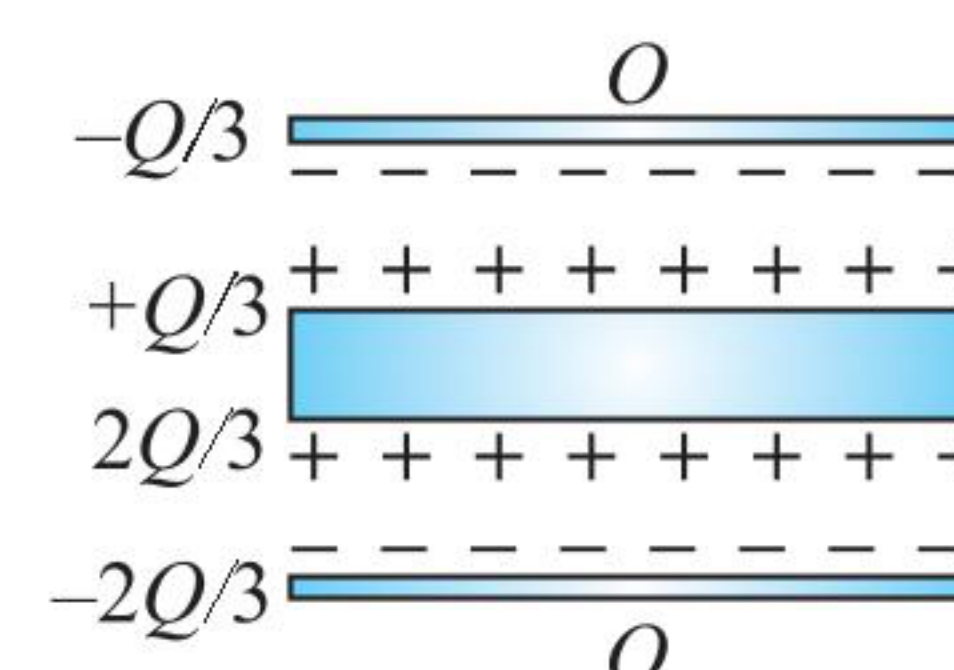


When both switches are closed the circuit can be redrawn as shown. Moving from A to B

$$0 + \frac{x}{C} - \frac{(Q - x)}{2C} = 0 \text{ or } x = \frac{Q}{3}$$



Final charges on each plate will be

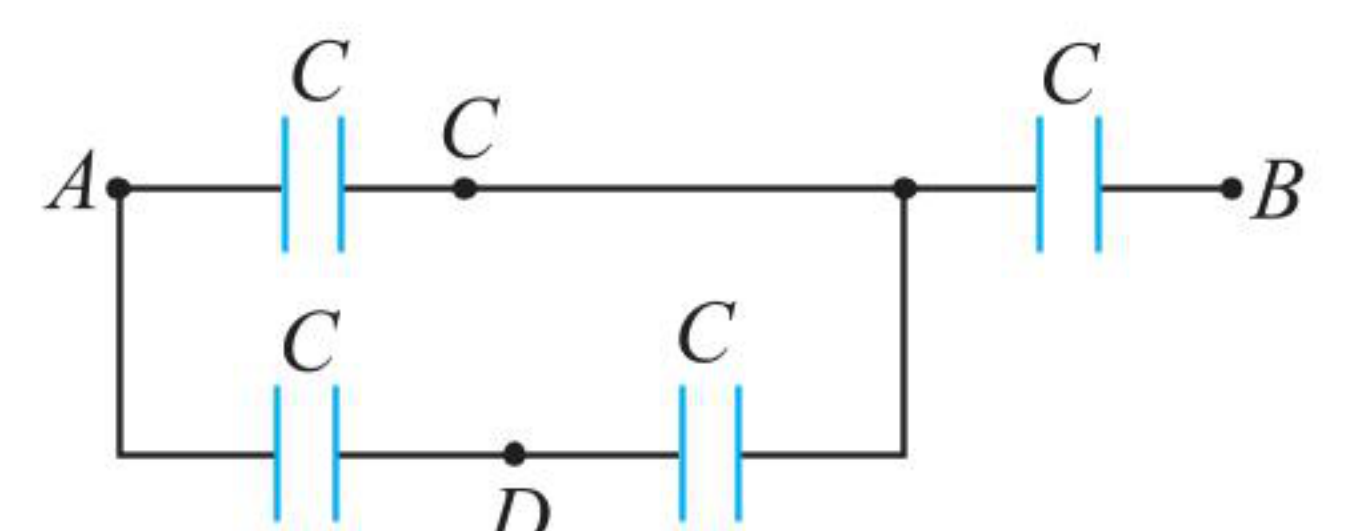


Hence charge moving from top plate to ground is $Q - \left(-\frac{Q}{3}\right) = \frac{4Q}{3}$ and from bottom plate to ground is $-Q - \left(-\frac{2Q}{3}\right) = -\frac{Q}{3}$

25. (2), (3), (4)

$$C = \frac{A\epsilon_0}{d}$$

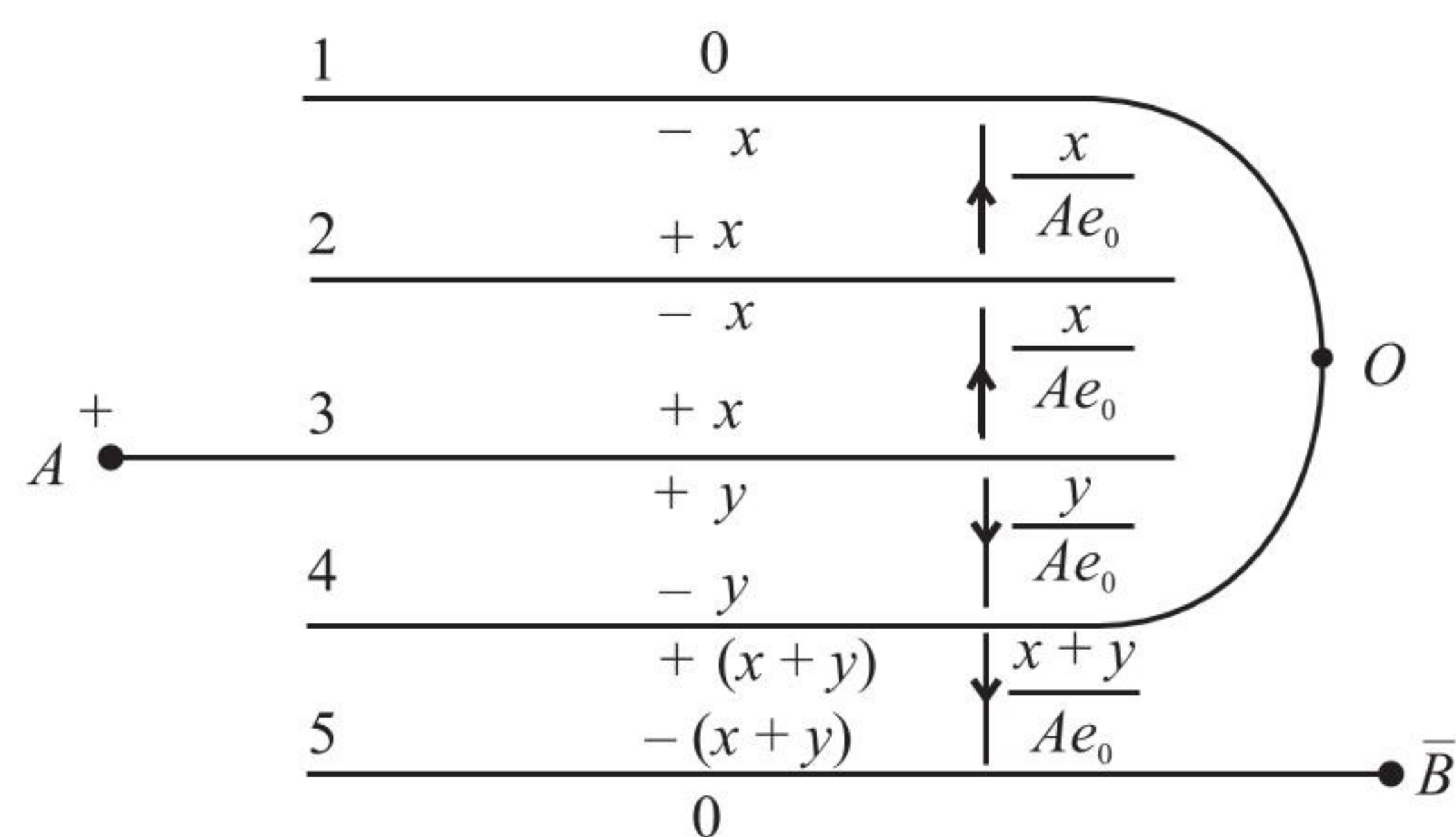
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$



or $C_{eq} = \frac{3C}{5} = \frac{3A\epsilon_0}{5d}$

Alternative method

$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$



$$C_{eq} = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

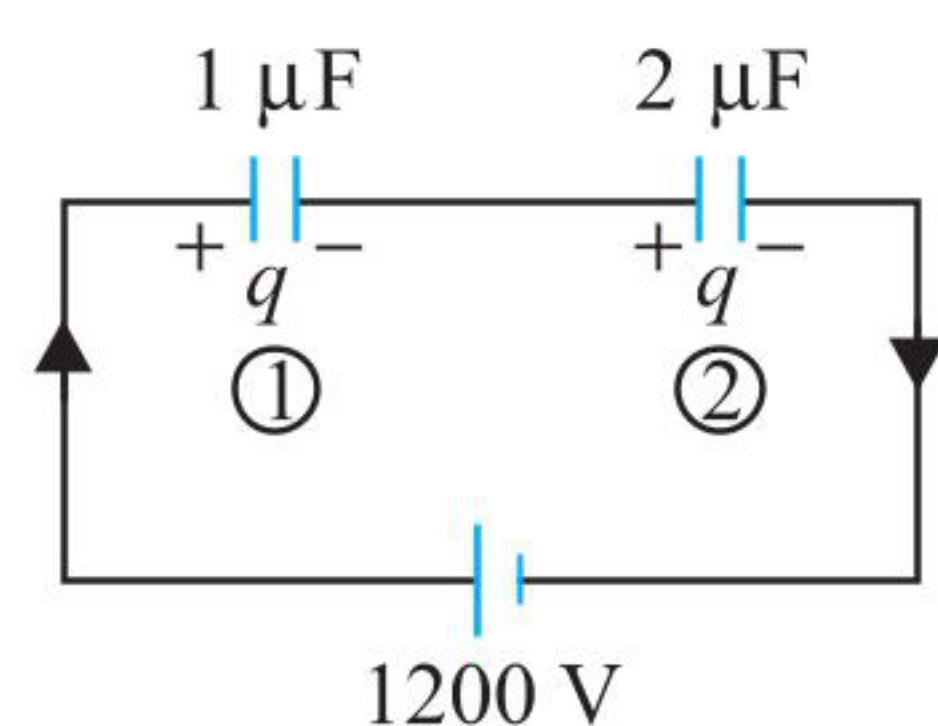
Potential of 1 and 4 is same

$$\frac{y}{A\epsilon_0} = \frac{2x}{A\epsilon_0} \quad \text{or} \quad y = 2x$$

$$V = \left(\frac{2y+x}{A\epsilon_0} \right) d, \quad C_{eq} = \frac{(x+2x)A\epsilon_0}{(5x)d} = \frac{3A\epsilon_0}{5d}$$

Linked Comprehension Type

1. (2), 2. (4)



$$1200 - \frac{q}{1} - \frac{q}{2} = 0 \quad \text{or} \quad q = 800 \mu\text{C}$$

Charge on each capacitor is 800 μC.

$$V_1 = \frac{q}{C_1} = \frac{800}{1} = 800 \text{ V}$$

$$V_2 = \frac{q}{C_2} = \frac{800}{2} = 400 \text{ V}$$

(i) $q'_1 + q'_2 = 1600$

Also, $\frac{q'_1}{1} - \frac{q'_2}{2} = 0$

or $q'_2 = 2q'_1$

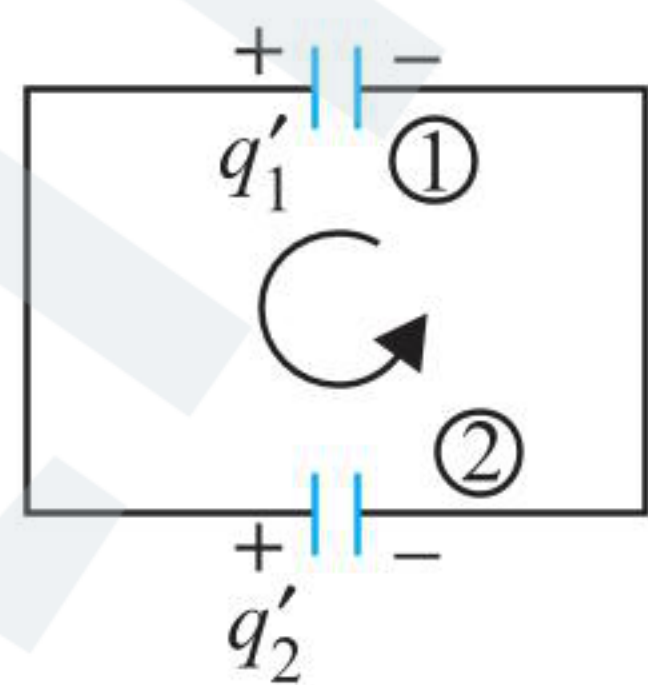
From Eq. (i), $3q'_1 = 1600$ or $q'_1 = \frac{1600}{3} \mu\text{C}$

$$3q'_2 = \frac{3200}{3} \mu\text{C}$$

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{1600}{3} \text{ V}$$

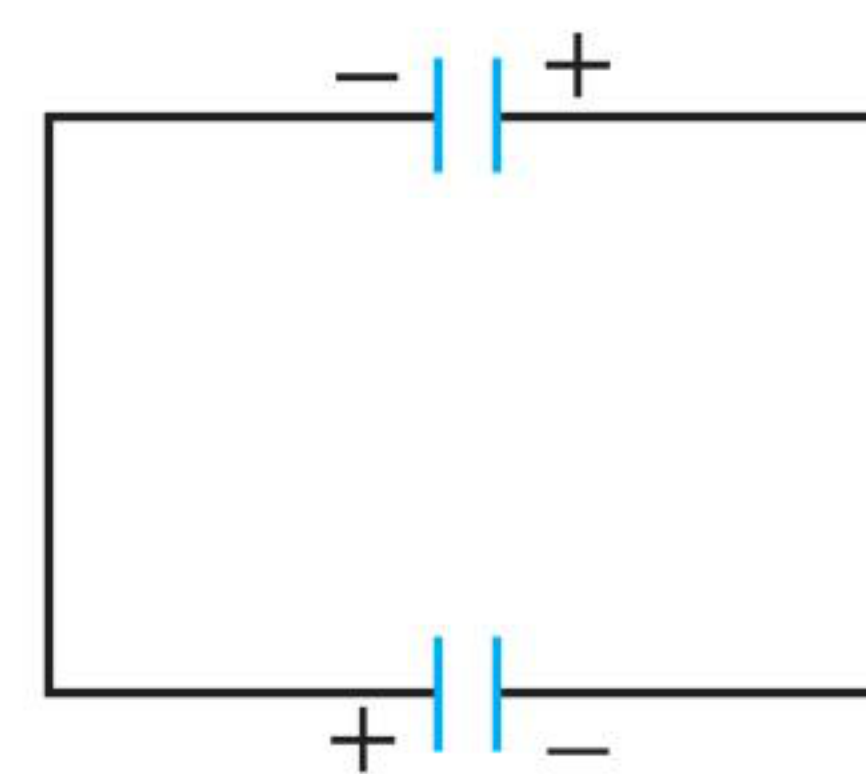
(ii) $q'_1 + q'_2 = 800 - 800 = 0$

Also, $\frac{q'_1}{1} - \frac{q'_2}{2} = 0$



...(i)

...(ii)



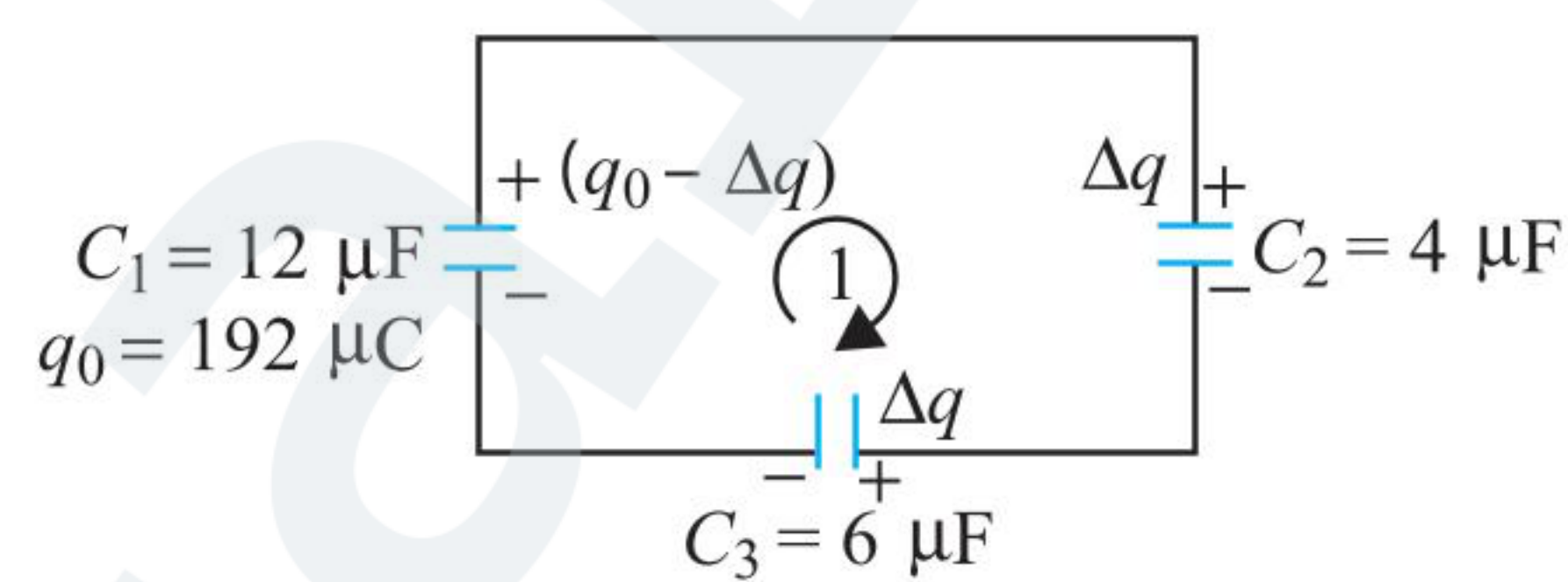
From Eq. (ii), $q'_1 = q'_2 = 0$.

Therefore, potential difference across each capacitor is zero.

3. (3), 4. (2), 5. (1)

In the close loop,

$$\frac{q_0 - \Delta q}{C_1} - \frac{\Delta q}{C_2} - \frac{\Delta q}{C_2} = 0$$



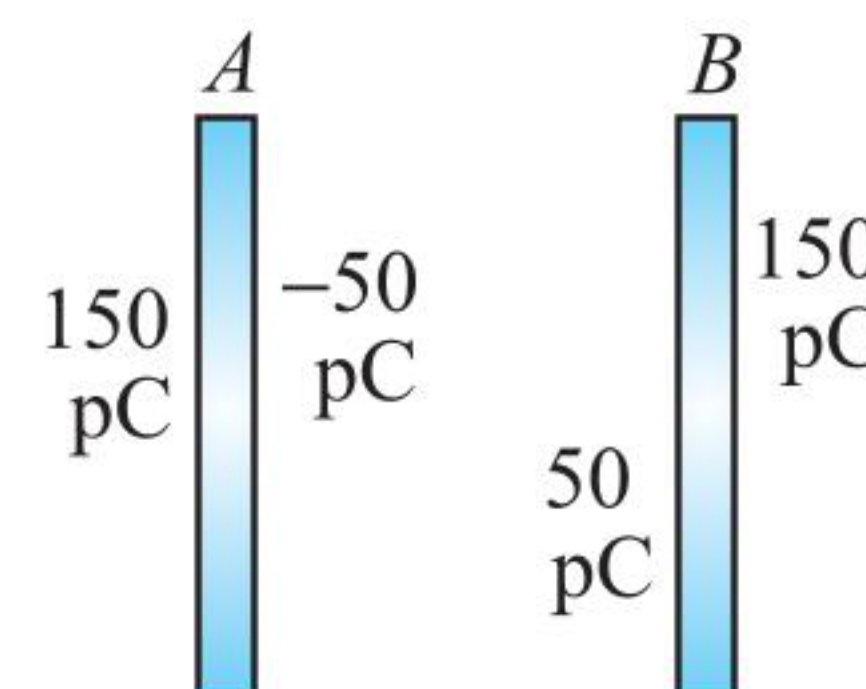
$$\frac{q_0}{C_1} = \Delta q \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]$$

$$\Delta q = \frac{q_0}{C_1 \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]} = \frac{192}{[1+3+2]} = 32 \mu\text{C}$$

Potential difference across $C_2 = \frac{\Delta q}{C_2} = 8 \text{ V}$

Potential difference across $C_1 = \frac{(192 - 32)}{12} = \frac{160}{12} = \frac{40}{3} \text{ V}$

6. (2), 7. (1)



$$C = \frac{\epsilon_0 A}{d} = \frac{8.8 \times 10^{-12} \times 5 \times 10^{-3}}{8.8 \times 10^{-3}} = 5 \times 10^{-12} \text{ F}$$

Charge on the plate after connection with the battery is

$$q = CV = 5 \times 10^{-12} \times 10 = 50 \text{ pC}$$

Charge supplied by the battery is

$$\Delta q = (50 + 50) \text{ pC} = 100 \text{ pC}$$

Energy supplied by the battery is

$$U_{\text{battery}} = \Delta q V = 100 \times 10 = 1000 \text{ pJ} = 10^{-9} \text{ J}$$

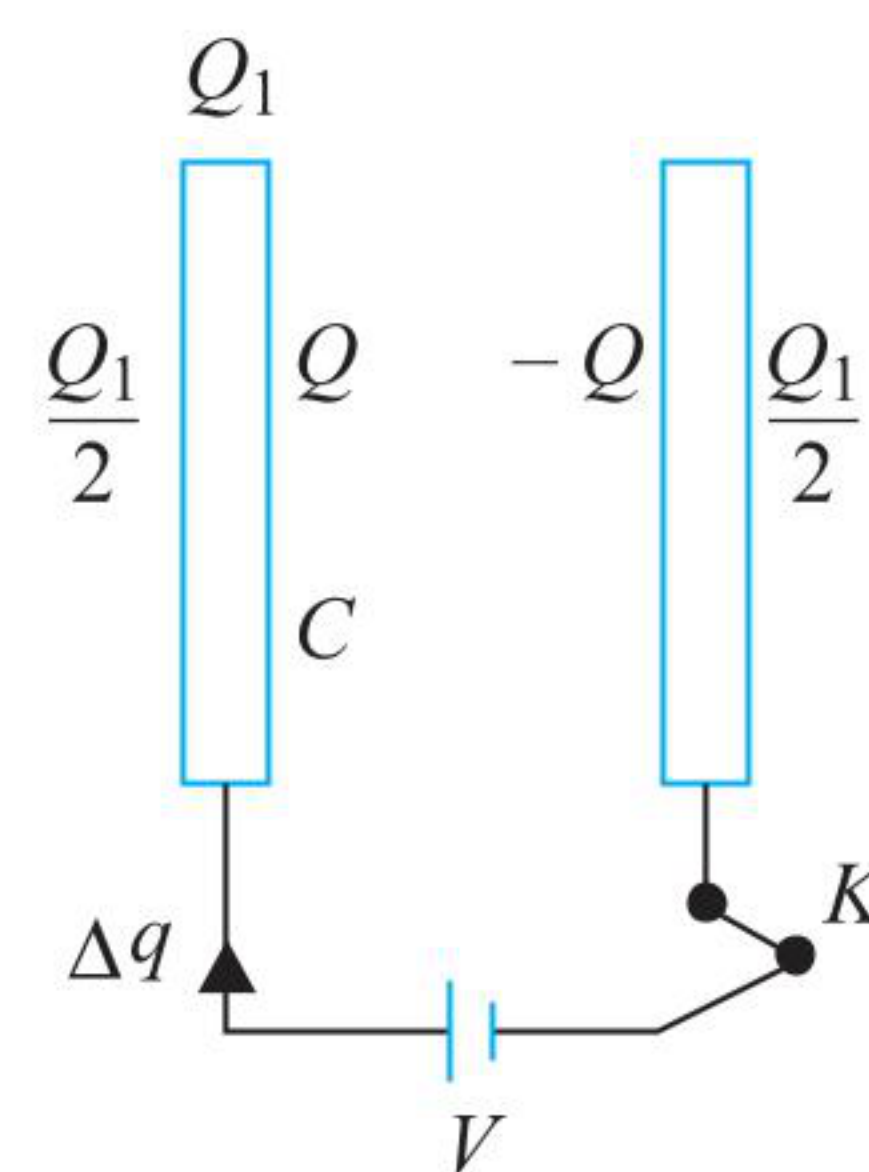
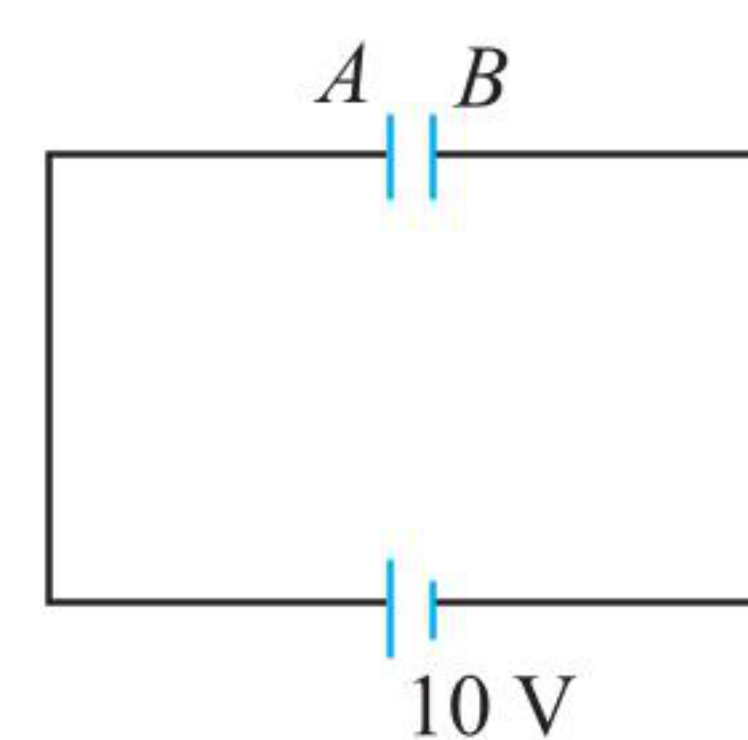
8. (4), 9. (4)

$$Q = CV$$

Charge flow is $\Delta q = \left(Q + \frac{Q_1}{2} \right) - Q_1$

$$Q - \frac{Q_1}{2} = CV - \frac{Q_1}{2}$$

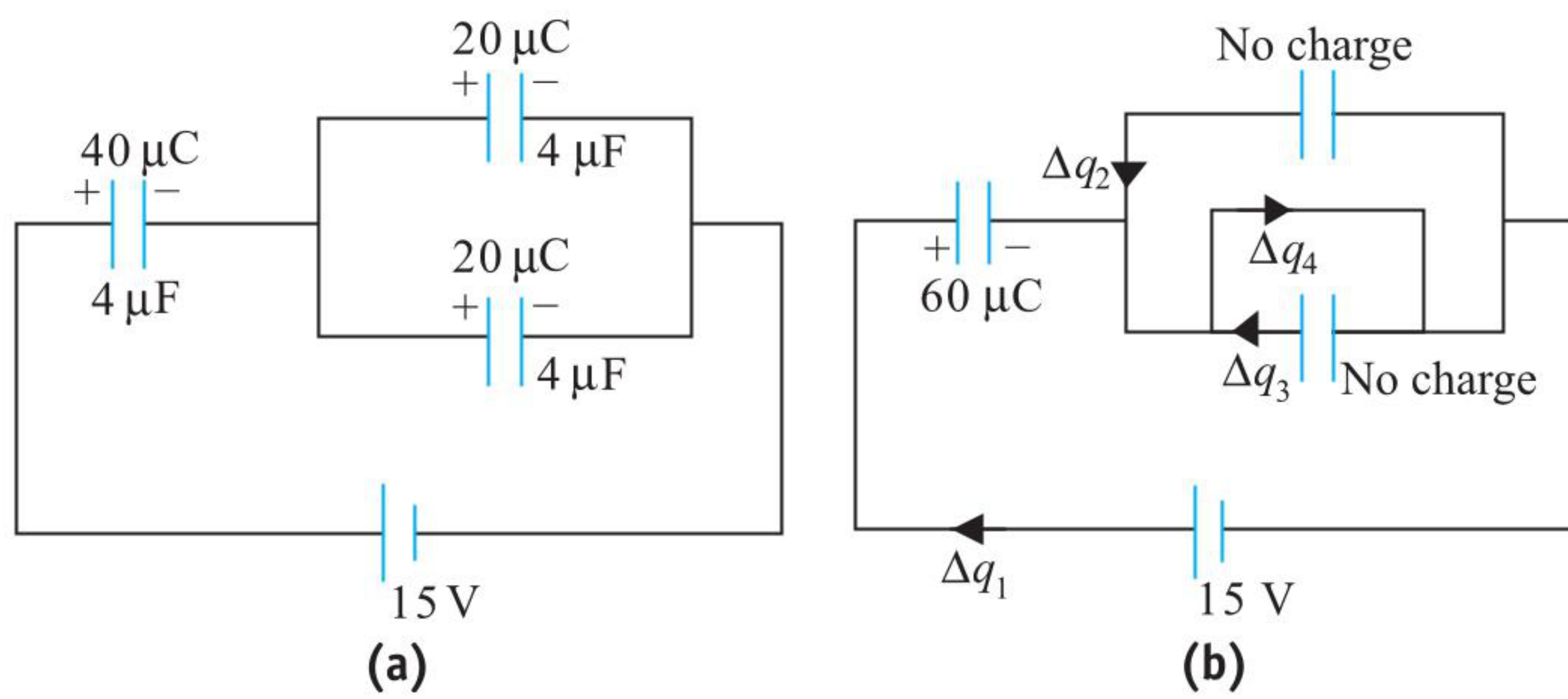
and charge on the inner surface of the left plate is $Q = CV$.



10. (1), 11. (2)

Figure (a) corresponds to when switch is open.

Figure (b) corresponds to when switch is closed.



Various charges flowing after closing the switch are as follows:

$$\Delta q_1 = 60 - 40 = 20 \mu\text{C}$$

$$\Delta q_2 = 20 - 0 = 20 \mu\text{C}$$

$$\Delta q_3 = 20 - 0 = 20 \mu\text{C}$$

$$\Delta q_4 = \Delta q_1 + \Delta q_2 + \Delta q_3 = 60 \mu\text{C}$$

12. (3), 13. (1), 14. (4), 15. (2), 16. (3)

Potential difference across C_4 is V_4 . As discussed above

$$V_4 = V_1$$

$$V_4 = 17.5 \text{ V}$$

Consider any branch, say, XAY.

Since C_1 and C_2 are in series, charge on C_2 is also $35 \mu\text{C}$.

Using $Q = CV$ for C_2

$$C_2 = C = \frac{35}{V_2} = \frac{35}{12.5} = 2.8 \mu\text{F}$$

Equivalent capacitance of branch XAY is the series equivalent of

$$C_1 = 2 \mu\text{F} \text{ and } C_2 = 2.8 \mu\text{F}, \text{ i.e., } \frac{2 \times 2.8}{(2 + 2.8)} = 1.17 \mu\text{F}$$

Equivalent capacitance of branch XBY is the series equivalent of $C_3 = C = 2.8 \mu\text{F}$ and $C_4 = 2 \mu\text{F}$, i.e., $1.17 \mu\text{F}$. These branches are in parallel between X and Y. Hence, equivalent capacitance between X and Y is $1.17 + 1.17 = 2.34 \mu\text{F}$.

Capacitor C_5 is connected in the part of circuit between X and A either in series or parallel with $C_1 = 2 \mu\text{F}$. Let the equivalent capacitance of C_1 and C_5 , i.e., the equivalent capacitance between X and A be C'_1 .

Capacitor C_6 is connected between A and B. Obviously, the circuit then becomes a Wheatstone bridge. Further, since equivalent capacitance between X and Y is independent of the value of C_6 , it implies that the bridge is in the balanced condition and potentials at A and B are now equal, so that

$$\frac{C'_1}{C_3} = \frac{C_2}{C_4}$$

$$\text{or } \frac{C'_1}{2.8} = \frac{2.8}{2} \quad (C_2 = 2.8 \mu\text{F}, \text{ as determined earlier})$$

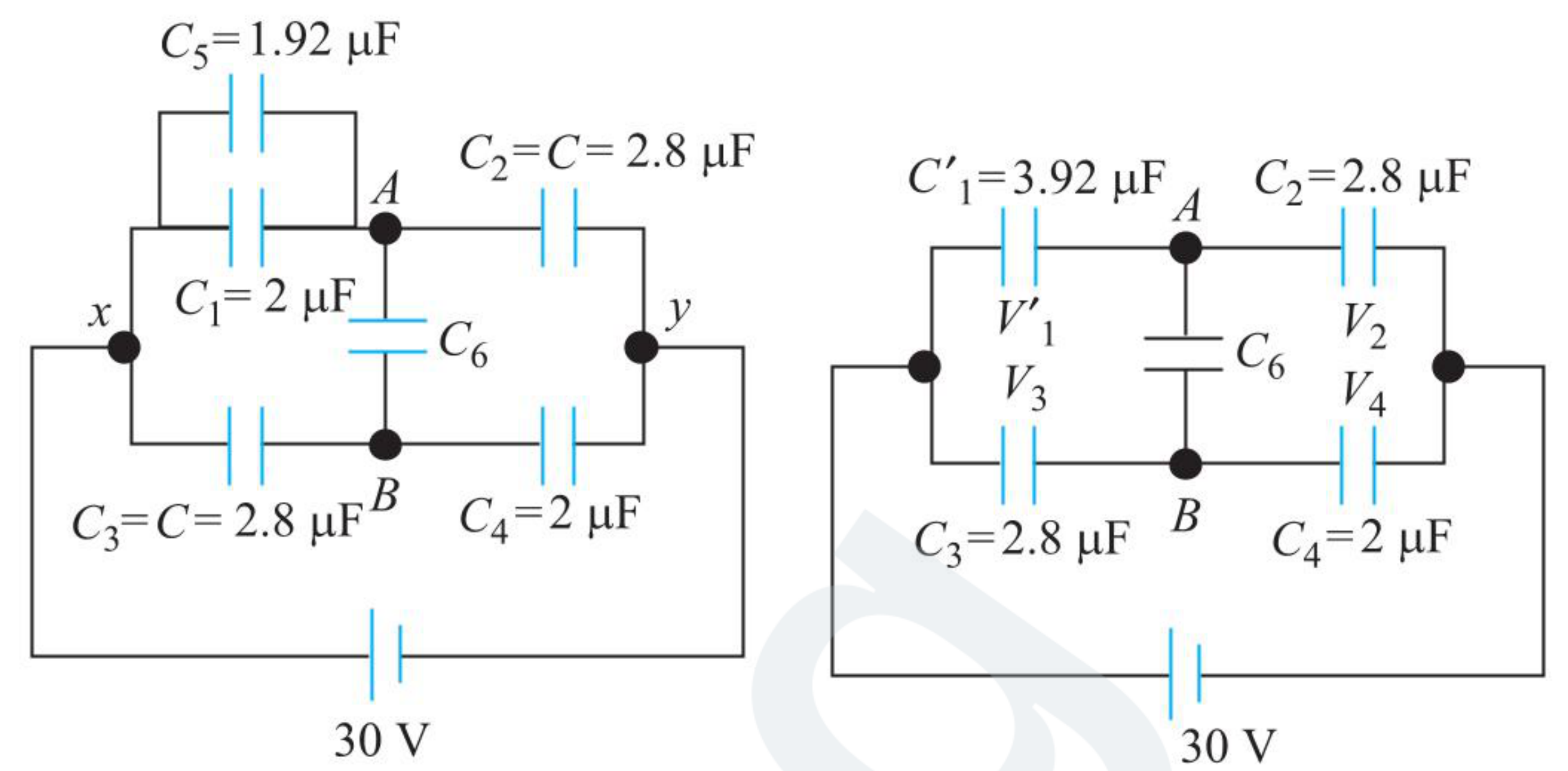
$$\text{or } C'_1 = 3.92 \mu\text{F}$$

C'_1 is the equivalent capacitance of $C_1 = 2 \mu\text{F}$ and C_5 . Since $C'_1 > C_1$, we can conclude that C_1 and C_5 could not be connected in series. We know that series equivalent capacitance is less than each individual capacitance. Hence, C_1 and C_5 are in parallel, so

$$C'_1 = C_1 + C_5 \text{ or } C_5 = C'_1 - C_1 = 3.92 - 2 = 1.92 \mu\text{F}$$

$$V'_1 + V_2 = 30$$

...(iv)



Since the bridge is in a balanced condition, A and B are at same potential and C_6 has zero charge. C'_1 and C_2 are in series. Therefore, charge on C'_1 and C_2 has the same value, so that potential difference will be in the inverse ratio of capacitance

$$[Q = CV \text{ or } V = \frac{Q}{C} \text{ hence for same } Q, V \propto \frac{1}{C}]$$

$$\frac{V'_1}{V_2} = \frac{2.8}{3.92} \approx 0.71 \quad \text{or} \quad V'_1 = 0.71 V_2$$

From Eq. (iv), $0.71 V_2 + V_2 = 30$ or $V_2 = 17.5 \text{ V}$

Hence $V'_1 = 30 - V_2 = 12.5 \text{ V}$

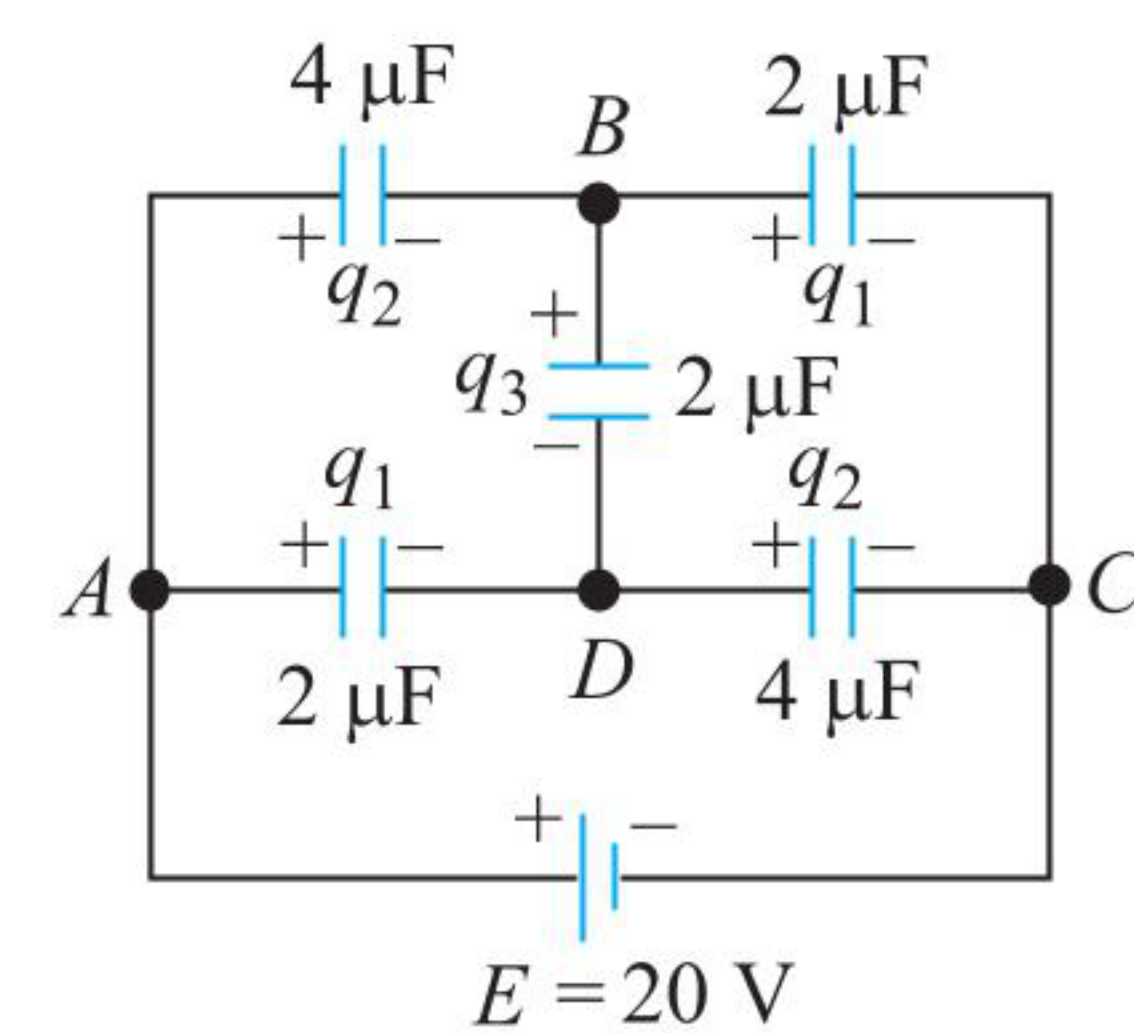
Since C'_1 is parallel equivalent of C_1 and C_5 , so potential difference across each is $V'_1 = 12.5 \text{ V}$.

Hence, charge on C_5 is

$$C_5 V'_1 = 1.92 \times 12.5 = 24 \mu\text{C}$$

17. (2), 18. (1)

If $2 \mu\text{F}$ were not present between B and D, then potential drop across upper $4 \mu\text{F}$ will be less than potential drop across lower $2 \mu\text{F}$, i.e.,



$$V_A - V_B < V_A - V_D \Rightarrow V_B > V_D$$

Now if $2 \mu\text{F}$ is connected between B and D, charge will flow from B to D and finally $V_B > V_D$.

$V_A = 20 \text{ V}$. At junction B:

$$q_2 = q_1 + q_3$$

$$\text{or } (V_A - V_B)4 = (V_B - V_D)2 + (V_B - V_C)2$$

$$\text{or } 4(V_A - V_B) + 2(V_D - V_B) = 2V_B$$

At junction D:

$$q_2 = q_1 + q_3$$

$$\text{or } 4(V_D - V_C) = 2(V_A - V_D) + 2(V_B - V_D)$$

$$\text{or } 2(V_A - V_D) + 2(V_B - V_D) = 4V_D$$

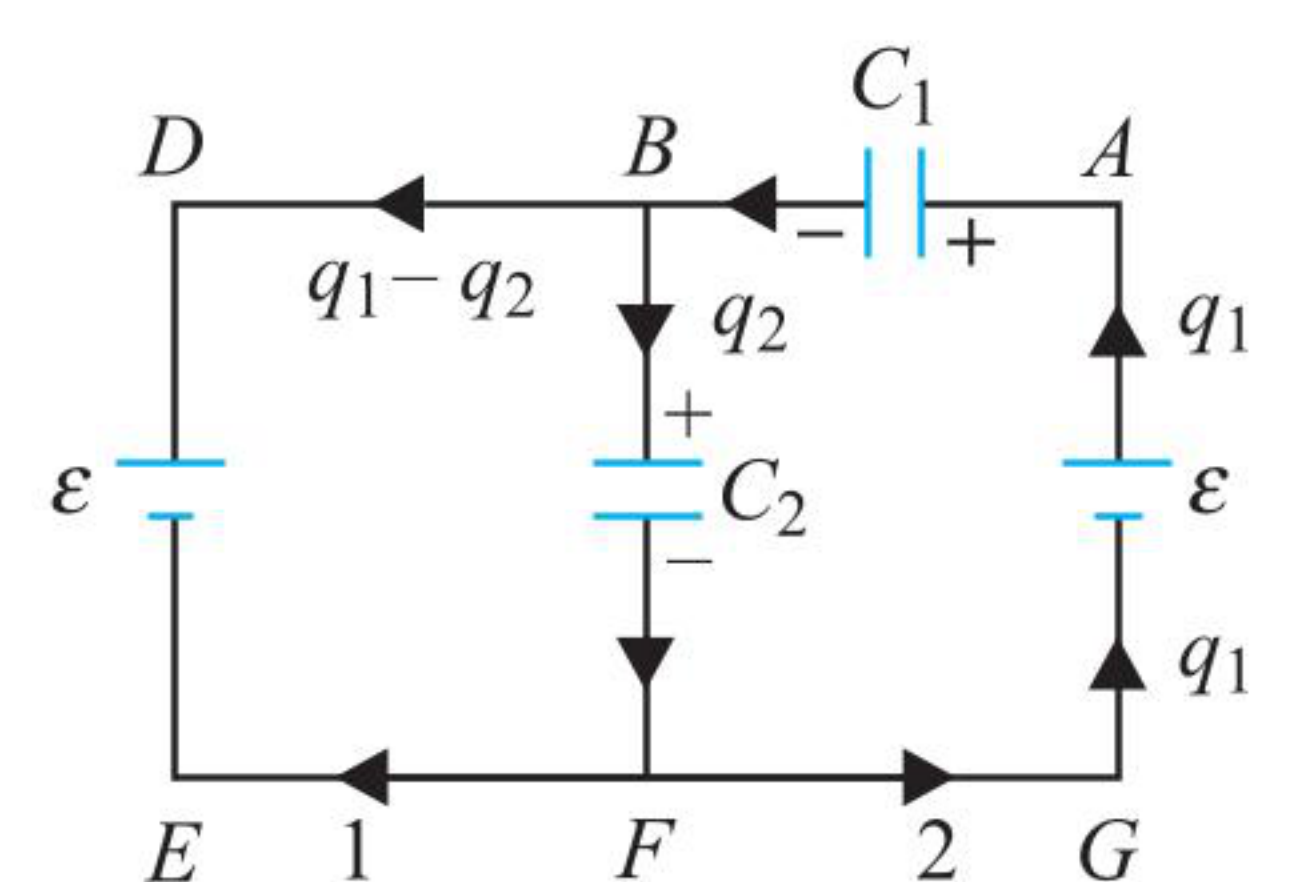
19. (2), 20. (4)

Charge on capacitors C_1 and C_2 before closing the switch S (figure) is

$$q_0 = \left(\frac{C_1 C_2}{C_1 + C_2} \right) \epsilon$$

After closing S, charge in C_2 (final charge) is

$$q_2 = C_2 \epsilon$$



In loop $ABDEFGA$, $\varepsilon - \frac{q_1}{\varepsilon} - \varepsilon = 0$ or $q_1 = 0$.

Final charge on C_1 , $q_1 = 0$. To make final charge in C_1 , the charge q_0 will flow toward battery.

Hence, charge flow toward direction 2 is

$$\Delta q_2 = -\left(\frac{C_1 C_2}{C_1 + C_2}\right)\varepsilon$$

To make charge on capacitor C_2 to final value q_2 , the charge flow into capacitor is

$$\Delta q = C_2 \varepsilon - \frac{C_1 C_2}{C_1 + C_2} \varepsilon = C_2 \varepsilon \left[1 - \frac{C_1}{C_1 + C_2}\right]$$

$$\Delta q_B = \frac{C_2^2 \varepsilon}{C_1 + C_2}$$

At junction F ,

$$\Delta q_1 = \Delta q - \Delta q_2 = \frac{C_2 \varepsilon}{C_1 + C_2} [C_2 + C_1] = C_2 \varepsilon$$

Hence, charge flowing in the direction of 1 is $\Delta q = C_2 \varepsilon$.

Alternative method: (See figure)

In loop $ABDEG$, $-\frac{(q_0 + \Delta q_1)}{C_1} - \varepsilon + \varepsilon = 0$

$$\text{or } \Delta q_1 = -q_0 = -\frac{C_1 C_2 \varepsilon}{C_1 + C_2}$$

In loop $BFEDB$,

$$-\frac{(q_0 + \Delta q_1)}{C_2} + \varepsilon = 0$$

$$\text{or } \Delta q = C_2 \varepsilon - q_0$$

At junction F ,

$$\Delta q_2 = \Delta q_1 - \Delta q_1 = -q_0 - (C_2 \varepsilon - q_0) = -C_2 \varepsilon$$

Hence charge flow, in the direction of:

$$(1) \text{ is } -\frac{C_1 C_2 \varepsilon}{C_1 + C_2}$$

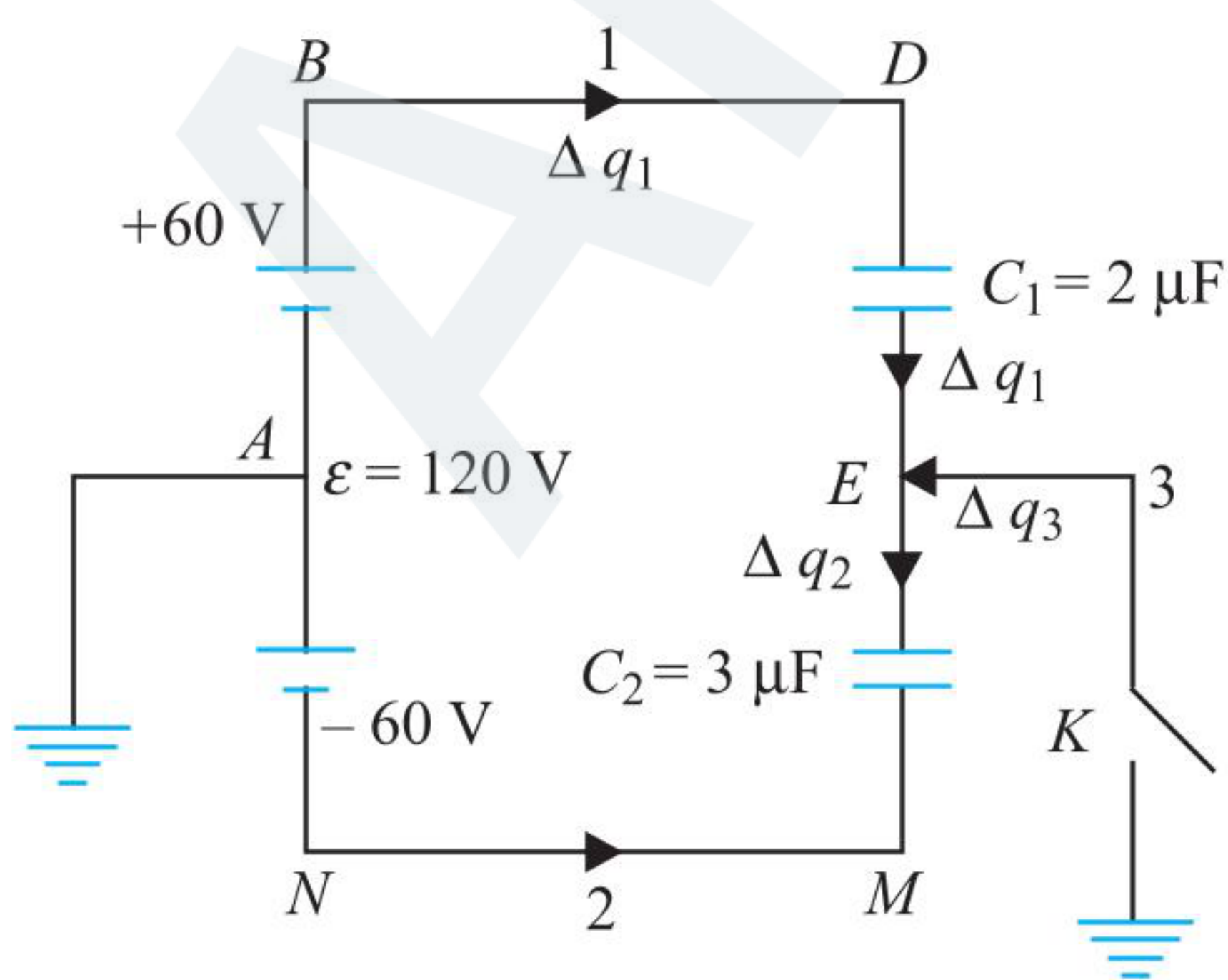
$$(2) \text{ is } C_2 \varepsilon$$

21. (1), 22. (2), 23. (4)

Before earthing charge on each capacitor (See figure),

$$q_0 = 120 \times \frac{6}{5}$$

$$q_0 = 24 \times 6 = 144 \mu\text{C}$$



After earthing, we get

$$\text{Path } ABDE, 0 + 60 - \frac{(q_0 + \Delta q_1)}{2} = 0$$

$$\Delta q_1 = 120 - q_0 = 24 \mu\text{C}$$

$$\text{Path } ANME, 0 - 60 + \frac{(q_0 + \Delta q_2)}{3} = 0$$

$$\text{or } \Delta q_2 = 180 - q_0 = 36 \mu\text{C}$$

$$\text{At junction } E, \Delta q_1 + \Delta q_3 = \Delta q_2$$

$$\text{or } \Delta q_3 = \Delta q_2 - \Delta q_1 = 36 + 24 = 60 \mu\text{C}$$

Hence, the charge flow in the direction

(1) is $-24 \mu\text{C}$

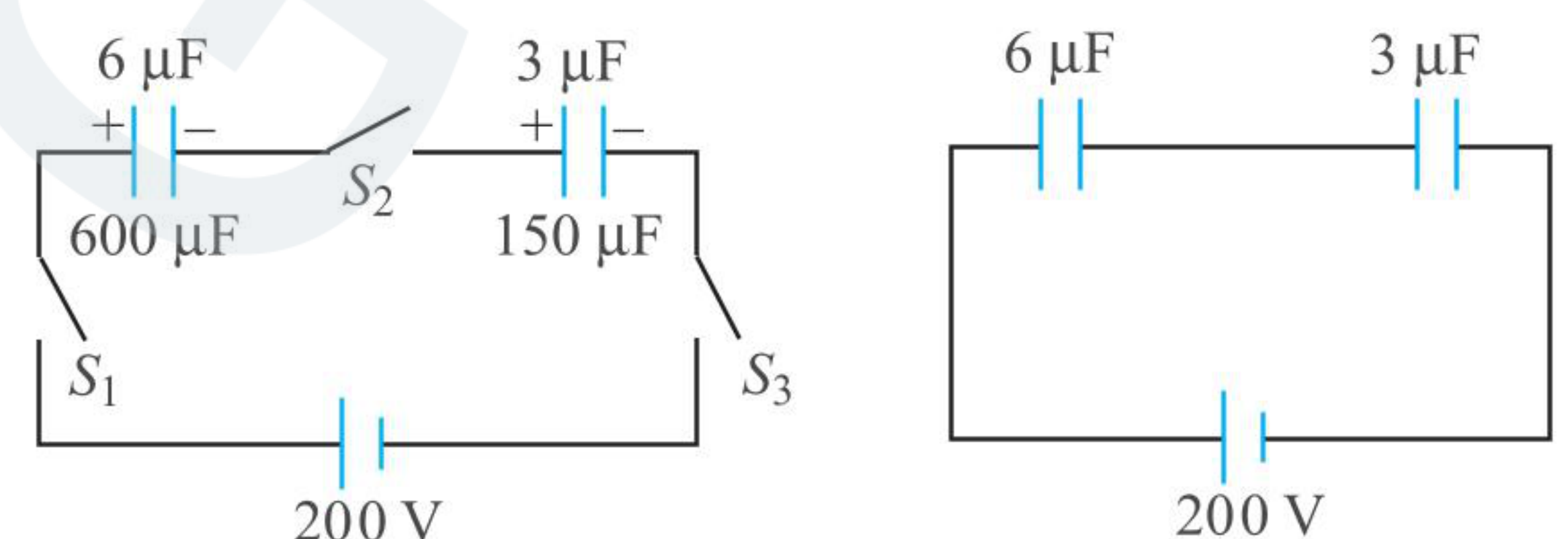
(2) is $-36 \mu\text{C}$

(3) is $+60 \mu\text{C}$

24. (4), 25. (2), 26. (4)

Plates 2 and 3 are joined together and they are neither connected to any of the terminals of the battery nor to any other source of charge. So, they jointly form an isolated system.

Before closing the switches, charges are shown in figure. After closing the switch, let q charge goes from battery. Then charge on each will increase by q .



Applying Kirchhoff's voltage law, we get

$$200 - \left(\frac{600 + q}{6}\right) - \left(\frac{150 + q}{3}\right) = 0$$

$$\text{or } q = 100 \mu\text{C}$$

So charge on $6 \mu\text{F}$ capacitor will be $700 \mu\text{C}$ and $3 \mu\text{F}$ capacitor will be $250 \mu\text{C}$.

From all the switches $100 \mu\text{C}$ of charge will flow.

27. (1), 28. (2), 29. (3)

When $C_3 \rightarrow \infty$, it means distance between the plates of C_3 is zero or both plates will be at the same potential. Then C_2 will be shorted. Entire potential V will be across C_1 . Hence, $V = V_1 = 10 \text{ V}$.

From graph, when $C_3 = 0$, $V_1 = 2 \text{ V}$. So, C_3 will act like open switch. C_1 and C_2 will be in series. Potential different across C_2 is $V_2 = 10 - V_1 = 8 \text{ V}$.

$$q = C_1 V_1 = C_2 V_2 \text{ or } C_{12} = C_{28} \text{ or } C_1 = 4C_2$$

$$V_1 = 4 \text{ V}, V_2 = 10 - 4 = 6 \text{ V}$$

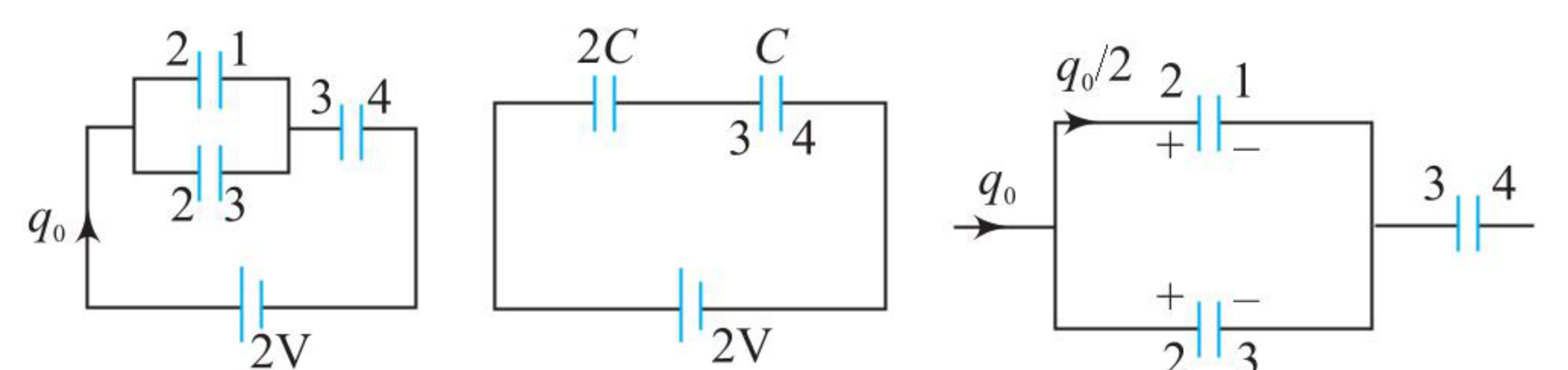
$$q = C_1 V_1 = (C_2 + C_3) V_2$$

$$\text{or } C_{14} = (C_2 + C_3)6 \text{ or } 16C_2 = 6C_2 + 6C_3$$

$$\text{or } C_3 = 5 C_{2/3}$$

30. (4), 31. (4), 32. (1)

When both the switches are closed, circuit is



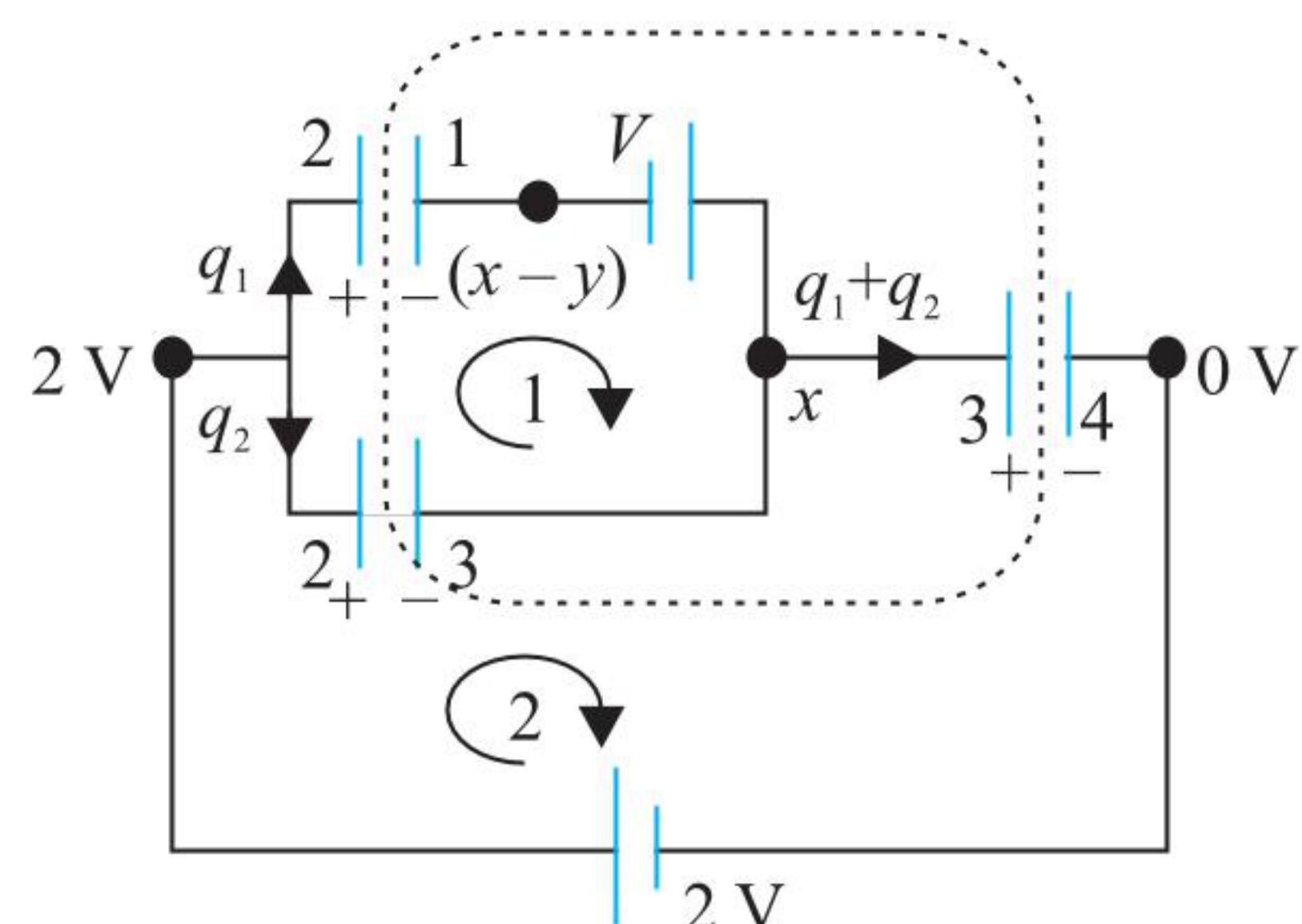
Equivalent capacitance

$$C_{eq} = \frac{2C}{3} = \frac{2\varepsilon_0 A}{3d}$$

Charge supplied by battery = Charge on plate '2'

$$q_0 = \left(\frac{2C}{3} \right) (2V) = \frac{4CV}{3} = \frac{4\epsilon_0 AV}{3d}$$

where switch Sw_2 is opened, circuit is



$$\left[\frac{(x-V)}{-2V} \right] C + (x-2V)C + xC = 0$$

or $x = \frac{5}{3}V$

Charge on plate 4 is $-\frac{5}{3}CV = -\frac{5\epsilon_0 AV}{3d}$

Alternative method:

$$\frac{-q_1}{C} + V + \frac{q_2}{C} = 0 \quad \text{or} \quad -q_1 + q_2 = -CV \quad \dots(i)$$

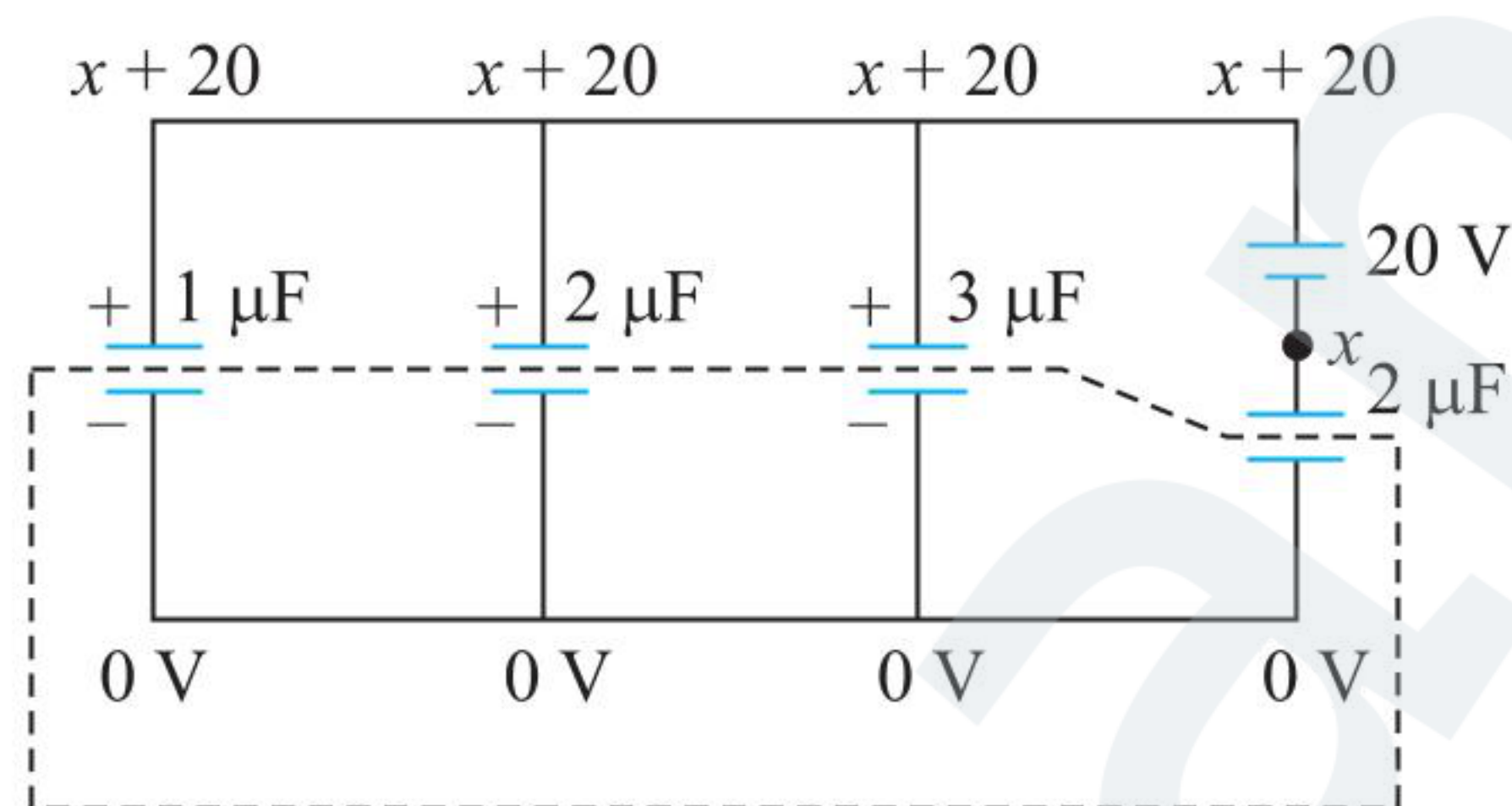
$$\frac{-q_1}{C} - \frac{(q_1 + q_2)}{C} + 2V = 0$$

or $q_1 + 2q_2 = 2CV \quad \dots(ii)$

$$q_1 = \frac{4CV}{3}, q_2 = \frac{CV}{3}$$

33. (3), 34. (1)

The circuit can be redrawn as



Conserving charges on lower plates, we get

$$-[(x+20)1 + (x+20)2 + (x+20)3 + 2x] = -60$$

or $x = -\frac{15}{2}V$

$$|Q_{1\mu F}| = 12.5 \mu C$$

$$|Q_{2\mu F}| = 25 \mu C$$

$$|Q_{3\mu F}| = 37.5 \mu C$$

$$|Q_{2\mu F}| = 15 \mu C$$

Matrix Match Type

1. i. → a., c.; ii. → b.; iii. → a., c.; iv. → d.

- By inserting dielectric slab, capacitance of 1 increases thereby increasing charge on capacitor. Thus, more charge flows through the battery. Energy stored in capacitor also increases.
- By increasing separation between the plates, capacitance of C_1 decreases. Charge on C_2 also decreases.

(iii) By shorting capacitor 1, only capacitor 2 remains in the circuit. Potential difference across C_2 increases, thereby increasing charge on 2 as well as energy stored.

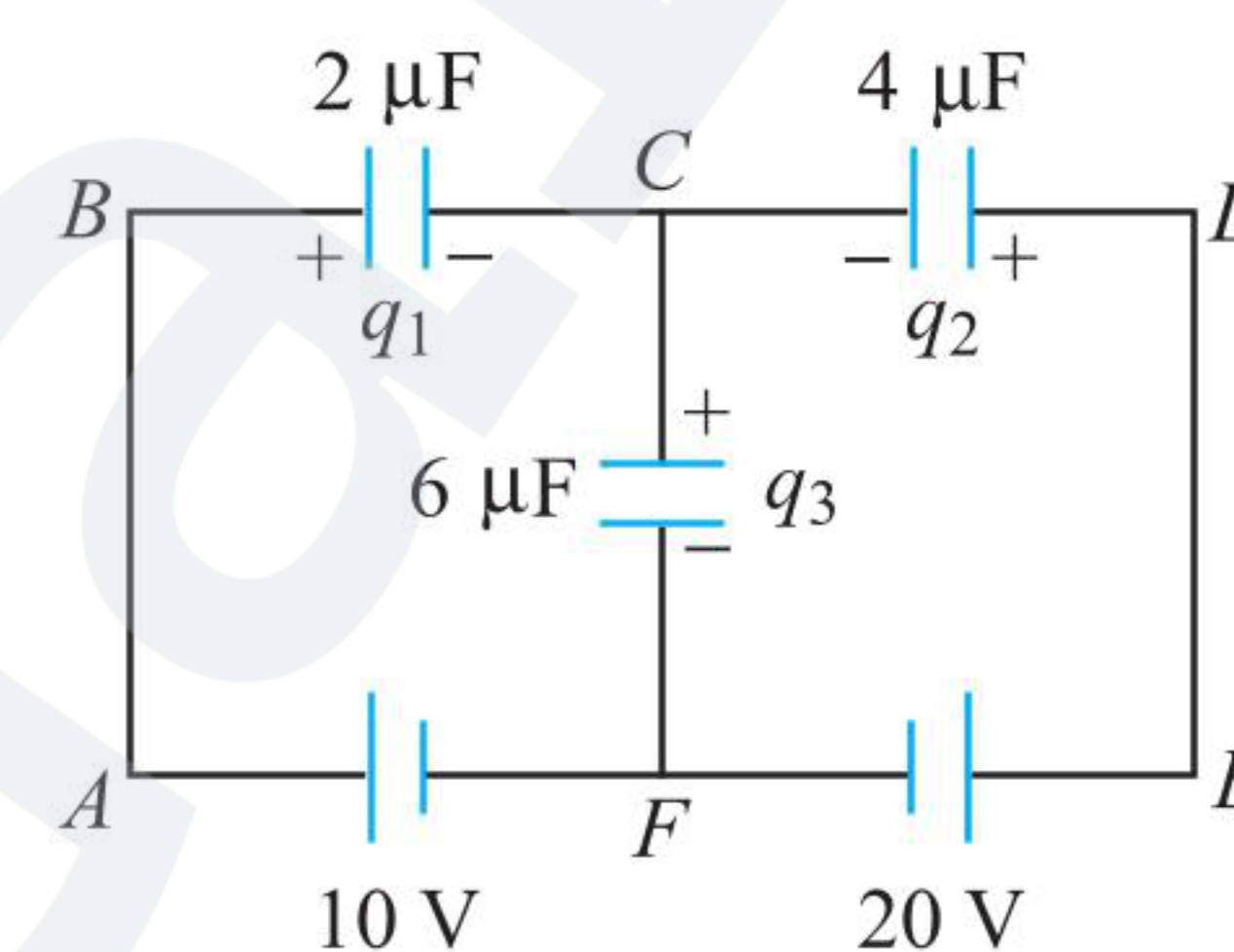
(iv) By earthing plate of capacitor 1, potentials will change but there will be no change in potential difference, making no overall change in the circuit.

2. i. → b.; ii. → c.; iii. → a.; iv. → d.

The charges on three plates that are in contact add to zero, because these plates taken together form an isolated system, which cannot receive charges from the batteries. Thus, $q_3 - q_1 - q_2 = 0$. Applying Kirchhoff's law in loop $ABCF$ and $CDEFC$, we get

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

or $q_3 + 3q_1 = 60$



and $20 - \frac{q_2}{4} - \frac{q_3}{6} = 0$

or $3q_2 + 2q_3 = 240 \quad \dots(iii)$

Solving the above three equations, we have

$$q_1 = \frac{10}{3} \mu C, q_2 = \frac{140}{3} \mu C, q_3 = 50 \mu C$$

Potential difference across 6 μF capacitor is

$$\frac{q_3}{6} = \frac{50 \mu C}{6 \mu F} = \frac{25}{3} V$$

3. i. → b.; ii. → a.; iii. → d.; iv. → c.

These five plates constitute four identical capacitors in parallel, each of capacity $\epsilon_0 A/d$. Now, as plate 1 is connected to the positive terminal of the battery and is a part of one capacitor only, so charge on it is

$$q_1 = + \left(\frac{\epsilon_0 AV}{d} \right)$$

So, i. → b.

However, plate 4 is connected to the negative terminal of the battery and is common to the two identical capacitors in parallel. So,

$$q_4 = - \frac{2\epsilon_0 AV}{d}$$

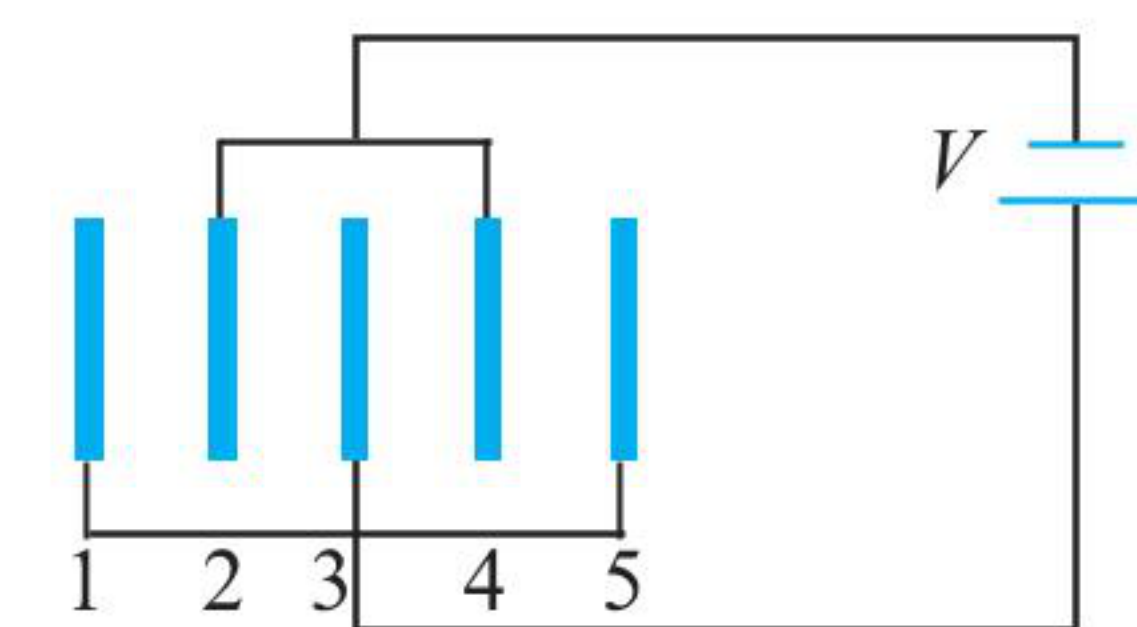
Hence, ii. → a.

From the circuit, it is obvious that between the plates 2 and 3, the battery is connected, so the potential difference will be V . Plates 1 and 5 get connected through connecting wire, so the potential difference between 1 and 5 is zero.

Hence, iii. → d. iv. → c.

4. i. → b., d.; ii. → a., c., d.; iii. → a., c., d.; iv. → a., c., d.

Charge on the outer surfaces of the plates of capacitor will be always zero. If $V = E$, then no charge will flow in the circuit and hence no thermal energy will be dissipated. But if $V \neq E$, then charge will flow in the circuit and thermal energy will be dissipated.



5. i. → a., c.; ii. → b.; iii. → b., d.; iv. → b.

(i) Initial potential difference across C_1

$$V_1 = \frac{4V}{2+4} = \frac{2V}{3}$$

On doubling the distance between plates, C_1 becomes half. So, final potential difference across C_1

$$V'_1 = \frac{4V}{1+4} = \frac{4V}{5}$$

This increases by a factor $\frac{V'_1}{V_1} = \frac{4V/5}{2V/3} = \frac{6}{5}$

(ii) Across C_2

$$\text{Initially } V_2 = \frac{2V}{2+4} = \frac{V}{3}$$

$$\text{Finally } V'_2 = \frac{1V}{1+4} = \frac{V}{5}$$

This decreases by a factor of

$$\frac{V'_2}{V_2} = \frac{V/5}{V/3} = \frac{3}{5}$$

(iii) Energy in C_1

$$\text{Initially } U_1 = \frac{1}{2} \times 2 \left(\frac{2V}{3} \right)^2 = \frac{4V^2}{9}$$

$$\text{Finally } U'_1 = \frac{1}{2} \times 1 \times \left(\frac{4V}{5} \right)^2 = \frac{8V^2}{25}$$

This decreases by a factor

$$\frac{U'_1}{U_1} = \frac{8V^2/25}{4V^2/9} = \frac{18}{25}$$

(iv) Energy in C_2

$$\text{Initially } U_2 = \frac{1}{2} \times 4 \left(\frac{V}{3} \right)^2 = \frac{2V^2}{9}$$

$$\text{Finally } U'_2 = \frac{1}{2} \times 4 \left(\frac{V}{5} \right)^2 = \frac{2V^2}{25}$$

This decreases by a factor $\frac{U'_2}{U_2} = \frac{9}{25}$

6. i. → d.; ii. → d.; iii. → a.; iv. → a.;

(i) Voltage gets distributed among all capacitors.

(ii) The one with least capacitance

$$\Rightarrow \frac{C}{3}$$

(iii) The one with maximum capacitance

$$\Rightarrow 3C$$

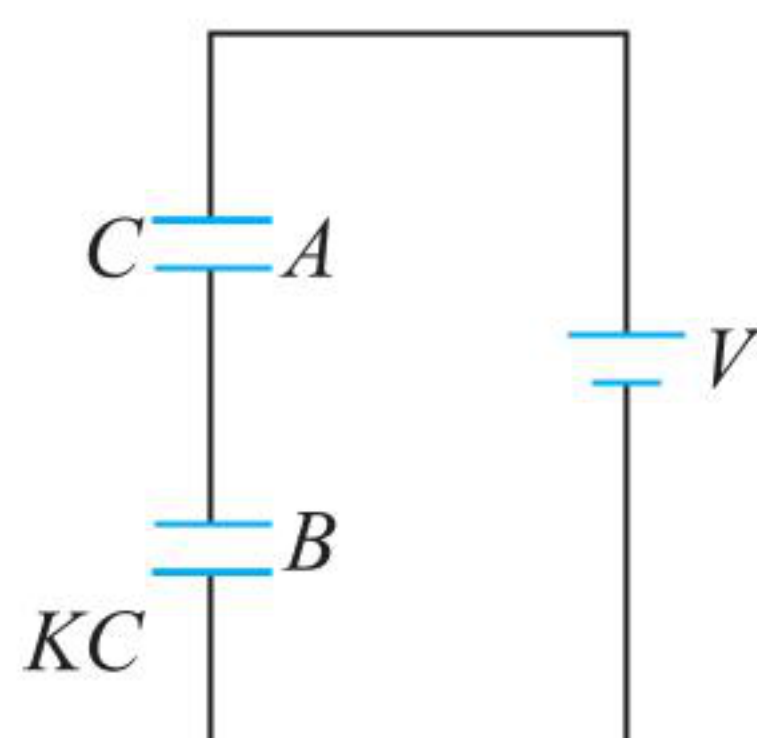
7. i. → b., d.; ii. → b., d.; iii. → b., d.; iv. → b., d.

As slab is inserted, capacitance of B increases and hence total capacitance of system increases as a result change on both increase.

$$Q = \frac{KC}{1+K} \times V$$

$$\text{or } V_A = \frac{Q}{C} = \frac{KV}{1+K} > \frac{V}{2}$$

$$V_B = \frac{Q}{KC} = \frac{V}{1+K} < \frac{V}{2}$$



8. i. → a.; ii. → b.; iii. → b.; iv. → c.

$$O = \frac{KQ}{2R} + \frac{K_C 2Q}{3R} + \frac{Kq_B}{2R}$$

$$q_B = -\left(Q + \frac{4Q}{3}\right) = -\frac{7Q}{3}$$

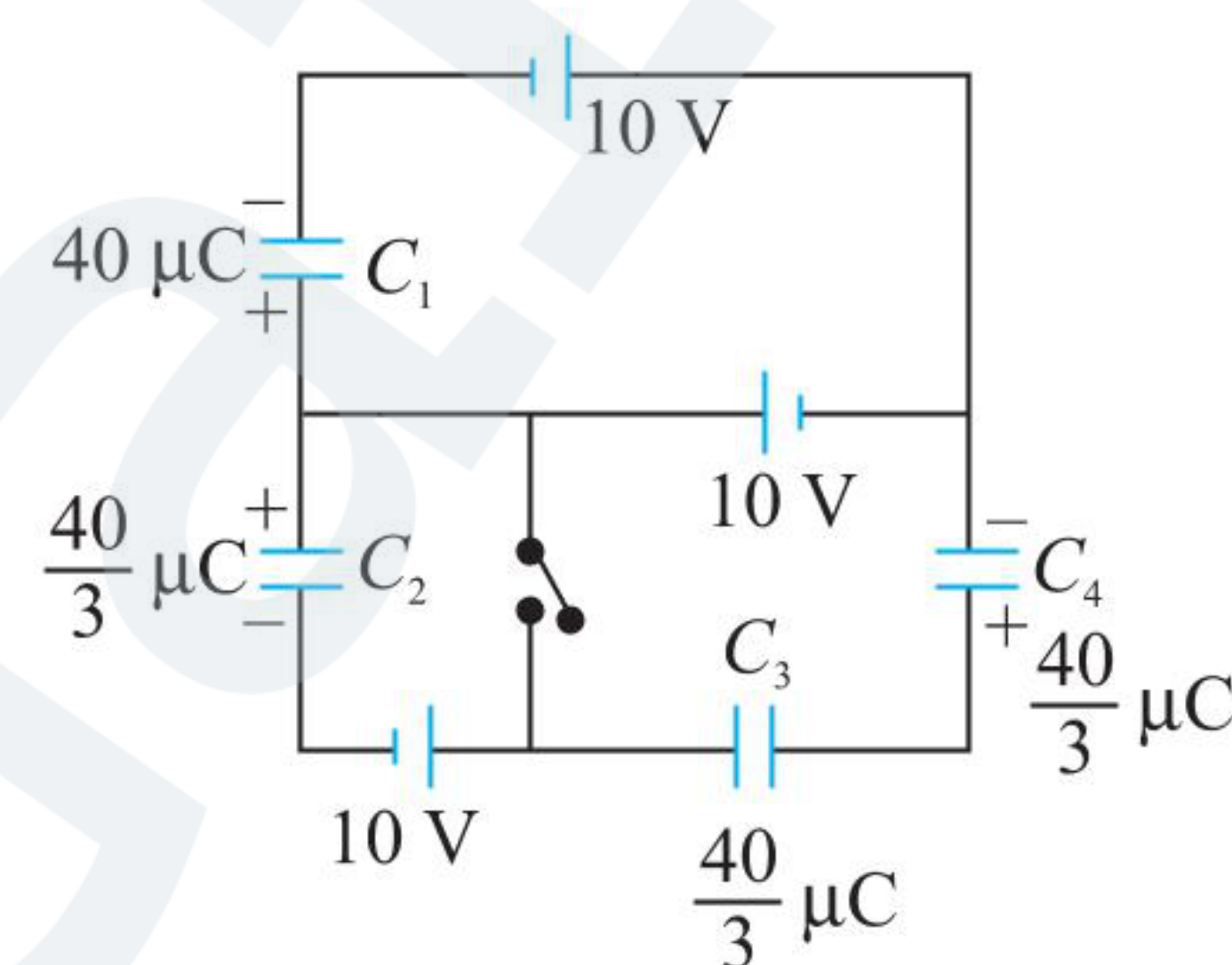
$$q_1 = -Q_A = -q$$

$$q_2 = -\frac{7Q}{3} + q = -\frac{4q}{3}$$

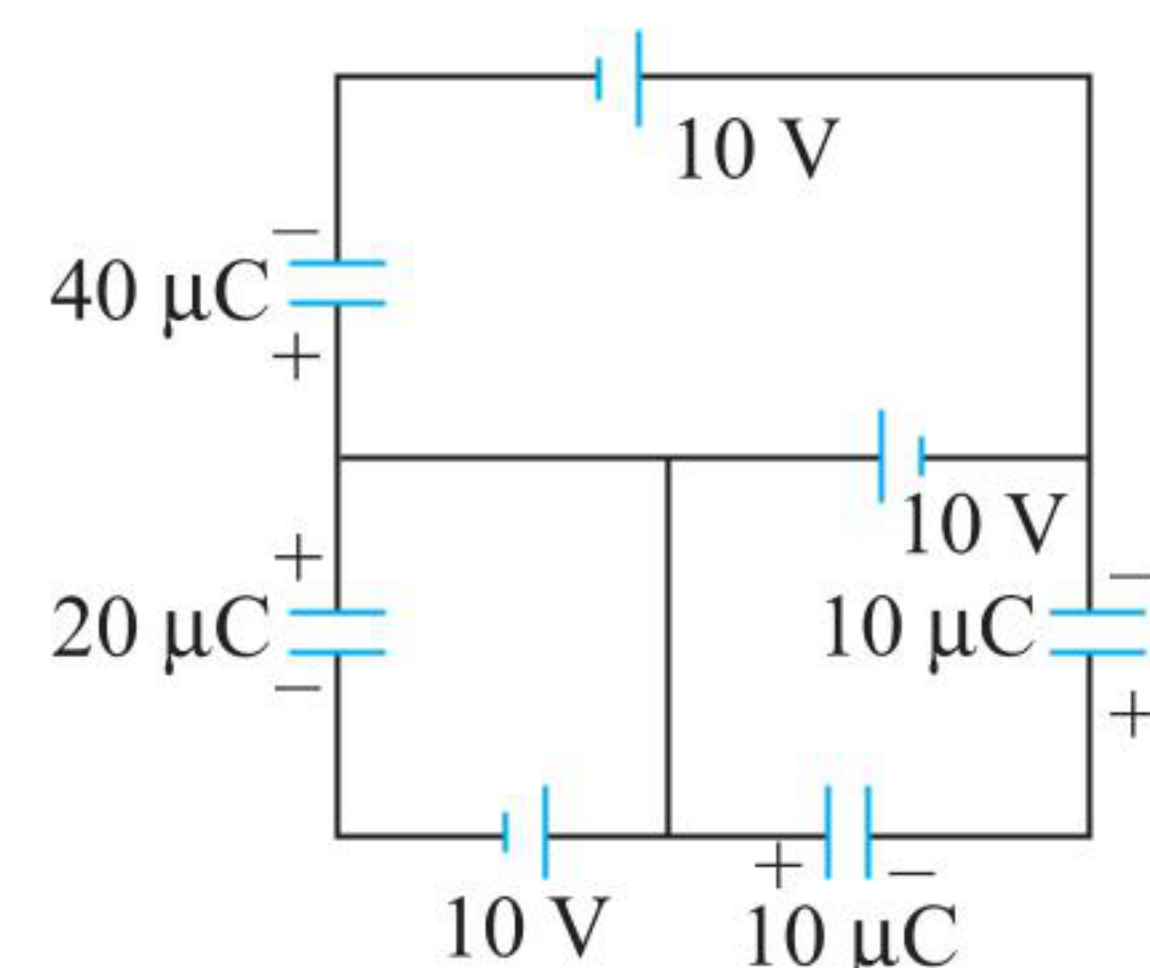
$$q_3 = \frac{4q}{3} \text{ and } q_3 + q_4 = 2q \text{ or } q_4 = -\frac{2q}{3}$$

9. i. → c.; ii. → a.; iii. → b.; iv. → b.

Before closing the switch:



After closing the switch



So charge on C_1 remains same, on C_2 increases, and on C_3 and C_4 decreases.

10. i. → a.; ii. → c.; iii. → b.; iv. → a.

The initial charge on capacitor is

$$q_i = CV_i = 1 \times 2 = 2 \mu\text{C}$$

The final charge on capacitor is

$$q_f = CV_f = 1 \times 4 = 4 \mu\text{C}$$

Net charge crossing the cell of emf 4 V is

$$q_f - q_i = 4 - 2.2 \mu\text{C}$$

The magnitude of work done by cell of emf 4 V is

$$W = (q_f - q_i) 4 = 8 \mu\text{J}$$

The gain in potential energy of capacitor is

$$\Delta U = \frac{1}{2} C (V_f^2 - V_i^2) = \frac{1}{2} \times [4^2 - 2^2] \mu\text{J} = 6 \mu\text{J}$$

Net heat produced in circuit is

$$\Delta H = W - U = 8 - 6 = 2 \mu\text{J}$$

Numerical Value Type

1. (4) To determine charge on $6 \mu\text{F}$, we can remove $3 \mu\text{F}$ from the circuit. After this,

$$C_{\text{eq}} = 2 \mu\text{F}$$

$$Q = C_{\text{eq}} V = 2 \times 2 = 4 \mu\text{C}$$

2. (4) $\Delta V_{AP} = \frac{Q}{3}$, $\Delta V_{PB} = \frac{Q}{6}$ [$\because (4+2)\mu\text{F} = 6\mu\text{F}$]

$$\text{Also, } \frac{Q}{3} + \frac{Q}{6} = 12 \text{ or } Q = 24 \mu\text{C}$$

$$\Delta V_{AP} = \frac{24}{3} = 8 \text{ or } V_A - V_P = 8$$

$$\text{or } 12 - V_P = 8 \text{ or } V_P = 4 \text{ V}$$

$$3. (2) \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}} = \frac{4}{3} \left(\frac{\epsilon_0 A}{d} \right). \text{ Solve to get } k = 2.$$

4. (6) Volume of eight drops = Volume of big drop

$$8 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \text{ or } r = \frac{R}{2}$$

$$C = 4\pi\epsilon_0 R = 12 \mu\text{F}$$

Thus, capacitance of smaller drop is

$$C = 4\pi\epsilon_0 r = \frac{4\pi\epsilon_0 R}{2} = \frac{12}{2} = 6 \mu\text{F}$$

$$5. (4) \text{ Initial energy: } E = \frac{1}{2} C_0 V_0^2$$

(i) $E_1 = 2E$, because charge remains the same and the capacitance becomes half.

(ii) $E_2 = E/2$, because potential remains the same and the capacitance becomes half. So

$$\frac{E_1}{E_2} = 4$$

6. (9) Maximum charge C_1 can hold

$$Q_1 = C_1 V_1 = 1 \times 10^{-6} \times 6 \times 10^3 = 6 \times 10^{-3} \text{ C}$$

and maximum charge C_2 can hold

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 4 \times 10^3 = 8 \times 10^{-3} \text{ C}$$

When connected in series, both will have equal charges and so each can have charge Q_1 , which is smaller of the two. In this case, voltage across C_1 is 6 kV. And voltage across C_2 is

$$\frac{Q_1}{C_2} = \frac{6 \times 10^{-3}}{2 \times 10^{-6}} = 3 \text{ kV}$$

Therefore, the maximum voltage across the system is $6 + 3 = 9 \text{ kV}$.

7. (22) Let $C_1 = C$ and $C_2 = 2C$ and charges on different capacitors have been shown. Net charge on isolated system should be zero. Hence,

$$q_1 - q_2 - q_3 = 0 \quad \dots(i)$$

For left loop,

$$E - \frac{q_3}{2C} - \frac{q_1}{C} = 0 \quad \dots(ii)$$

$$\text{or } E - \frac{q_3}{2C} - \left(\frac{q_2 + q_3}{C} \right) = 0 \text{ (using } q_1 = q_2 + q_3 \text{)}$$

$$\text{or } E - \frac{q_3}{2C} - \frac{q_2}{C} - \frac{q_3}{C} = 0$$

$$\text{or } E - \frac{q_2}{C} - \frac{3q_3}{2C} = 0 \quad \dots(iii)$$

For right loop,

$$E - \frac{q_2}{2C} - \frac{q_3}{2C} = 0$$

$$\text{or } 2E - \frac{q_2}{C} + \frac{q_3}{C} = 0 \quad \dots(iv)$$

Solving Eqs. (iii) and (iv), we get

$$q_3 = \frac{-2CE}{5}$$

$$\text{So } V_{MN} = \frac{-q_3}{2C} = \frac{2CE}{5 \times 2C} = \frac{E}{5} = \frac{110}{5} = 22 \text{ V}$$

8. (5) The distribution of charge is shown in figure in compliance with the point rule. Apply loop rules.

$$\text{Left loop: } -\frac{q_2}{5} + \frac{q_3}{0.75} + \frac{q_1}{15} = 0$$

$$\text{or } q_1 - 3q_2 + 20q_3 = 0 \quad \dots(i)$$

$$\text{Middle loop: } -\frac{q_2 + q_3}{15} - \frac{q_3}{0.75} + \frac{q_1 - q_3}{5} - \frac{q_3}{0.75} = 0$$

$$\text{or } 3q_1 - q_2 - 44q_3 = 0 \quad \dots(ii)$$

$$\text{Outermost loop: } 23 - \frac{q_2}{5} - \frac{q_2 + q_3}{15} - \frac{q_2}{5} = 0$$

$$\text{or } 345 = 7q_2 + q_3 \quad \dots(iii)$$

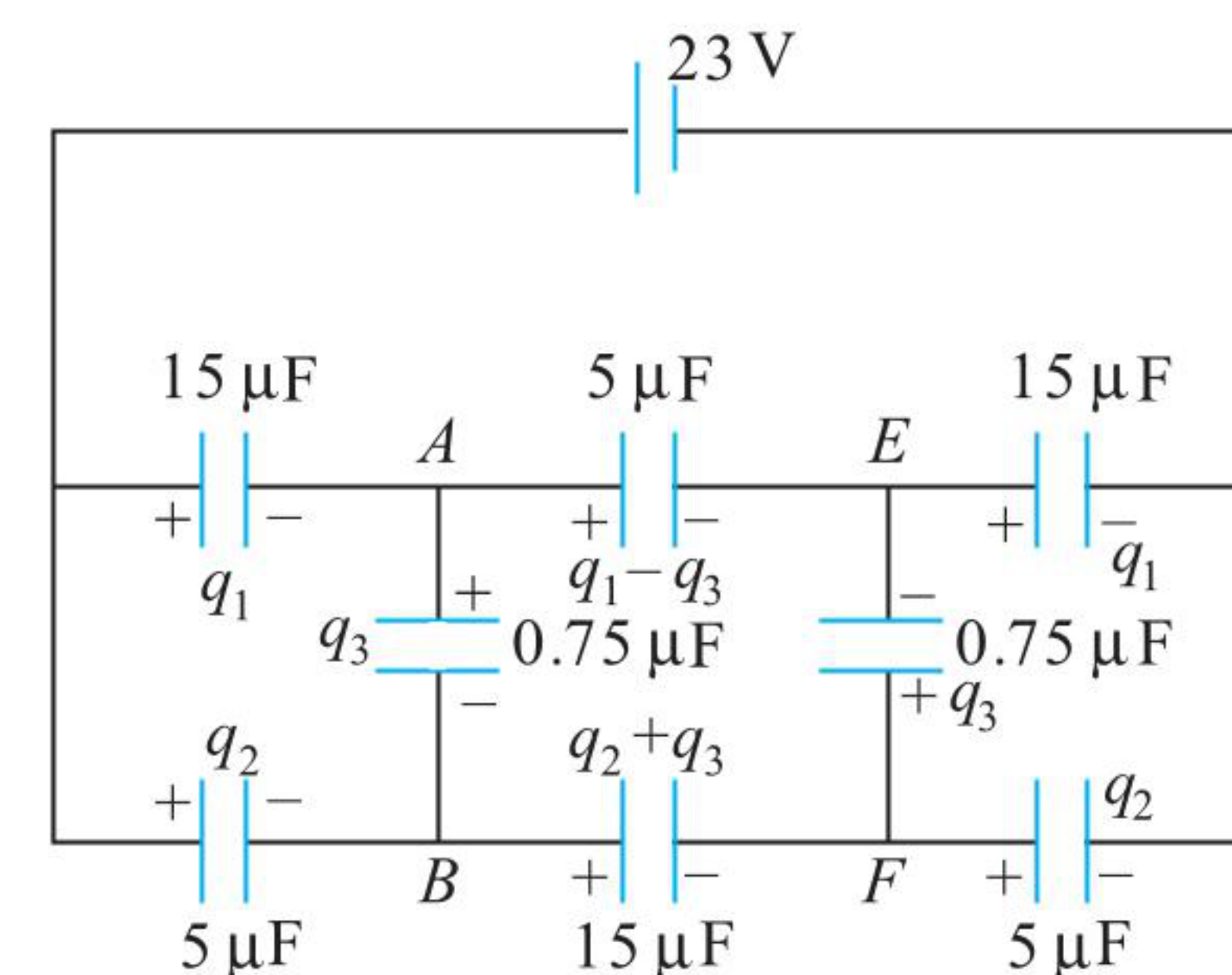
Solving for q_1 , q_2 , and q_3 , we get

$$q_1 = \frac{19 \times 345}{92}, q_2 = \frac{13 \times 345}{92}, q_3 = \frac{345}{92}$$

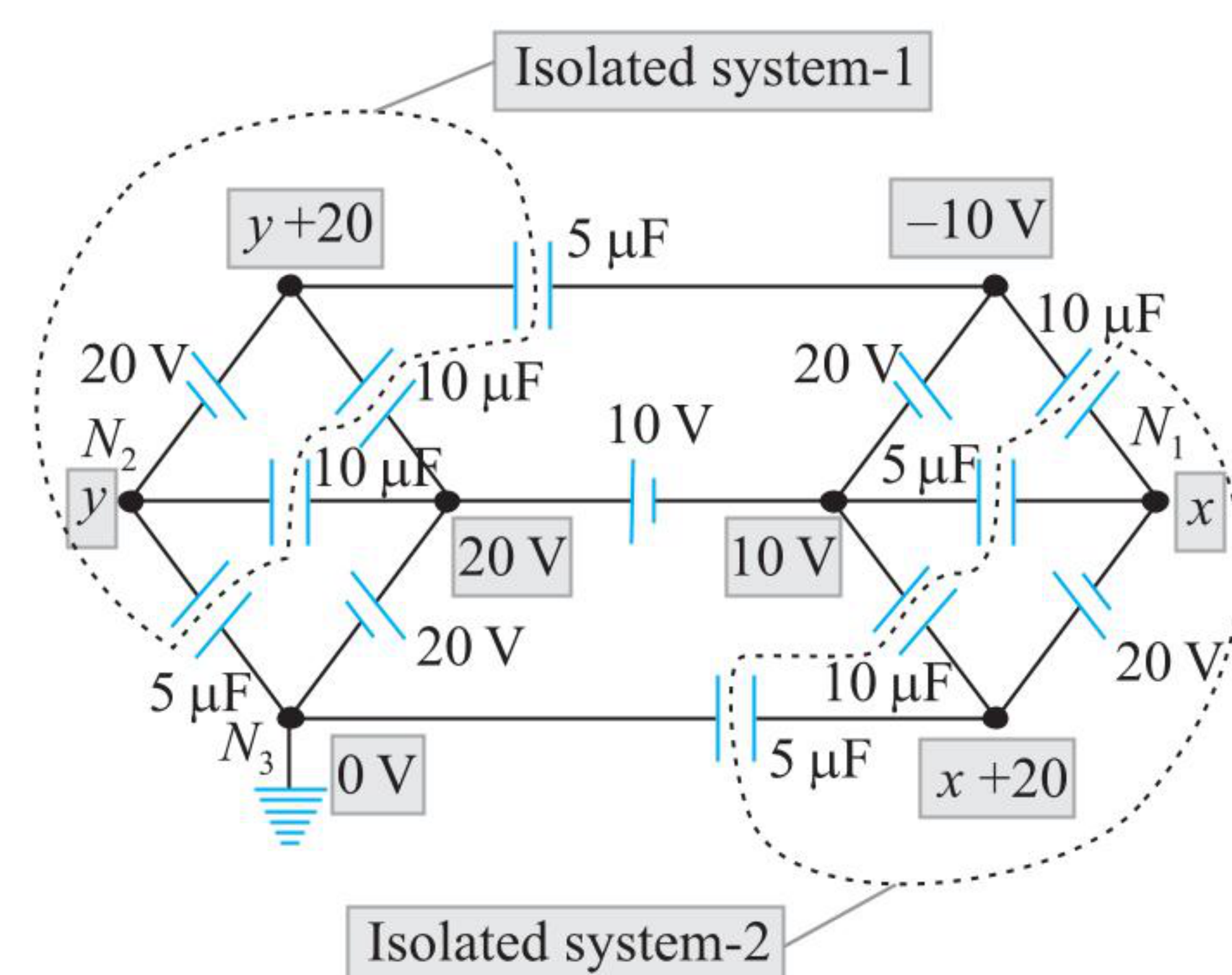
Therefore, potential difference between A and B is

$$\frac{q_3}{0.75} = \frac{345}{92} \times \frac{4}{3} = 5 \text{ V}$$

Potential difference between E and F is also 5 V but in the opposite direction.



9. (10) We assign node N_3 as 0 V and N_1 , N_2 as x , y volt, respectively. Conservation of charge in isolated system 1.



$$5(y - 0) + 10(y - 20) + 10[(y + 20) - 20] + 5[(y + 20) - (-10)] = 0$$

$$\text{or } 30y = 50 \text{ or } y = \frac{5}{3} \text{ V}$$

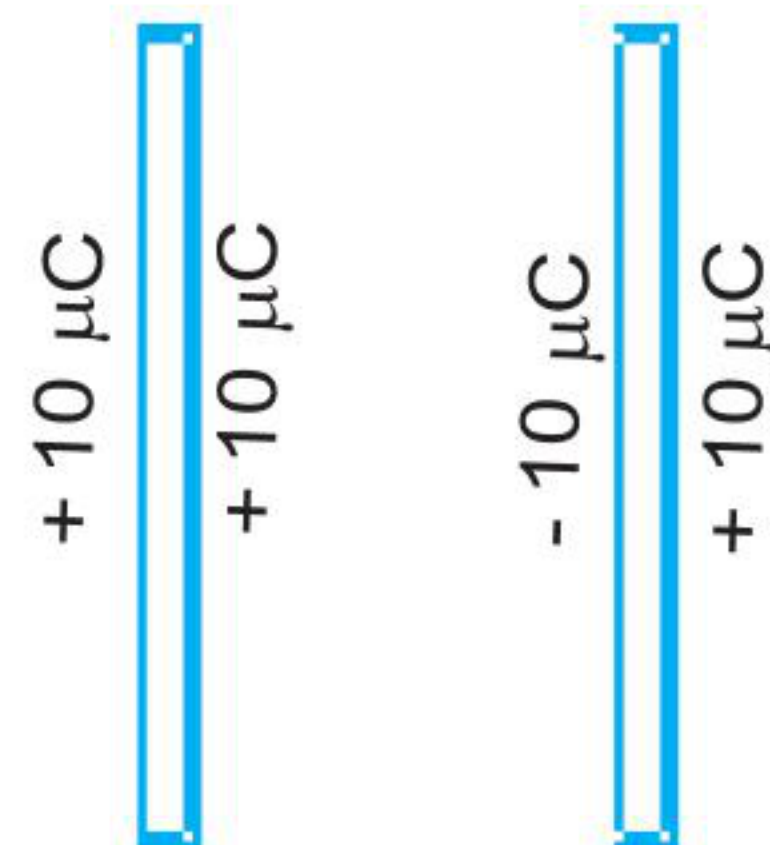
Conservation of charge in isolated system 2.

$$5[(x + 20) - 0] + 10[(x + 20) - 10] + 5[x - 10] + 10[x - (-10)] = 0$$

$$\text{or } 30x = -250 \text{ or } x = \frac{-25}{3} \text{ V}$$

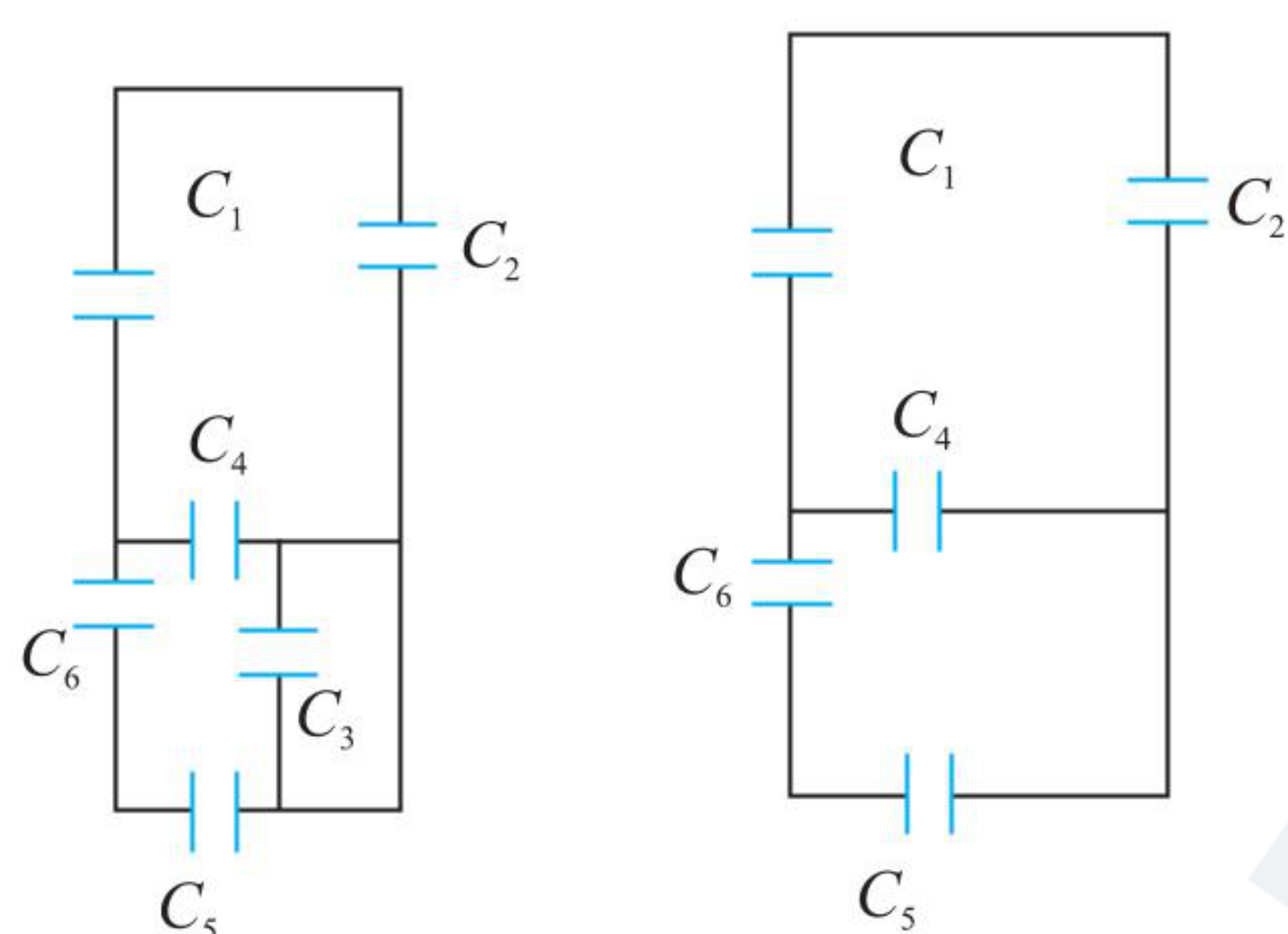
$$\text{Thus, } V_A - V_B = \frac{5}{3} - \left(\frac{-25}{3} \right) = 10 \text{ V}$$

10. (1) Given that capacitance $C = 10 \mu\text{F}$ and charge $= 20 \mu\text{C}$. But the effective charge $q = 10 \mu\text{C}$.



Potential difference is $q/C = 1 \text{ V}$.

11. (2) On joining two capacitors, charge on each will be half of the previous charge on the first capacitor. So energy stored on each capacitor will be one-fourth of energy stored in the previously in the first capacitor, that is, 1 J. Hence, the net energy stored in both will be $1 + 1 = 2 \text{ J}$.
12. (1.60) We begin with the circuit given, assigning names to each identical capacitor. C_1 begins with an unknown initial charge Q_0 , and achieves a charge Q after the circuit is connected and has re-established equilibrium.



As C_3 is clearly shorted, the potential difference across its plates is 0, and as such the capacitor will contain no charge and will not affect the circuit. We can therefore remove it from the schematic. This simplified circuit can be broken down into even simpler equivalent circuits. Leaving C_1 where it is, and recognizing C_5 and C_6 in series with each other (yielding an equivalent capacitance of $1/2 C$), and together in parallel with C_4 .

Acknowledging the combined capacitors are equal, C_{456} must have an equivalent capacitance of $3/2 C$. As C_{456} is simply in series with C_2 , the resultant equivalent circuit has simply one capacitor (C_{2456}) in series with C_1 , where C_{2456} must have an equivalent capacitance of $3/5 C$.

Since $Q = CV$, $V_1 = Q/C$

$$V_{2456} = 5Q'/3C,$$

where V_1 is the potential difference across C_1 , V_{2456} is the potential difference across C_{2456} , and Q' is the charge on C_{2456} .

For equilibrium conditions, $V_1 = V_2$. Therefore,

$$Q/C = 5Q'/3C, \quad 3CQ = 5CQ'$$

$$Q' = (3/5)Q.$$

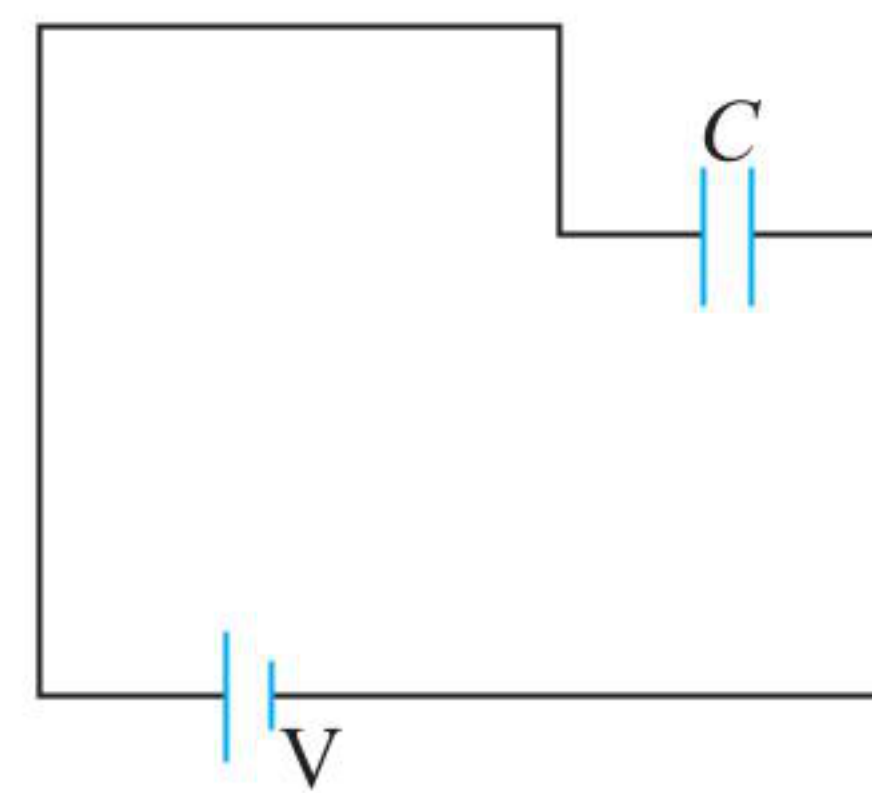
By the conservation of charge,

$$Q_0 = Q + Q' = Q + (3/5)Q = (8/5)Q.$$

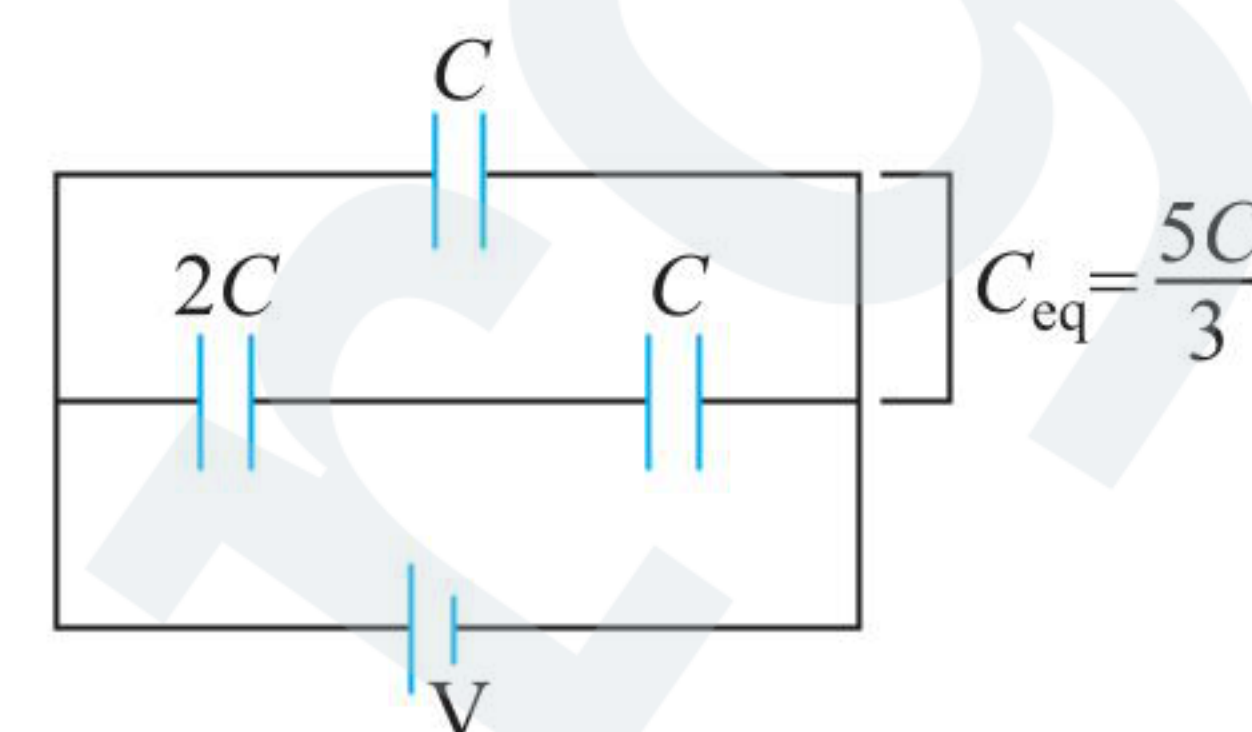
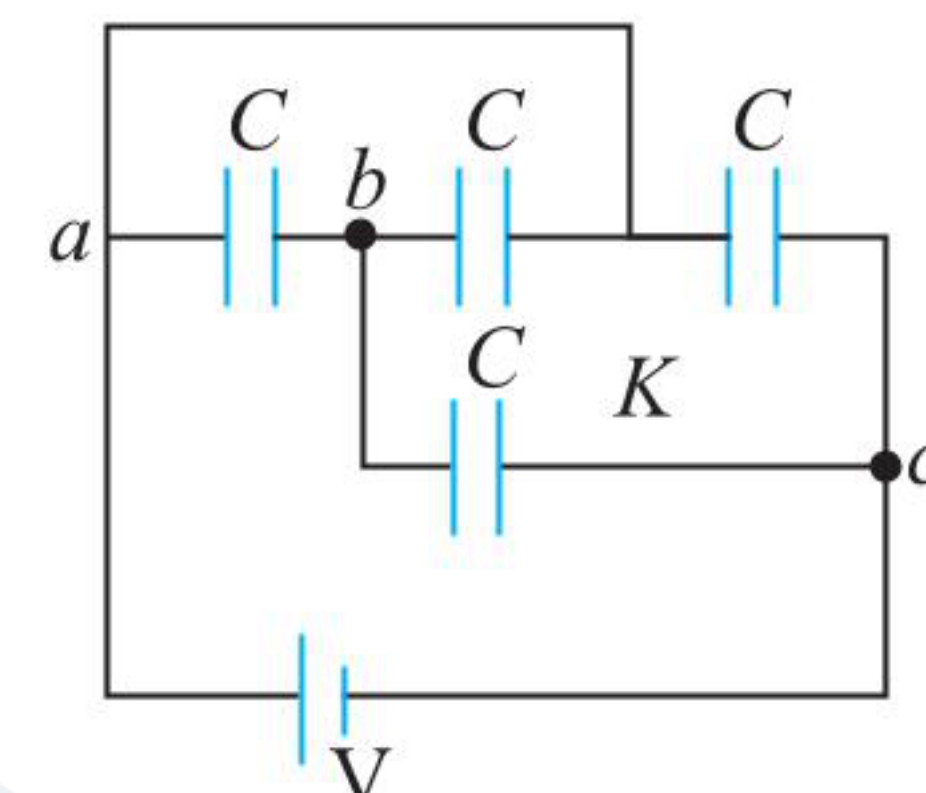
The initial charge on the capacitor was $8/5$ of the final charge on the capacitor after the switch was closed.

13. (100) We can redraw the circuit with K is open and when K is closed as shown in figures below

With key open circuit is



With key closed



Initial and final energies stored in capacitors are

$$U_i = \frac{1}{2} CV^2 \Rightarrow U_f = \frac{1}{2} \left(\frac{5C}{3} \right) V^2$$

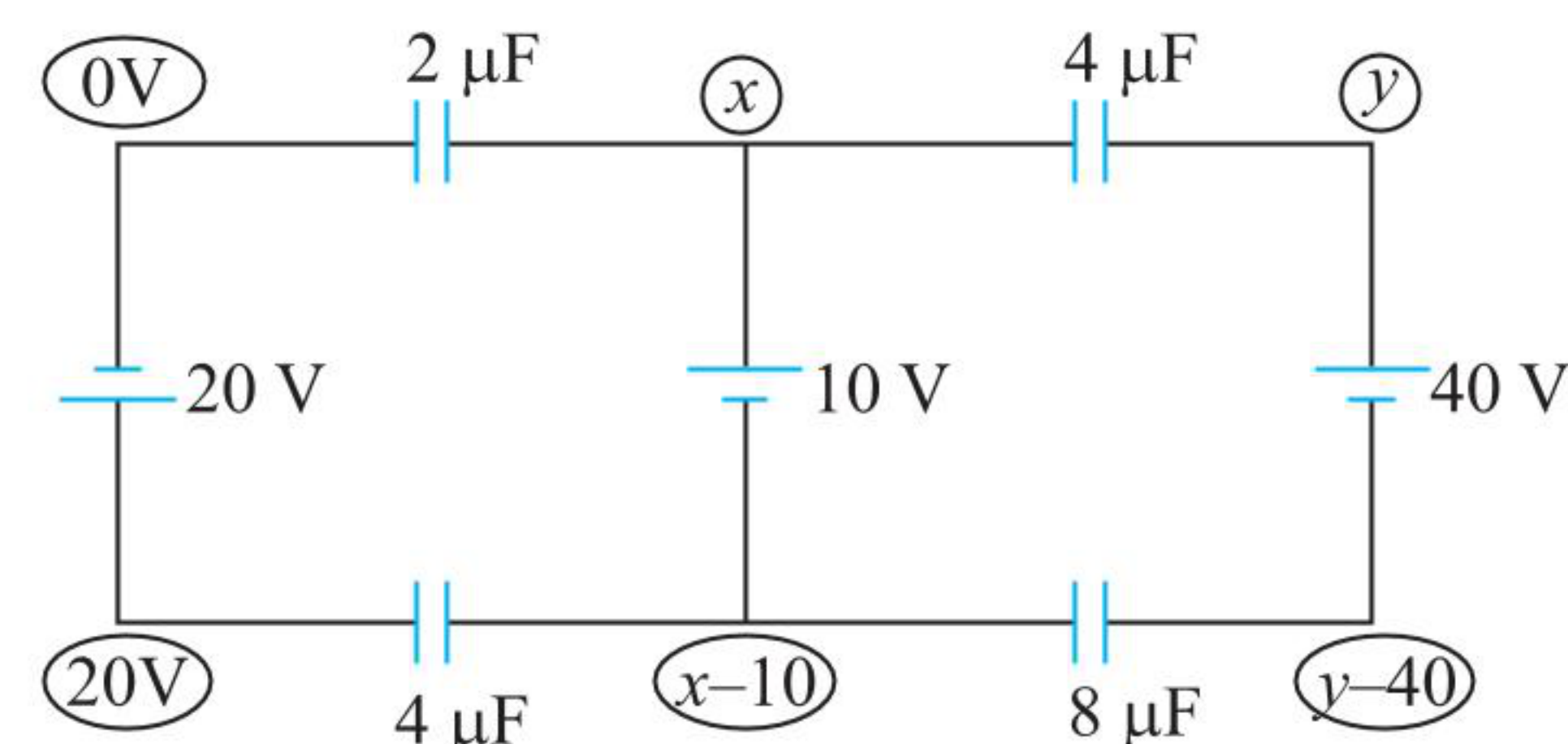
Change in stored energy is given as

$$\Delta U = U_f - U_i = \frac{1}{2} \left(\frac{5C}{3} - C \right) V^2$$

$$\Rightarrow \Delta U = \frac{1}{2} \left(\frac{2C}{3} \right) V^2 = \frac{1}{3} CV^2$$

$$\Rightarrow \Delta U = \frac{1}{3} \times 3 \times 10^{-6} \times 10^2 = 100 \mu\text{J}$$

14. (40) In the circuit we distribute the potentials as shown in figure below with reference to a zero potential considered at negative terminal of 20 V battery.



Writing nodal equations for x and y gives

$$2x + 4(x - y) + 4(x - 10 - 20) + 8(x - 10 - y + 40) = 0$$

$$9x - 6y = -60 \Rightarrow 3x - 2y = -20 \quad \dots(i)$$

$$\text{and } 4(y - x) + 8(y - 40 - x + 10) = 0$$

$$3y - 3x = 60 \Rightarrow y - x = 20 \quad \dots(ii)$$

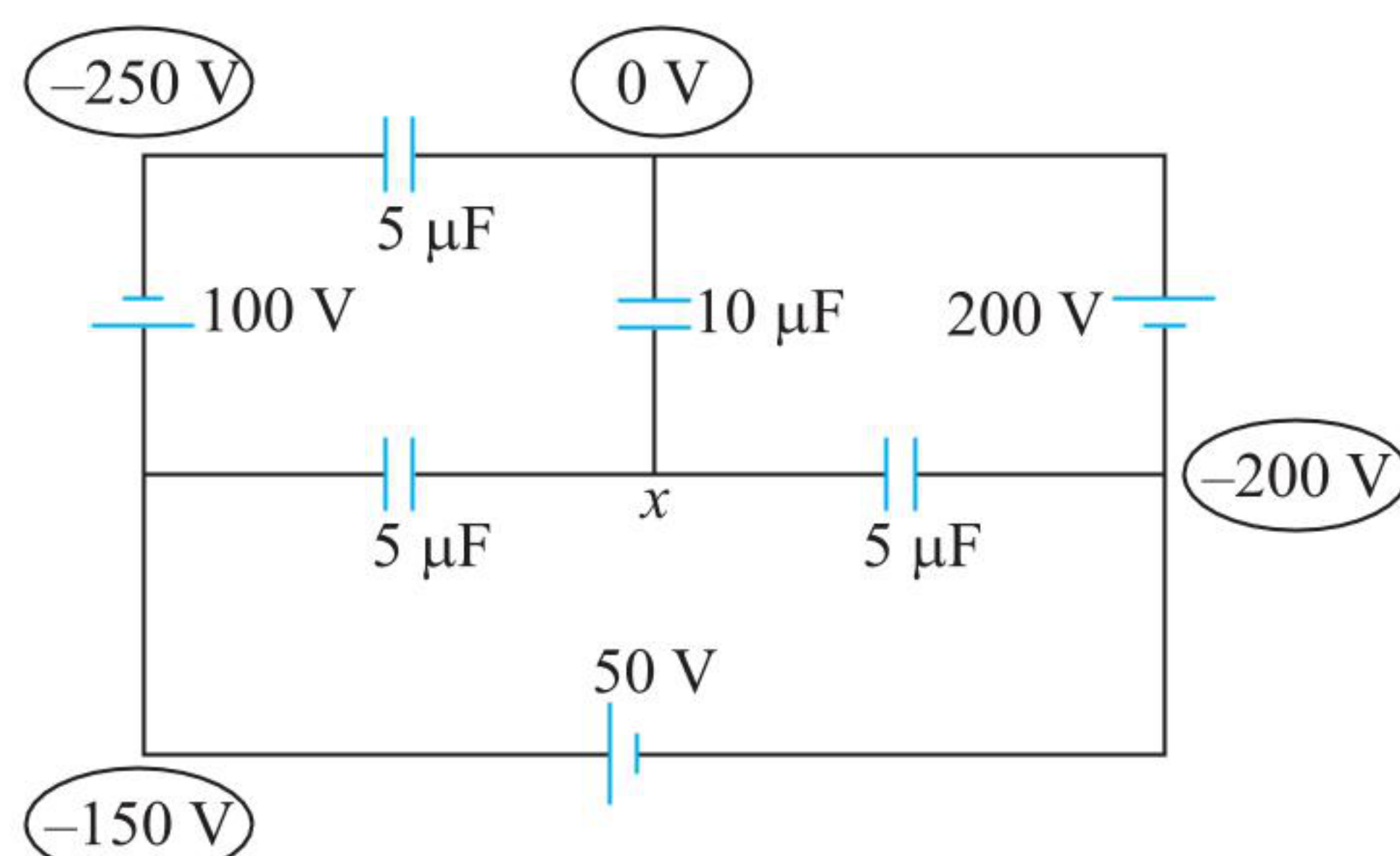
Solving Eqs. (i) and (ii) gives

$$x = 20\text{V}$$

This gives charge on $2 \mu\text{F}$ capacitor as

$$q_{2\mu\text{F}} = 2x = 2 \times 20 = 40 \mu\text{C}$$

15. (700) We distribute the potentials in the circuit shown in figure below.



Writing nodal equation for x gives

$$10x + 5(x + 200) + 5(x + 150) = 0$$

$$\Rightarrow 4x = -350 \Rightarrow x = -70 \text{ V}$$

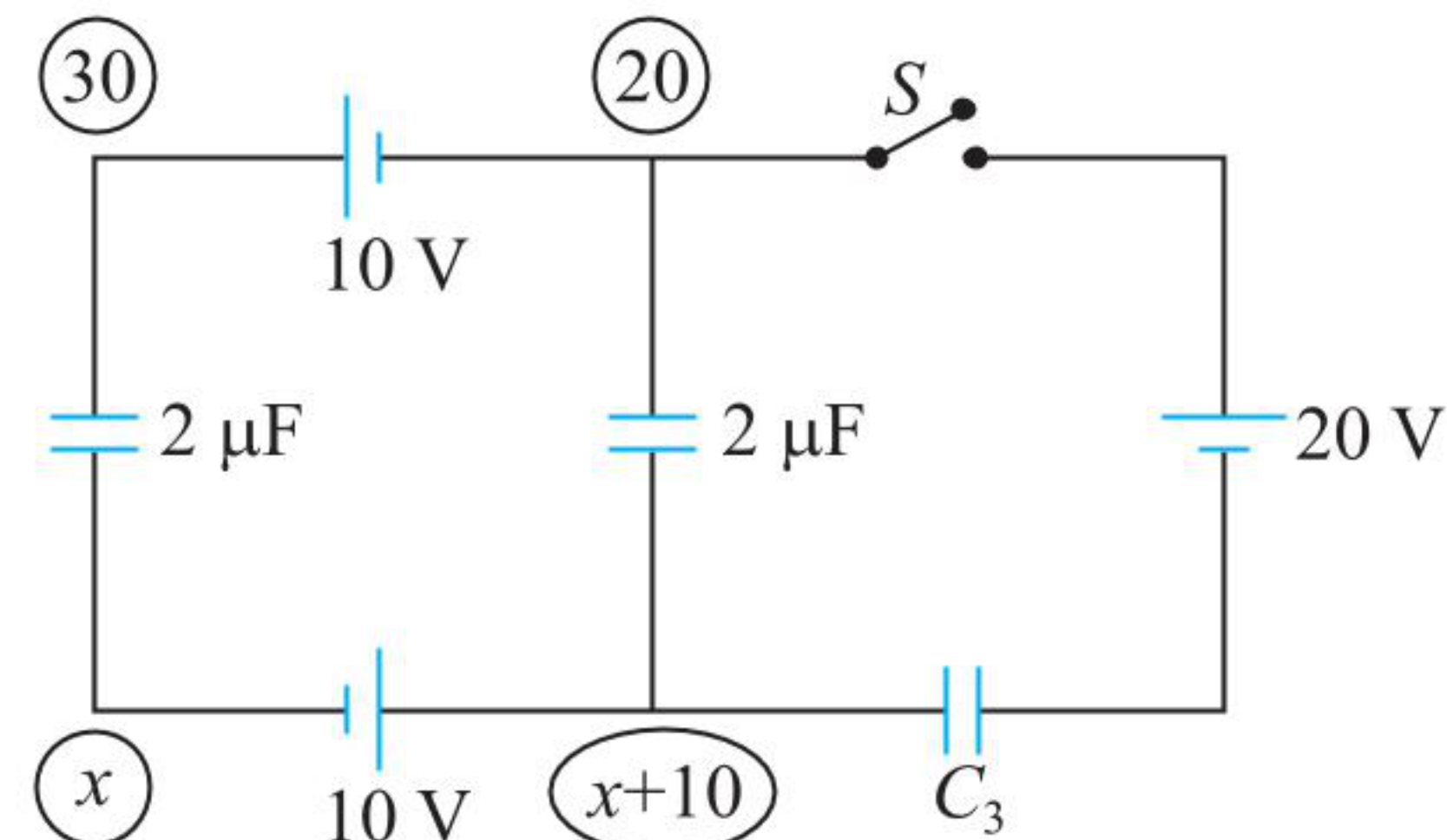
Thus charge on $10 \mu\text{F}$ capacitor is given as

$$q_{10 \mu\text{F}} = 10 \times 70 = 700 \mu\text{C}$$

16. (0.30) When switch is open charge on capacitors are

$$q_{C_1} = q_{C_2} = 1 \times 20 = 20 \mu\text{C} \Rightarrow q_{C_3} = 0$$

After closing the switch circuit is shown figure,



Writing nodal equation for x gives

$$2(x + 10) + 2(x + 10 - 20) + 2(x - 30) = 0$$

$$3x = 30 \Rightarrow x = 10 \text{ V}$$

Thus find charge on capacitors is

$$q_{C_1} = 2 \times 20 = 40 \mu\text{C} \Rightarrow q_{C_2} = 0$$

$$q_{C_3} = 2 \times 20 = 20 \mu\text{C}$$

Thus heat produced on closing the switch is

$$H = \frac{\Delta q_1^2}{2C_1} + \frac{\Delta q_2^2}{2C_2} + \frac{\Delta q_3^2}{2C_3} = 300 \mu\text{J} = 0.3 \text{ mJ}$$

17. (24) For area of the plate to be minimum, the separation between the plates must be minimum. The separation between the plates is limited by the breakdown strength of the dielectric.

For air capacitor $\frac{V}{d_{\min}} = E_{\text{air}}$ [E_{air} = Breakdown field for air]

$$\therefore d_{\min} = \frac{V}{E_{\text{air}}}$$

$$\text{Now } \frac{\epsilon_0 A_{\min}}{d_{\min}} = C$$

$$\Rightarrow A_{\min} = \frac{C}{\epsilon_0} \frac{V}{E_{\text{air}}}$$

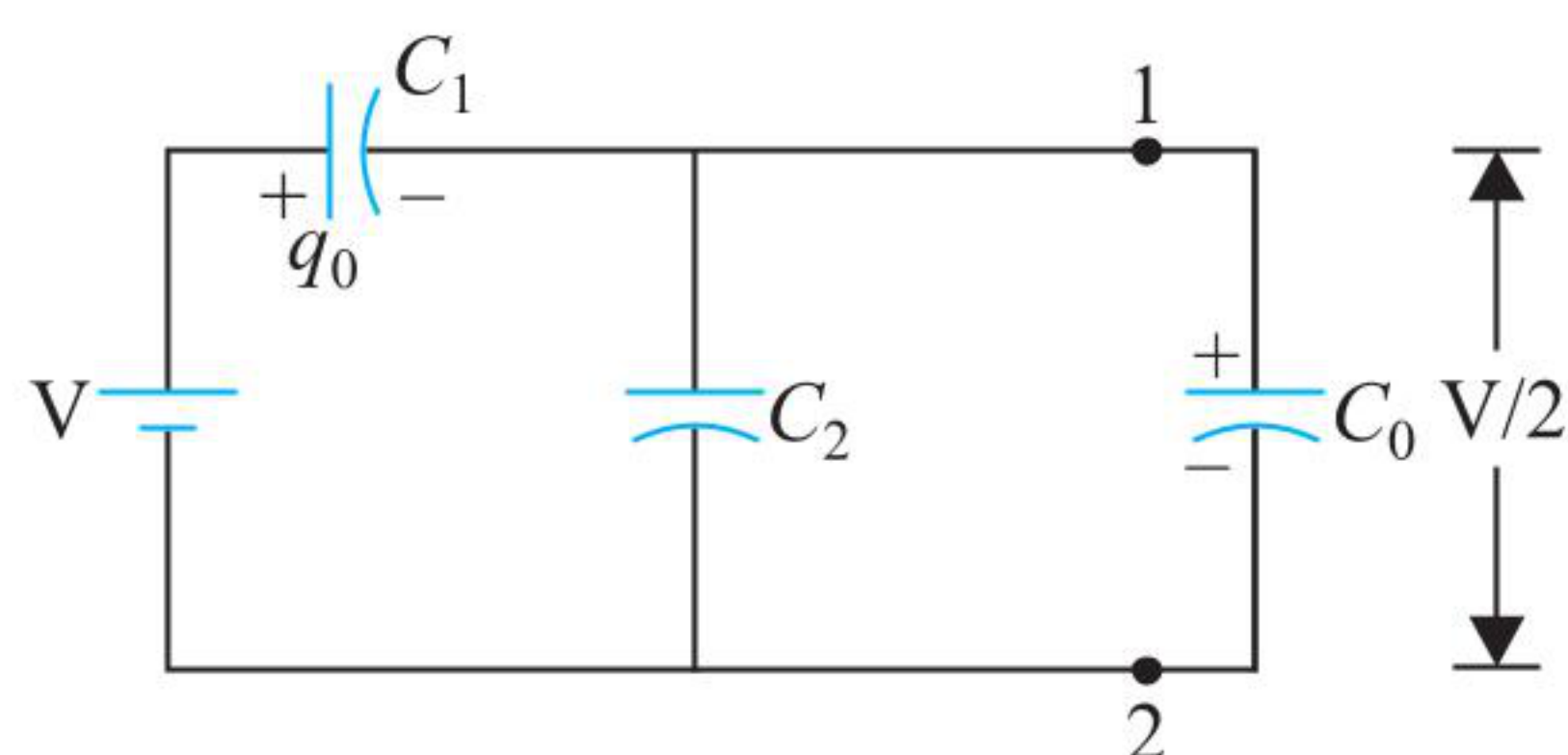
$$\Rightarrow A_1 = \frac{CV}{\epsilon_0 E_{\text{air}}}$$

With dielectric, similar calculation gives

$$A_2 = \frac{CV}{K\epsilon_0 E_{\text{dielect}}}$$

$$\therefore \frac{A_1}{A_2} = \frac{KE_{\text{dielect}}}{E_{\text{air}}} = 3 \times 8 = 24$$

18. (2) Let the equivalent capacitance be C_0 . Even if we remove one ladder, the equivalent will remain C_0 .



The potential drop across C_1 in figure shown is $\frac{V}{2}$.

$$\therefore \frac{q_0}{C_1} = \frac{V}{2} \Rightarrow q_0 = \frac{C_1 V}{2} \quad \dots(i)$$

$$\text{Also } q_0 = C_0 V \quad \dots(ii)$$

$$\therefore \frac{C_1}{2} = C_0 \Rightarrow C_1 = 2C_0 \quad \dots(iii)$$

From the given circuit

$$C_0 = \frac{(C_0 + C_2)C_1}{C_0 + C_2 + C_1}$$

$$\Rightarrow C_0^2 + C_2 C_0 - C_1 C_2 = 0$$

$$C_0 = \frac{-C_2 \pm \sqrt{C_2^2 + 4C_1 C_2}}{2}$$

Only positive sign is acceptable.

$$\therefore C_0 = \frac{-C_2 + \sqrt{C_2^2 + 4C_1 C_2}}{2}$$

$$\therefore \frac{C_1}{2} + \frac{C_2}{2} = \frac{\sqrt{C_2^2 + 4C_1 C_2}}{2} \quad \left[\because \frac{C_1}{2} = C_0 \right]$$

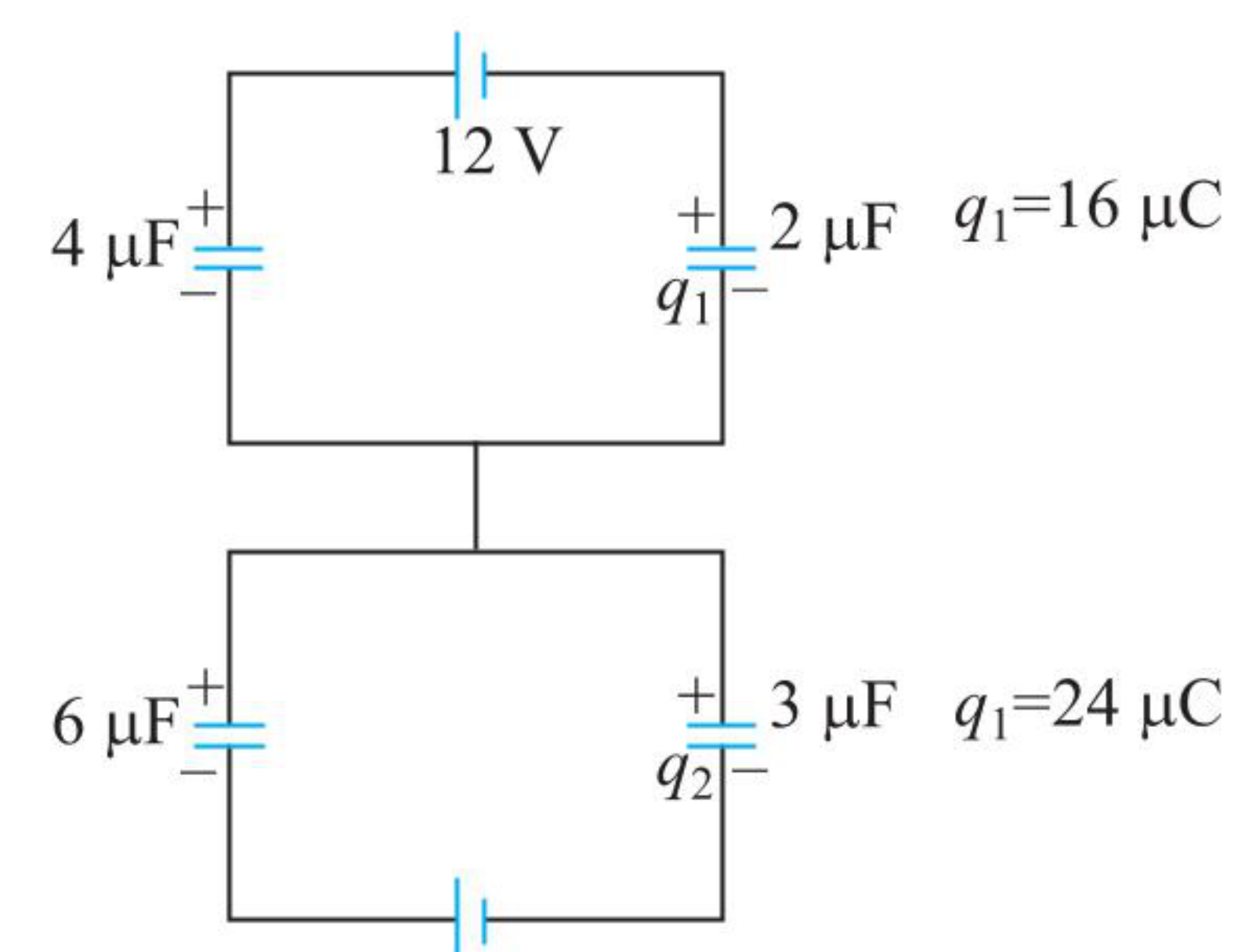
$$(C_1 + C_2)^2 = C_2^2 + 4C_1 C_2$$

$$C_1^2 = 2C_1 C_2 \Rightarrow C_1 = 2C_2$$

19. (240) With S open

Net emf = 0

$\therefore$ Charge on all capacitors = 0



With S closed

Energy stored in capacitors

$$U = 240 \mu\text{J}$$

Work done by cells = 480 μJ

Heat produced = 240 μJ

Archives

JEE Advanced

Single Correct Answer Type

$$1. (4) U_i = \frac{1}{2} (2)V^2 = V^2.$$

After connecting S to 2,

$$V_{\text{common}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{2V + 8 \times 0}{2 + 8} = \frac{V}{5}$$

$$U_f = \frac{1}{2} (2 + 8) \left(\frac{V}{5} \right)^2 = \frac{V^2}{5}$$

$$\begin{aligned}\text{Percent energy dissipated} &= \frac{U_i - U_f}{U_i} \times 100 \\ &= \frac{V^2 - \frac{V^2}{5}}{V^2} \times 100 = 80\%\end{aligned}$$

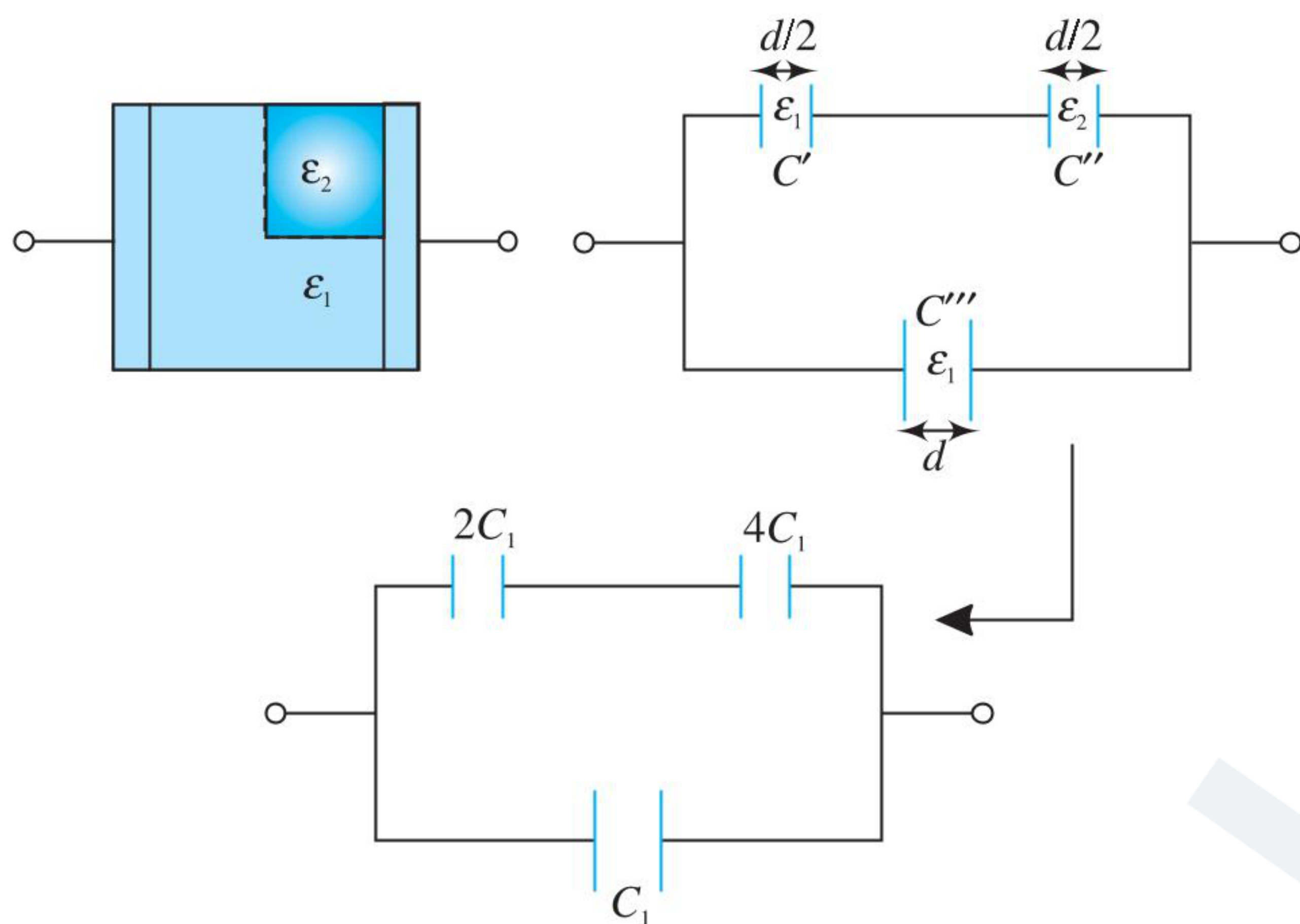
$$2. (3) \quad q_3 = \frac{C_3}{C_2 + C_3} Q$$

$$q_3 = \frac{3}{3+2} \times 80 = \frac{3}{5} \times 80 = 48 \mu\text{C}$$

3. (4) A capacitance of the capacitor without dielectric.

$$C_1 = \frac{\epsilon_0 A}{d}$$

Now consider the case with filled dielectric. The given capacitor can be divided into three capacitors as shown.



$$C' = \frac{2\epsilon_0 \frac{S}{2}}{d/2} = \frac{2\epsilon_0 S}{d} = 2C_1$$

$$C'' = \frac{4\epsilon_0 \frac{S}{2}}{d/2} = \frac{4\epsilon_0 S}{d} = 4C_1$$

$$C''' = \frac{2\epsilon_0 \frac{S}{2}}{d} = \frac{\epsilon_0 S}{d} = C_1$$

$$C_2 = \frac{C' C''}{C' + C''} + C''' = \frac{4\epsilon_0 S}{3d} + \frac{\epsilon_0 S}{d} = \frac{7\epsilon_0 S}{3d}$$

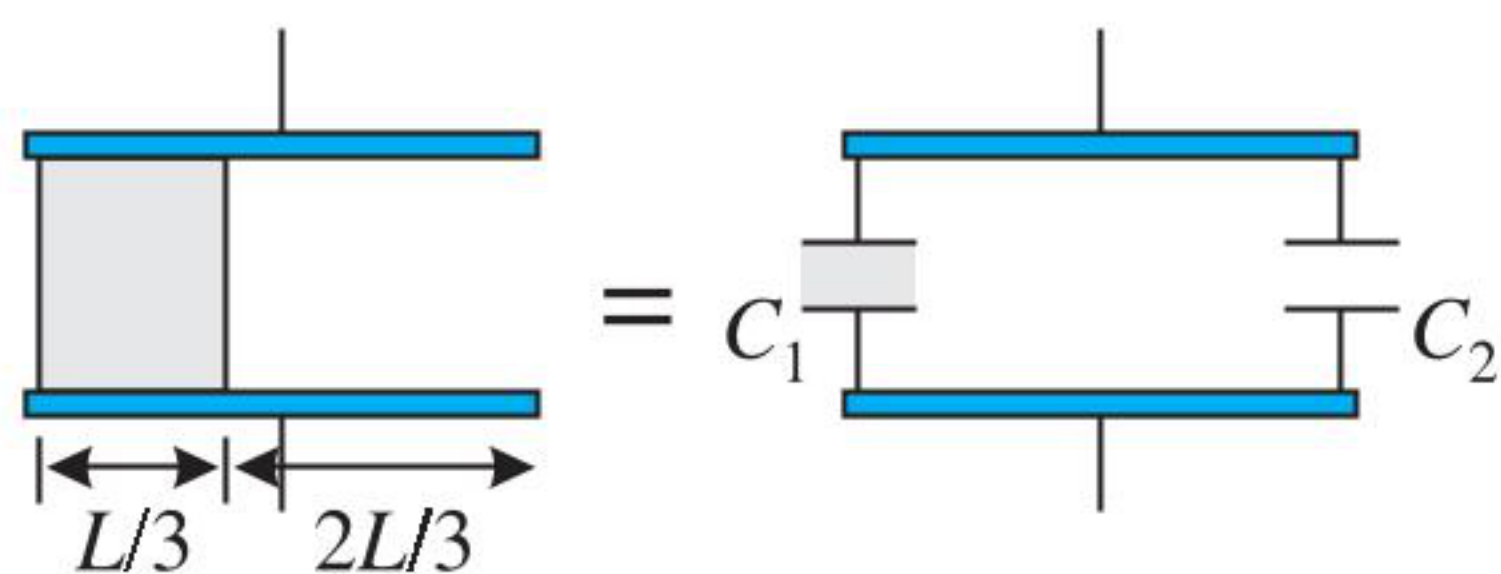
$$\text{This gives: } \frac{C_2}{C_1} = \frac{7}{3}$$

Multiple Correct Answers Type

1. (2),(4)

After switch S_1 is closed, C_1 is charged by $2CV_0$; when switch S_2 is closed, C_1 and C_2 both have upper plate charge CV_0 . When S_3 is closed, then upper plate of C_2 becomes charged by $-CV_0$ and lower plate by $+CV_0$.

2. (1),(4)



$$\text{If } C = \frac{\epsilon_0 A}{3d} \text{ then } C_1 = kc \quad C_2 = 2c$$

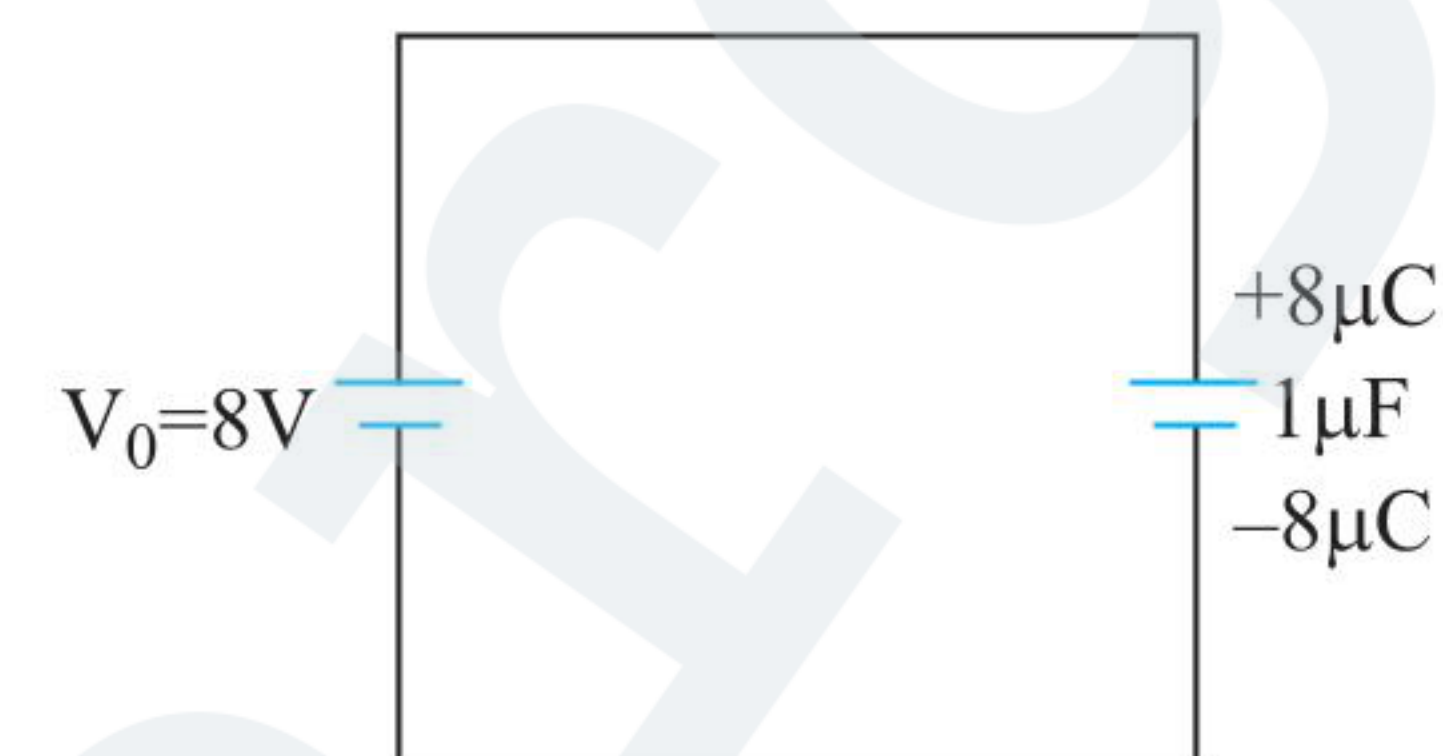
$$C_1 + C_2 = C \Rightarrow (k+2)c = C \Rightarrow c = \frac{C}{k+2}$$

$$\therefore C_1 = \frac{kc}{k+2}, C_2 = \frac{2c}{k+2} \Rightarrow \frac{C}{C_1} = \frac{2+k}{k}$$

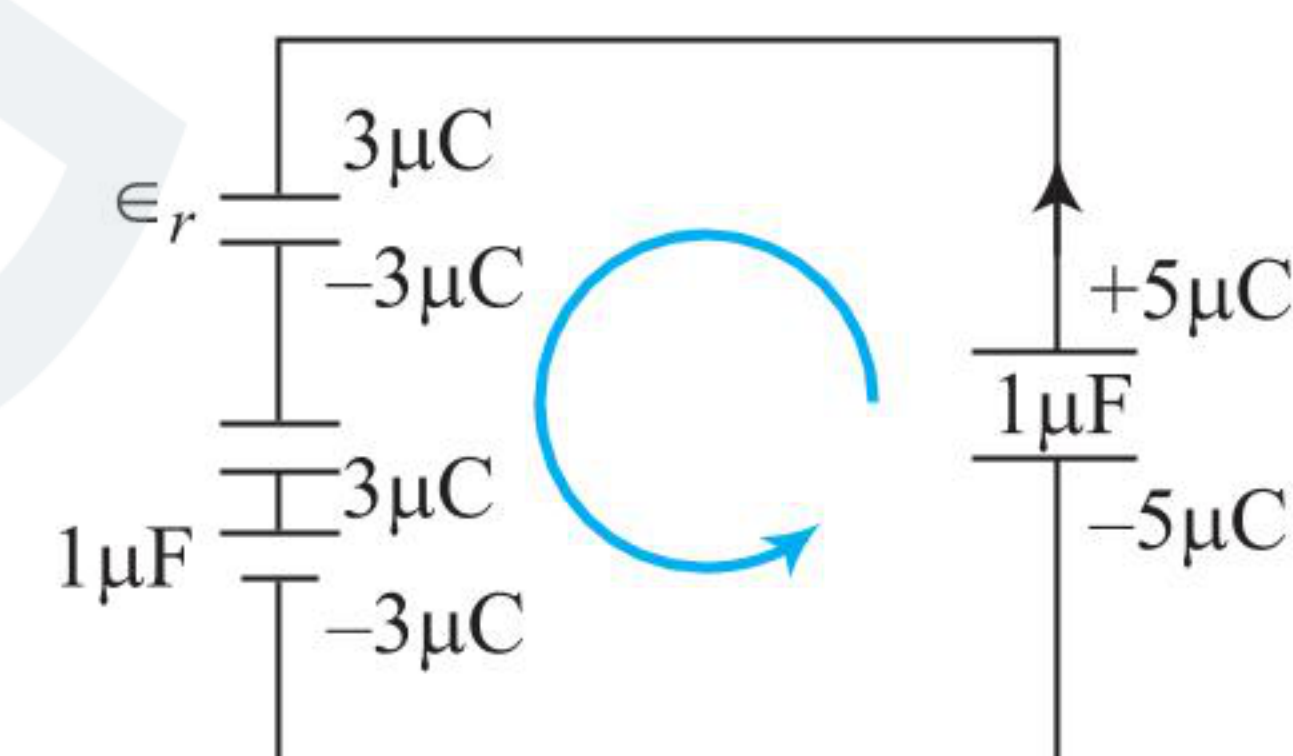
Numerical Value Type

1. (1.50) When S_1 is closed and S_2 is open charge in capacitor C_3

$$q_3 = C_3 V_0 = 1 \times 8 = 8 \mu\text{C}$$



Now S_1 is open and S_2 is closed. Final charge in C_3 is found to be $5 \mu\text{C}$, hence the change in capacitors C_1 and C_2 should be $3 \mu\text{C}$.



Applying loop rule

$$\frac{5}{1} - \frac{3}{\epsilon_r \cdot 1} - \frac{3}{1} = 0 \Rightarrow \frac{3}{\epsilon_r} = 2$$

$$\text{or } \epsilon_r = 1.50$$

2. (1.00)

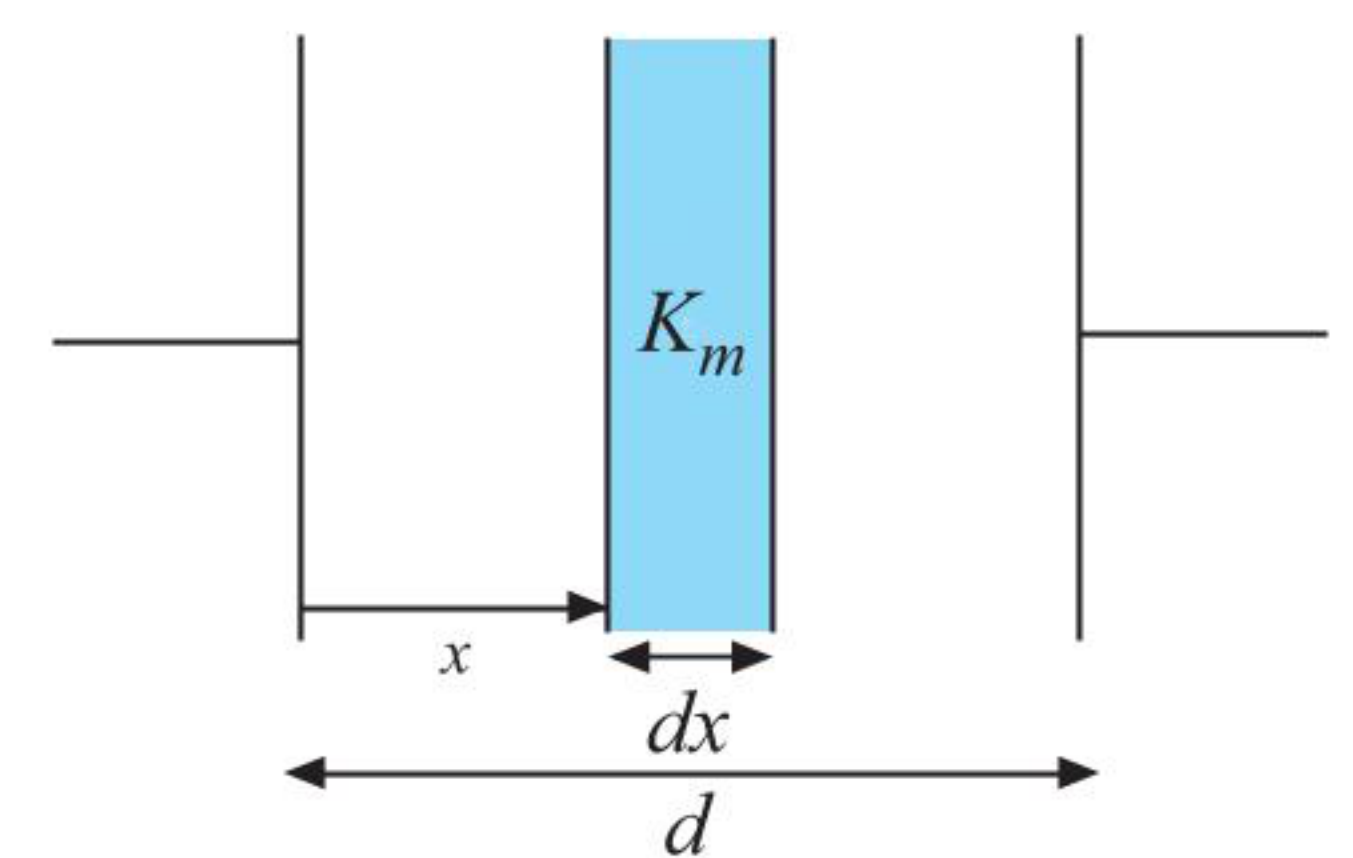
Let us consider the thickness of m^{th} layer be dx and it is located at a distance x from the left plate.

$$\text{Hence thickness of a layer } \delta = \frac{d}{N} = \frac{x}{m}$$

$$\text{or } \frac{m}{N} = \frac{x}{d}$$

$$K_m = K \left(1 + \frac{m}{N} \right)$$

$$\Rightarrow K_m = K \left(1 + \frac{x}{d} \right)$$



The capacitance of the m^{th} layer

$$dC = \frac{K_m A \epsilon_0}{dx}$$

$$\frac{1}{C_{\text{eq}}} = \int_0^d \frac{dx}{K_m A \epsilon_0} = \frac{1}{KA \epsilon_0} \int_0^d \frac{dx}{\left(1 + \frac{x}{d} \right)}$$

$$\Rightarrow \frac{1}{C_{\text{eq}}} = \frac{d}{KA \epsilon_0} \left[\ln \left(1 + \frac{x}{d} \right) \right]_0^d$$

$$\Rightarrow \frac{1}{C_{\text{eq}}} = \frac{d}{KA \epsilon_0} [\ln 2 - \ln(1)]$$

$$\Rightarrow C_{\text{eq}} = \frac{KA \epsilon_0}{d \ln 2} \Rightarrow \alpha = 1$$